STOCHASTIC MODEL FOR A SINGLE GRADE SYSTEM WITH TWO COMPONENTS FOR THRESHOLD AND CORRELATED INTER-DECISION TIMES

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ABSTRACT

In this paper, an organization with single grade system subjected to exodus of personnel due to policy decisions taken by it is considered. In order to avoid the crisis of the organization reaching a breakdown point, a suitable univariate policy recruitment based on shock model approach and cumulative damage process, is suggested. The recruitment is made, once additive loss of manpower on sequent occasions crosses the threshold level. The explicit expression for the expected time to recruitment time under the assumption that (i) the inter-arrival times between successive epochs of policy decisions are correlated, whereas the inter-arrival times between successive epochs of transfers are i.i.d random variables (ii) the threshold for the loss of manpower has two components namely the maximum allowable attrition and the maximum available manpower due to extra time work. The analytical results are substantiated with numerical illustrations. The Stochastic model discussed in the paper is not only applicable to industry as a whole but also in a wider context of other applicable areas.

Key Words: *Single grade system, Univariate policy of recruitment, shock model.*

INTRODUCTION

Frequent wastage or exit of personnel is common in many administrative and production oriented organizations. Whenever the organizations announce revised policies regarding sales target, revision of wages, incentives and perquisites, the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover in the organization. As

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the recruitment involves several costs, it is usual that the organization has the natural reluctance to go in for frequent recruitments. Once the total amount of wastage crosses a certain threshold level, the organization reaches an uneconomic status which otherwise be called the breakdown point and the recruitment is done at this point of time. The time to attain the breakdown point is an important characteristic for the management of the organization. Many models could be seen in Barthlomew [1], Barthlomew and Forbes [2], Researchers [7] and [8] have considered the problem of time to recruitment in a marketing organization under different conditions. In an organization, the depletion of manpower can occur due to two different cases:

i) whenever the policy decisions regarding pay, perquisites and work schedule are revised and

ii) due to transfer of personnel to the other organization of the same management.

In the presence of these two different sources of depletion, Elangovan et.al [3] have studied the problem of time to recruitment for an organization consisting of one grade and obtained the variance of the time to recruitment using univariate policy of recruitment when (i) the loss of man power in the organization due to the two sources of depletion and its threshold are independent and identically exponential random variables, (ii) inter-policy decision times and inter-transfer decision times form the same renewal process. The Authors [11] have discussed a stochastic model for estimating the expected time to recruitment under the assumptions that (i) the depletion of manpower is in terms of persons leaving at every epoch of decision making and at every epoch of transfer and hence they are in terms of discrete random variables, (ii) the recruitment is carried out as and when the total depletion crosses a level called the threshold which is a discrete random variable. Usha et.al [12] proposed a model to determine the expected time to recruitment under the assumption that the inter-arrival times between successive epochs of policy decisions are not independent but correlated, whereas the inter-arrival times between successive epochs of transfers are i.i.d random variables. Vijaysankar et.al [13] have constructed a stochastic model by assuming the threshold with two components namely the level of wastage which can be allowed and the manpower which is available from what is known as backup resource. Uma and Roja Mary [5] have considered a single grade system which is subjected to exodus of personnel due to policy decisions given by the organization and the explicit expression for the long run average cost is derived by considering survival time process, which is a geometric process and the threshold has two components. With the same conditions, but considering threshold with four components the explicit expression for the long run average cost using bivariate policy is derived by Uma and Roja Mary [6].Uma [9] has derived the mean time to recruitment by considering a Manpower system with two groups. In [10],Uma and Sudharani have determined the time to recruitment for a single grade manpower system with two sources of depletion and the renewal processes governing the inter-policy decisions times and that of the inter-transfer decisions times are the same. Also it is assumed that the threshold for the loss of manpower has two components namely the maximum allowable attrition and the maximum available manpower due to extra time work. The assumption in [10] that the inter-decision times and inter-transfer times forming the same renewal process is a severe restriction. This restriction is removed in this paper by considering different renewal processes for inter-decision times and inter-transfer times. This helps the organization to take decisions more comfort. The performance measures namely the mean time to recruitment has been derived.

