ON INTUITIONISTIC FUZZY REGULAR GENERALIZED SEMIPRE CONTINUOUS MAPPINGS

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Abstract:

In this paper we define the notion of intuitionistic fuzzy regular generalized semipre continuous mappings. After providing the preliminary definitions we proceed towards the notion and example of intuitionistic fuzzy regular generalized semipre continuous mappings. Further we discuss the liaison of the intuitionistic fuzzy regular generalized semipre continuous mapping and few of the already existing intuitionistic fuzzy continuous mappings. Furthermore, we proceed to enjoy with a few fascinating theorems concerning intuitionistic fuzzy semipre continuous mappings in intuitionistic fuzzy regular generalized semipre $T_{1/2}$ space which will be very useful in this research work.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy regular generalized semipre $T_{1/2}$ space, intuitionistic fuzzy regular generalized semipre continuous mappings.

1. Introduction

In 1965 Zadeh [15] established fuzzy sets which has now invaded almost all branches of Mathematics. Later the introduction of fuzzy topology was given by Chang [2] in 1967. This was followed by the notion of intuitionistic fuzzy sets by Atanassov [1] which was a breakthrough towards the evolution of intuitionistic fuzzy topology. Using this notion, Coker [3] constructed the basic concepts of intuitionistic fuzzy topological spaces. This was eventually followed by the introduction of intuitionistic fuzzy generalized semipreclosed sets by Santhi and Jayanthi [8] in 2010 which was simultaneously followed by the introduction of intuitionistic fuzzy generalized semi-pre continuous mappings [9] by the same authors. We now extend our idea towards intuitionistic fuzzy regular generalized semipre continuous mappings and study some of their properties.

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2. Preliminaries

Definition 2.1 [1]: An *intuitionistic fuzzy set* (IFS in short) A is an object having the form

$$A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the function $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A=\langle x, \mu_A, \nu_A\rangle$ instead of denoting $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2 [1]: Let A and B be two IFSs of the form

$$A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

and

$$B=\{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geqslant \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$

The intuitionistic fuzzy sets $0\sim = \langle x, 0, 1\rangle$ and $1\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3 [3] : An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0\sim, 1\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called the *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The compliment A^c of an IFOS A in IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4 [3] : Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* are defined by

$$\mathrm{int}(A) = \bigcup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$$

$$\mathrm{cl}(A) = \bigcap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$$

Note that for any IFS A in (X, τ), we have $\mathrm{cl}(A^c) = (\mathrm{int}(A))^c$ and $\mathrm{int}(A^c) = (\mathrm{cl}(A))^c$.

Definition 2.5 [5] : An. IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$

(ii) *intuitionistic fuzzy pre closed set* (IFPCS in short) if $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$

(iii) *intuitionistic fuzzy α closed set* (IFαCS in short) if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))) \subseteq A$

(iv) *intuitionistic fuzzy β closed set* (IFβCS in short) if $\mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) \subseteq A$

The respective complements of the above IFCSs are called their respective IFOSs. The family of all IFSCSs, IFPCSs, IFαCSs and IFβCSs (respectively IFSOSs, IFPOSs, IFαOSs and IFβOSs) of an IFTS (X, τ) are respectively denoted by IFSC(X), IFPC(X), IFαC(X), IFβC(X) (respectively IFSO(X), IFPO(X), IFαO(X), IFβO(X)).

Definition 2.6 [14] : An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i) *intuitionistic fuzzy semi-pre closed set* (IFSPCS in short) if there exists an IFPCS B such that $\mathrm{int}(B) \subseteq A \subseteq B$,

(ii) *intuitionistic fuzzy semi-pre open set* (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq \mathrm{cl}(B)$.

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X,τ) is denoted by IFSPC(X) (respectively IFSPO(X)).

Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not hold in general.

Definition 2.7 [8] : Let A be an IFS in an IFTS (X, τ). Then the *semi-pre interior* and the *semi-pre closure* of A are defined as

$$\mathrm{spint}(A) = \bigcup \{ G \mid G \text{ is an IFSPOS in } X \text{ and } G \subseteq A \},$$

$$\mathrm{spcl}(A) = \bigcap \{ K \mid K \text{ is an IFSPCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS A in (X, τ), we have $spcl(A^c) = (spint(A))^c$ and $spint(A^c) = (spcl(A))^c$.

Definition 2.8 [10] : An IFS A is an

(i) *intuitionistic fuzzy regular closed set* (IFRCS in short) if $A = cl(int(A))$

(ii) *intuitionistic fuzzy regular open set* (IFROS in short) if $A = int(cl(A))$

(iii) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS

(iv) *intuitionistic fuzzy regular generalized closed set* (IFRGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFROS.

Definition 2.9 [8] : An IFS A in an IFTS (X,τ) is said to be an *intuitionistic fuzzy generalized semi-pre closed set* (IFGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 2.10 [10] : An IFTS (X, τ) is said to be an *intuitionistic fuzzy $T_{1/2}$ space* ($IFT_{1/2}$ in short) if every IFGCS in (X, τ) is an IFCS in (X, τ).

Definition 2.11 [8] : If every IFGSPCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an *intuitionistic fuzzy semi-pre $T_{1/2}$ space* ($IFSPT_{1/2}$ space in short).

Definition 2.12 [13] : If every IFRGSPCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an *intuitionistic fuzzy regular semipre $T_{1/2}$ space* ($IFRSPT_{1/2}$ in short).

Definition 2.13 [4] : An *intuitionistic fuzzy point* (IFP in short), written as $p_{(\alpha,\beta)}$, is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & if\ x = p, \\ (0, 1) & otherwise. \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leqslant \mu_A$ and $\beta \geqslant \nu_A$.

Definition 2.14 [10] : Two IFSs are said to be *q-coincident* ($A_q B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.15 [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy continuous* (IF continuous in short) mapping if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.

Definition 2.16 [7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(I) *intuitionistic fuzzy semi continuous* (IFS continuous in short) mapping if $f^{1}(B) \in IFSO(X)$ for every $B \in \sigma$

(ii) *intuitionistic fuzzy α-continuous* (IFα-continuous in short) mapping if $f^{1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$

(iii) *intuitionistic fuzzy pre continuous* (IFP continuous in short) mapping if $f^{-1}(B) \in IFPO(X)$ for every $B \in \sigma$.

Every IF continuous mapping is an IFα-continuous mapping and every IFα-continuous mapping is an IFS continuous mapping as well as an IFP continuous mapping, but the separate converses may not be true in general.

Definition 2.17 [10] : Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy generalized continuous* (IFG continuous in short) mapping if $f^{1}(B) \in IFGC(X)$ for every IFCS $B \in Y$.

Every IF continuous mapping is an IFG continuous mapping but the converse may not be true in general.

Definition 2.18 [14] : Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy semi-pre continuous* (IFSP continuous in short) mapping if $f^{1}(B) \in IFSPO(X)$ for every $B \in \sigma$.

Every IFS continuous mapping and IFP continuous mappings are IFSP continuous mapping but the converses may not be true in general[14].

Definition 2.19 [9] : A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy generalized semi-pre continuous* (IFGSP continuous for short) mapping if $f^{1}(V)$ is an IFGSPCS in (X, τ) for every IFCS V of (Y, σ).

Corollary 2.20 [3] : Let A, $A_i (1 \in J)$ be intuitionistic fuzzy sets in X and B, $B_j (j \in K)$ be intuitionistic fuzzy sets in Y and $f: X \to Y$ be a function.