MODEL DESCRIPTION

Consider an organization having single grade in which decisions are taken at random epoch. The depletion of man-hours occurs at every decision making epoch and also due to transfer of personnel to the other organization of the same management. It is proposed to determine the expected time to recruitment under the assumption that the inter-arrival times between successive epochs of policy decisions are not independent but correlated, whereas the inter-arrival times between successive epochs of transfers are independent and identically distributed random variables. The threshold for the loss of manpower has two components namely the maximum allowable attrition and the maximum available manpower due to extra time work. If the total loss of man-hours crosses the sum of the threshold and the available man-hours due to extra time work the break down occurs. The process that generates the loss of man-hours and the threshold put together is linearly independent. Recruitment takes place only at decision points and the time of recruitment is negligible. The recruitment is made whenever the cumulative loss of man-hours exceeds the threshold of the organization. The analytical results are numerically illustrated and the influences of nodal parameters on the performance measure are studied and relevant conclusions are presented.

ASSUMPTIONS

(i) The interarrival times between decision epochs are random variables having exponential distribution with parameter a. These random variables are constantly correlated and exchangeable but not independent.

(ii) The depletion of man power also occurs due to transfer of personnel and the inter-arrival times between the successive epochs of transfer are i.i.d random variables having exponential distribution with parameter λ.

(iii) As and when the total depletion due to the above two reasons crosses the sum of the threshold and backup resource available, the breakdown occurs.

(iv) The two types of depletion are linear and additive.

NOTATIONS

X_i - a continuous random variable representing the amount of depletion of man-hours due to the i^{th} epoch of policy decisions. X_i's are i.i.d random variables having exponential distribution with parameter α.

Y_j - a continuous random variable representing the amount of depletion of man-hours due to the j^{th} event of transfer of personnel. Y_j's are i.i.d random variables having exponential distribution with parameter μ.

$$h(.) = \text{p.d.f of } X_i$$

$$k(.) = \text{p.d.f of } Y_j$$

u_i - a random variable denoting the inter-arrival times between decision epochs. u_i's are constantly correlated and exchangeable random variables having exponential distribution with parameter a.

v_j - a random variable representing the inter-arrival times between the successive epochs of transfer. v_j's are i.i.d random variables having exponential distribution with parameter λ.

$f(.)=$ p.d.f of u_i

$g(.)=$ p.d.f of v_j

Z - a continuous random variable representing the threshold level and is assumed to be exponential distribution with parameters θ_1 and θ_2.

$F_m(.)$ = c.d.f of $\sum_{i=0}^{m} u_i$, the sum of interarrival times between decision making epochs.

$G_n(.)$ = c.d.f of $\sum_{j=1}^{n} v_j$, the sum of interarrival times between the epochs of transfers.

T - a random variable representing the time to recruitment in the organizations.

$\overline{X} = X_1 + X_2 + ... + X_m$ and $\overline{Y} = Y_1 + Y_2 + ... + Y_n$

$L(t) = P(T < t)$ = c.d.f of the time to recruitment of the system.

$S(t)=$ Survivor function $= P(T > t)$.

$L^*(s)=$ Laplace Transform of $L(t)$.

$h^*(.)=$ Laplace Transform of $h(.)$.

$k^*(.)=$ Laplace Transform of $k(.)$.

MAIN RESULTS

The probability that the total depletion of man power on 'm' occasions of decision and 'n' occasions of transfer of personnel does not exceed the threshold level is given as

$$P[\bar{X} + \bar{Y} < Z] = \int_0^\infty Q_{\bar{x}+\bar{y}}(z)\frac{\theta_1\theta_2}{\theta_1 - \theta_2}[e^{-\theta_2 z} - e^{\theta_1 z}]\,dz$$

$$= \frac{\theta_1\theta_2}{\theta_1-\theta_2}\left\{\int_0^\infty Q_{\bar{x}+\bar{y}}(z)\, e^{-\theta_2 z}dz - \int_0^\infty Q_{\bar{x}+\bar{y}}(z)\, e^{-\theta_1 z}dz\right\}$$