Then

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

c) $A \subseteq f^{-1}(f(A))$ [If f is injective, then $A = f^{-1}(f(A))$]

d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective, then $B = f(f^{-1}(B))$]

e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

g) $f^{-1}(0\sim) = 0\sim$

h) $f^{-1}(1\sim) = 1\sim$

i) $f^{-1}(B^c) = (f^{-1}(B))^c$

3. INTUITIONISTIC FUZZY REGULAR GENERALIZED SEMIPRE CONTINUOUS MAPPINGS

Continuity is considered to be one of the core concepts of topology. In this section we give the notion of Intuitionistic fuzzy regular generalized semipre continuous mapping and discuss few interesting theorems.

Definition 3.1 A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy regular generalized semipre continuous (IFRGSP continuous in short) mapping if $f^{-1}(V)$ is an IFRGSPCS in (X, τ) for every IFCS V in (Y, σ).

Example 3.2 Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle x, (0.6, 0.7), (0.4, 0.2)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Then, IFPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, IFSPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Now $G_2^c = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFCS in Y. Therefore $f^{-1}(G_2^c) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$. We have $spcl(f^{-1}(G_2^c)) = f^{-1}(G_2^c)$. We have $f^{-1}(G_2^c) \subseteq G_1$. Hence $spcl(f^{-1}(G_2^c)) \subseteq G_1$, where G_1 is an IFROS in X. This implies $f^{-1}(G_2^c)$ is an IFRGSPCS in X. Therefore f is an IFRGSP continuous mapping.

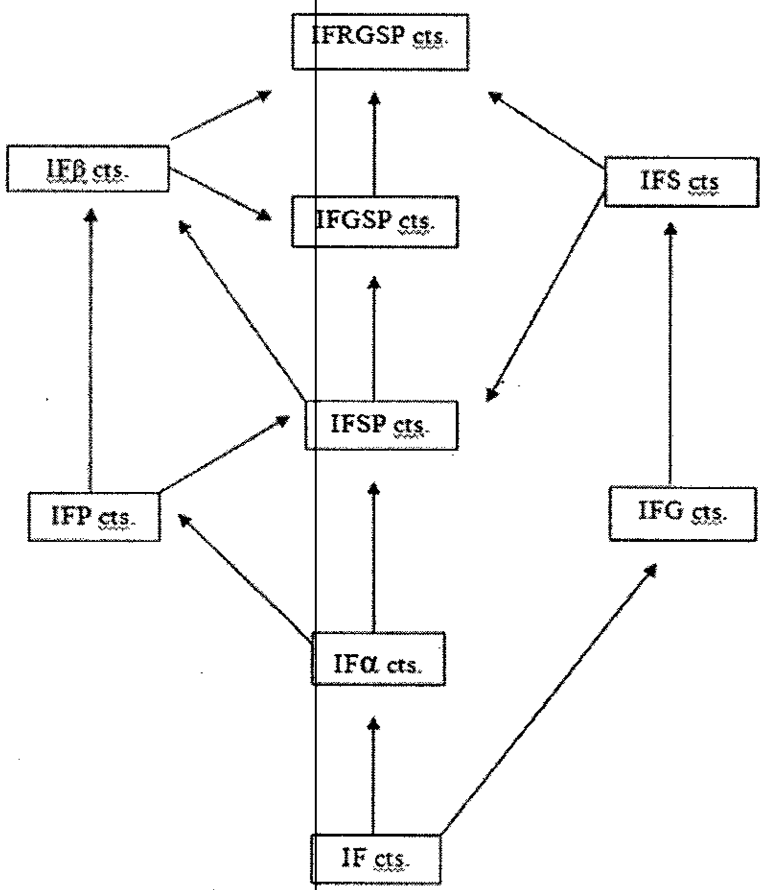
Remark 3.3 Every IF continuous mapping, IFG continuous mapping, IFS continuous mapping, IFP continuous mapping, IFSP continuous mapping, IFβ continuous mapping, IFα continuous mapping, IFGSP continuous mapping are IFRGSP continuous mapping but their converses need not be true in general.

Example 3.4 Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle x, (0.6, 0.7), (0.4, 0.2)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Then, IFPC(X) $= \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] /$ either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leqslant 1$ and $\mu_b + \nu_b \leqslant 1\}$. Therefore, IFSPC(X) $= \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \mu_a + \nu_a \leqslant 1$ and $\mu_b + \nu_b \leqslant 1\}$. Now $G_2^c = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFCS in Y. Therefore $f^{-1}(G_2^c) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$. We have $spcl(f^{-1}(G_2^c)) = f^{-1}(G_2^c)$. We have $f^{-1}(G_2^c) \subseteq G_1$. Hence $spcl(f^{1}(G_2^c)) \subseteq G_1$, where G_1 is an IFROS in X. This implies $f^{1}(G_2^c)$ is an IFRGSPCS in X. Therefore f is an IFRGSP continuous mapping. We have G_2 is IFOS in Y, but $f^{1}(G_2) = \langle x, (0.6, 0.7), (0.4, 0.2)\rangle$ is not an IFOS in X, since $int(f^{-1}(G_2)) = G_1 \neq f^{-1}(G_2)$. Now $f^{-1}(G_2^c) \subseteq G_1$ but $cl(f^{-1}(G_2^c)) = G_1^c \nsubseteq G_1$. Therefore $f^{-1}(G_2^c)$ is not an IFGCS. Hence f is an IFRGSP continuous mapping but neither IF continuous mapping nor IFG continuous mapping.

Example 3.5 Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ and $G_2 = \langle x, (0.5, 0.3), (0.5, 0.7)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Then, IFPC(X) $= \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \mu_b < 0.6$ whenever $\mu_a \geqslant 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geqslant 0.6$, $\mu_a + \nu_a \leqslant 1$ and $\mu_b + \nu_b \leqslant 1\}$. Therefore, IFSPC(X) $= \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geqslant 0.6$, $\mu_a + \nu_a \leqslant 1$ and $\mu_b + \nu_b \leqslant 1\}$. Now $G_2^c = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFCS in Y. Therefore $f^{-1}(G_2^c) = \langle x, (0.5, 0.7), (0.5, 0.3)\rangle$. We have $spcl(f^{-1}(G_2^c)) = f^{-1}(G_2^c)$ We have $f^{-1}(G_2^c) \subseteq G_1$. Hence $spcl(f^{-1}(G_2^c)) \subseteq G_1$, where G_1 is an IFROS in X. This implies $f^{-1}(G_2^c)$ is an IFRGSPCS in X. Therefore f is an IFRGSP continuous mapping. We have $int(cl(f^{-1}(G_2^c))) = int(1\sim) = 1\sim \nsubseteq f^{-1}(G_2^c)$ which implies $f^{-1}(G_2^c)$ is not an IFSCS in X. Therefore f is not an IFS continuous mapping. Also we have $cl(int(f^{-1}(G_2^c))) = cl(G_1) = 1\sim \nsubseteq f^{-1}(G_2^c)$ which implies $f^{-1}(G_2^c)$ is not an IFPCS in X. Hence f is not an IFP continuous mapping. Furthermore $f^{-1}(G_2^c)$ is not an IFSPCS in X, since there exists no IFPCS B in X such that $int(B) \subseteq f^{-1}(G_2^c) \subseteq B$. Therefore f is not an IFSP continuous mapping. Also we have $int(cl(int(f^{-1}(G_2^c)))) = int(cl(G_1)) = int(1\sim) = 1\sim \nsubseteq f^{1}(G_2^c)$. Hence $f^{-1}(G_2^c)$ is not an IFβCS in X. Thus f is not an IFβ continuous mapping. Further we have $cl(int(cl(f^{-1}(G_2^c)))) = cl(int(1\sim)) = cl(1\sim) =$

$1_\sim \not\subseteq f^{-1}(G_2^c)$. Therefore $f^{-1}(G_2^c)$ is not an IFαCS in X. Hence f is not an IFα-continuous mapping.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts.' means continuous.