$$= \frac{\theta_1\theta_2}{\theta_1-\theta_2}[Q^*_{\bar{x}+\bar{y}}(\theta_2) - Q^*_{\bar{x}+\bar{y}}(\theta_1)]$$

$$= \frac{\theta_1\theta_2}{\theta_1-\theta_2}Q^*_{\bar{x}+\bar{y}}(\theta_2) - \frac{\theta_1\theta_2}{\theta_1-\theta_2}Q^*_{\bar{x}+\bar{y}}(\theta_1)]$$

We know that, $F_m^*(s) = \frac{1}{s} f_m^*(s)$

Using this result, we get

$$P[\bar{X} + \bar{Y} < Z] = \frac{\theta_1}{\theta_1 - \theta_2} q^*_{\bar{x}+\bar{y}}(\theta_2) - \frac{\theta_2}{\theta_1 - \theta_2} q^*_{\bar{x}+\bar{y}}(\theta_1)$$

$$= \frac{\theta_1}{\theta_1 - \theta_2} q^*_{\bar{x}}(\theta_2) q^*_{\bar{y}}(\theta_2) - \frac{\theta_2}{\theta_1 - \theta_2} q^*_{\bar{x}}(\theta_1) q^*_{\bar{y}}(\theta_1),$$ Since X & Y are independent and X_i and Y_j are i.i.d random variables

$$= \frac{\theta_1}{\theta_1 - \theta_2} [q^*(\theta_2)]^m [q^*(\theta_2)]^n - \frac{\theta_2}{\theta_1 - \theta_2} [q^*(\theta_1)]^m [q^*(\theta_1)]^n$$

$$= \frac{\theta_1}{\theta_1 - \theta_2} [h^*(\theta_2)]^m [k^*(\theta_2)]^n - \frac{\theta_2}{\theta_1 - \theta_2} [h^*(\theta_1)]^m [k^*(\theta_1)]^n \quad \text{______ (1)}$$

$$S(t) = P(T > t)$$

= Probability that there are exactly 'm' occasions of policy making and 'n' occasions of transfer and the total depletion does not cross the threshold Z in (0,t]

$$\frac{\theta_1}{\theta_1 - \theta_2} \left\{ \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)][h^*(\theta_2)]^m \sum_{n=0}^{\infty} [G_n(t) - G_{n+1}(t)][k^*(\theta_2)]^n \right\} -$$

$$\frac{\theta_2}{\theta_1 - \theta_2} \left\{ \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)][h^*(\theta_1)]^m \sum_{n=0}^{\infty} [G_n(t) - G_{n+1}(t)][k^*(\theta_1)]^n \right\} \quad \text{_____ (2)}$$

$$S(t) = \frac{\theta_1}{\theta_1 - \theta_2} \left\{ \left[1 - (1 - h^*(\theta_2)) \sum_{m=1}^{\infty} F_m(t) [h^*(\theta_2)]^{m-1} \right] \left[1 - (1 - k^*(\theta_2)) \sum_{n=1}^{\infty} G_n(t) [k^*(\theta_2)]^{n-1} \right] \right\}$$

$$- \frac{\theta_2}{\theta_1 - \theta_2} \left\{ \left[1 - (1 - h^*(\theta_1)) \sum_{m=1}^{\infty} F_m(t) [h^*(\theta_1)]^{m-1} \right] \left[1 - (1 - k^*(\theta_1)) \sum_{n=1}^{\infty} G_n(t) [k^*(\theta_1)]^{n-1} \right] \right\}$$