Theorem 3.6 A mapping $f:(X,\tau)\to(Y,\sigma)$ is an IFRGSP continuous mapping if and only if the inverse image of each IFOS in Y is an IFRGSPOS in X.

Proof: The proof is obvious since $f^{-1}(A^c) = (f^{-1}(A))^c$.

Theorem 3.7 If $f:(X,\tau)\to(Y,\sigma)$ is an IFRGSP continuous mapping then for each IFP $p(\alpha,\beta)$ of X and each $A \in \sigma$ such that $f(p(\alpha,\beta)) \in A$, there exists an IFRGSPOS B of X such that $p(\alpha,\beta) \in B$ and $f(B) \subseteq A$.

Proof: Let $p(\alpha,\beta)$ be an IFP of X and $A \in \sigma$ such that $f(p(\alpha,\beta)) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis B is an IFRGSPOS in X such that $p(\alpha,\beta) \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 3.8 If $f:(X,\tau)\to(Y,\sigma)$ is an IFRGSP continuous mapping then for each IFP $p(\alpha,\beta)$ of X and each $A\in\sigma$ such that $f(p(\alpha,\beta))_q A$, there exists an IFRGSPOS of X such that $p(\alpha,\beta)_q B$ and $f(B)\subseteq A$.

Proof: Let $p(\alpha,\beta)$ be an IFP of X and $A\in\sigma$ such that $f(p(\alpha,\beta))_q A$. Put $B=f^{1}(A)$. Then by hypothesis B is an IFRGSPOS in X such that $p(\alpha,\beta)_q B$ and $f(B)=f(f^{-1}(A))\subseteq A$.

Theorem 3.9 Let : $(X,\tau)\to(Y,\sigma)$ be an IFRGSP continuous mapping, Then f is an IFSP continuous mapping if X is an IFRSP$T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Since every IFCS is an IFGSPCS[8], V is an IFGSPCS in Y. $f^{1}(V)$ is an IFRGSPCS in X, as f is an IFRGSP continuous mapping. Again since X is an IFRSP$T_{1/2}$ space, $f^{1}(V)$ is an IFSPCS in X. Hence f is an IFSP continuous mapping.

Theorem 3.10 Let $f:(X,\tau)\to(Y,\sigma)$ be an IFRGSP continuous mapping and $g:(Y,\sigma)\to(Z,\gamma)$ be an IFG continuous mapping if Y is an IF$T_{1/2}$ space, then $g\circ f\ :\ (X,\tau)\to(Z,\gamma)$ is an IFRGSP continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ be an IFGCS in Y, as g is a IFG continuous mapping. Since Y is an IF$T_{1/2}$ space, $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFRGSPCS in X, as f is an IFRGSP continuous mapping. Hence $g\circ f$ is an IFRGSP continuous mapping.

Theorem 3.11 Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IFRSP$T_{1/2}$ space:

(i) f is an IFRGSP continuous mapping

(ii) If B is an IFOS in Y then $f^{1}(B)$ is an IFRGSPOS in X

(iii) $f^{-1}(int(B))\subseteq cl(int(cl(f^{-1}(B))))$ for every IFS B in Y

Proof: (i) $\Leftrightarrow$ (ii) is obviously true by Theorem 3.6.

(ii) $\Rightarrow$ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFRGSPOS in X, by hypothesis. Since X is an IFRSP$T_{1/2}$ space $f^{-1}(int(B))$ is an IFSPOS in X. Therefore $f^{-1}(int(B))\subseteq cl(int(cl(f^{-1}(int(B)))))\subseteq cl(int(cl(f^{-1}(B))))$.

(iii) $\Rightarrow$ (i) Let B be an IFOS in Y. Then int(B) = B. By hypothesis $f^{1}(B)\subseteq cl(int(cl(f^{1}(B))))$. This implies $f^{1}(B)$ is an IFβOS in X. Therefore it is an IFRGSPOS[13] in X and hence f is an IFRGSP continuous mapping, by Theorem 3.6.