______________ (3)

Now,

$$L(t) = P(T < t) = 1 - S(t)$$

$$= 1 -$$

$$\frac{\theta_1}{\theta_1-\theta_2}\left\{\left[1-(1-h^*(\theta_2))\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_2)]^{m-1}\right]\left[1-(1-k^*(\theta_2))\sum_{n=1}^{\infty} G_n(t)\,[k^*(\theta_2)]^{n-1}\right]\right\}+$$

$$\frac{\theta_2}{\theta_1-\theta_2}\left\{\left[1-(1-h^*(\theta_1))\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_1)]^{m-1}\right]\left[1-(1-k^*(\theta_1))\sum_{n=1}^{\infty} G_n(t)\,[k^*(\theta_1)]^{n-1}\right]\right\}$$

$$= \frac{\theta_1}{\theta_1-\theta_2}\left\{(1-h^*(\theta_2))\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_2)]^{m-1}+(1-k^*(\theta_2))\sum_{n=1}^{\infty} G_n(t)\,[k^*(\theta_2)]^{n-1}\right.$$

$$\left.-(1-h^*(\theta_2))(1-k^*(\theta_2))\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_2)]^{m-1}\sum_{n=1}^{\infty} G_n(t)\,[k^*(\theta_2)]^{n-1}\right\} +$$

$$\frac{\theta_2}{\theta_1-\theta_2}\left\{(1-h^*(\theta_1))\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_1)]^{m-1}+(1-k^*(\theta_1))\sum_{n=1}^{\infty} G_n(t)\,[k^*(\theta_1)]^{n-1}\right.$$

$$\left.-(1-h^*(\theta_1))(1-k^*(\theta_1))\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_1)]^{m-1}\sum_{n=1}^{\infty} G_n(t)\,[k^*(\theta_1)]^{n-1}\right\}$$

———— (4)

The pdf $l(t)$ is given by,

$$l(t) = \frac{\theta_1}{\theta_1-\theta_2}\left\{(1-h^*(\theta_2))\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_2)]^{m-1}+(1-k^*(\theta_2))\sum_{n=1}^{\infty} g_n(t)\,[k^*(\theta_2)]^{n-1}\right.$$

$$-(1-h^*(\theta_2))(1-k^*(\theta_2))\left[\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_2)]^{m-1}\sum_{n=1}^{\infty} G_n(t)\,[k^*(\theta_2)]^{n-1}\right.$$

$$\left.\left.+\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_2)]^{m-1}\sum_{n=1}^{\infty} g_n(t)\,[k^*(\theta_2)]^{n-1}\right]\right\} \quad +$$

$$\frac{\theta_2}{\theta_1-\theta_2}\Bigg\{\big(1-h^*(\theta_1)\big)\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_1)]^{m-1}+\big(1-k^*(\theta_1)\big)\sum_{n=1}^{\infty} g_n(t)\,[k^*(\theta_1)]^{n-1}-$$

$$\big(1-h^*(\theta_1)\big)\big(1-k^*(\theta_1)\big)\Bigg[\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_1)]^{m-1}\sum_{n=1}^{\infty} G_n(t)\,[k^*(\theta_1)]^{n-1}+$$

$$\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_1)]^{m-1}\sum_{n=1}^{\infty} g_n(t)\,[k^*(\theta_1)]^{n-1}\Bigg]\Bigg\} \qquad (5)$$

$$=\frac{\theta_1}{\theta_1-\theta_2}\Bigg\{\big(1-h^*(\theta_2)\big)\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_2)]^{m-1}+\big(1-k^*(\theta_2)\big)\sum_{n=1}^{\infty}\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}[k^*(\theta_2)]^{n-1}$$

$$-\big(1-h^*(\theta_2)\big)\big(1-k^*(\theta_2)\big)\Bigg[\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_2)]^{m-1}\sum_{n=1}^{\infty}\int_0^t\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}[k^*(\theta_2)]^{n-1}dt$$

$$+\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_2)]^{m-1}\sum_{n=1}^{\infty}\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}[k^*(\theta_2)]^{n-1}\Bigg]\Bigg\}\quad +$$

$$\frac{\theta_2}{\theta_1-\theta_2}\Bigg\{\big(1-h^*(\theta_1)\big)\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_1)]^{m-1}+\big(1-k^*(\theta_1)\big)\sum_{n=1}^{\infty}\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}[k^*(\theta_1)]^{n-1}$$