Theorem 3.12 Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IFRSP$T_{1/2}$ spaces:

(i) f is an IFRGSP continuous mapping

(ii) $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(spcl(B))$ for each IFCS B in Y

(iii) $f^{-1}(spint(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for each IFOS B of Y

(iv) $f(int(cl(int(A)))) \subseteq cl(f(A))$ for each IFS A of X

Proof: (i) $\Rightarrow$ (ii) Let B be an IFCS in Y. This implies B is an IFSPCS, since every IFCS is an IFSPCS[14]. Therefore spcl(B) = B. Further $f^{-1}(B)$ is an IFRGSPCS in X, by hypothesis. Since X is an IFRSP$T_{1/2}$ space, $f^{-1}(B)$ is an IFSPCS in X. Furthermore since every IFSPCS is an IFβCS[6], we get $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(B) = f^{-1}(spcl(B))$, as spcl(B) = B.

(ii) $\Rightarrow$(iii) can be easily proved by taking complement in (ii).

(iii) $\Rightarrow$(iv) Let A be an IFS in X. Then B = f A $\subseteq f^{-1}(B)$. Here int(f(A)) = int(B) is an IFOS in Y. Then (iii) implies that $f^{-1}(spint(int(B))) \subseteq cl(int(cl(f^{-1}(int(B))))) \subseteq cl(int(cl(f^{-1}(B))))$. Now $(cl(int(cl(A^c))))^c \subseteq (cl(int(cl(f^{-1}(B)^c))))^c \subseteq (f^{-1}(spint(int(B^c))))^c$. Therefore $int(cl(int(A))) \subseteq f^{-1}(spcl(cl(B)))$. Now $f(int(cl(int(A)))) \subseteq f(f^{-1}(spcl(cl(B)))) \subseteq cl(B) = cl(f(A))$.

(iv) $\Rightarrow$ (i) Let B be any IFCS in Y, then $f^{-1}(B)$ is an IFS in X. By hypothesis $f(int(cl(int(f^{1}(B)))))$ $\subseteq cl(f(f^{-1}(B))) \subseteq cl(B) = B$. Now $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(f(int(cl(int(f^{-1}(B)))))) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFβCS and hence it is an IFRGSPCS in X[12]. Thus f is an IFRGSP continuous mapping.

Theorem 3.13 A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IFRGSP continuous mapping if $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof: Let A be an IFOS in Y then A^c is an IFCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(cl(A^c)) = f^{-1}(A^c)$, since A^c is an IFCS. Now $(int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(A^c) = (f^{-1}(A))^c$. This implies $f^{-1}(A) \subseteq int(cl(int(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFαOS in X and hence it is an IFRGSPOS in X[13]. Therefore f is an IFRGSP continuous mapping, by Theorem 3.6.

Theorem 3.14 Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y where X is an IFRSP$T_{1/2}$ space. Then f is an IFRGSP continuous mapping if and only if $int(cl(int(f^{1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof: Necessity: Let f be an IFRGSP continuous mapping then $f^{-1}(B)$ is an IFRGSPCS in X for every IFCS B in Y. Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By Definition 3.1, $f^{-1}(cl(A))$ is an IFRGSPCS in X. Since X is an IFRSP$T_{1/2}$ space, $f^{-1}(cl(A))$ is an IFSPCS. Since every IFSPCS is an IFβCS[6],we get $f^{-1}(cl(A))$ is an IFβCS in X. Therefore $int(cl(int(f^{-1}(cl(A))))) \subseteq f^{-1}(cl(A))$. Now $int(cl(int(f^{-1}(A)))) \subseteq int(cl(int(f^{-1}(cl(A))))) \subseteq f^{-1}(cl(A))$.

Sufficiency: Let A be an IFCS in Y. By hypothesis, $int(cl(int(f^{-1}(A)))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. This implies $f^{1}(A)$ is an IFβCS in X and hence it is an IFRGSPCS[12]. Thus f is an IFRGSP continuous mapping.

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