$$-\big(1-h^*(\theta_1)\big)\big(1-k^*(\theta_1)\big)\Bigg[\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_1)]^{m-1}\sum_{n=1}^{\infty}\int_0^t\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}[k^*(\theta_1)]^{n-1}dt$$

$$+\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_1)]^{m-1}\sum_{n=1}^{\infty}\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}[k^*(\theta_1)]^{n-1}\Bigg]\Bigg\}$$

$$= \frac{\theta_1}{\theta_1-\theta_2}\left\{(1-h^*(\theta_2))\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_2)]^{m-1}\,e^{-\lambda t(1-k^*(\theta_2))} + (1-k^*(\theta_2))\lambda e^{-\lambda t(1-k^*(\theta_2))} - (1-h^*(\theta_2))(1-k^*(\theta_2))\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_2)]^{m-1}\lambda e^{-\lambda t(1-k^*(\theta_2))}\right\} + \frac{\theta_2}{\theta_1-\theta_2}\left\{(1-h^*(\theta_1))\sum_{m=1}^{\infty} f_m(t)\,[h^*(\theta_1)]^{m-1}\,e^{-\lambda t(1-k^*(\theta_1))} + (1-k^*(\theta_1))\lambda e^{-\lambda t(1-k^*(\theta_2))} - (1-h^*(\theta_1))(1-k^*(\theta_1))\sum_{m=1}^{\infty} F_m(t)\,[h^*(\theta_1)]^{m-1}\lambda e^{-\lambda t(1-k^*(\theta_1))}\right\} \quad (6)$$

Taking Laplace Transform,

$$l^*(s) = \frac{\theta_1}{\theta_1-\theta_2}\left\{(1-h^*(\theta_2))\sum_{m=1}^{\infty} f_m^*(s+\lambda(1-k^*(\theta_2)))\,[h^*(\theta_2)]^{m-1}\left[1-\frac{\lambda(1-k^*(\theta_2))}{s+\lambda(1-k^*(\theta_2))}\right] + \frac{\lambda(1-k^*(\theta_2))}{s+\lambda(1-k^*(\theta_2))}\right\} + \frac{\theta_2}{\theta_1-\theta_2}\left\{(1-h^*(\theta_1))\sum_{m=1}^{\infty} f_m^*(s+\lambda(1-k^*(\theta_1)))\,[h^*(\theta_1)]^{m-1}\left[1-\frac{\lambda(1-k^*(\theta_1))}{s+\lambda(1-k^*(\theta_1))}\right] + \frac{\lambda(1-k^*(\theta_1))}{s+\lambda(1-k^*(\theta_1))}\right\} \quad (7)$$

$$\frac{dl^*(s)}{ds} =$$

$$\frac{\theta_1}{\theta_1-\theta_2}\left\{(1-h^*(\theta_2))\sum_{m=1}^{\infty} f_m^*(s+\lambda(1-k^*(\theta_2)))\,[h^*(\theta_2)]^{m-1}\left[\frac{\lambda(1-k^*(\theta_2))}{[s+\lambda(1-k^*(\theta_2))]^2}\right]-\frac{\lambda(1-k^*(\theta_2))}{[s+\lambda(1-k^*(\theta_2))]^2}\right\}$$

$$+$$

$$\frac{\theta_2}{\theta_1-\theta_2}\left\{(1-h^*(\theta_1))\sum_{m=1}^{\infty} f_m^*(s+\lambda(1-k^*(\theta_1)))\,[h^*(\theta_1)]^{m-1}\left[\frac{\lambda(1-k^*(\theta_1))}{[s+\lambda(1-k^*(\theta_1))]^2}\right]-\frac{\lambda(1-k^*(\theta_1))}{[s+\lambda(1-k^*(\theta_1))]^2}\right\}$$

$$-\left.\frac{dl^*(s)}{ds}\right]_{s=0} =$$

$$-\frac{\theta_1}{\theta_1-\theta_2}\left\{(1-h^*(\theta_2))\sum_{m=1}^{\infty} f_m^*(\lambda(1-k^*(\theta_2)))\,[h^*(\theta_2)]^{m-1}\left[\frac{1}{\lambda(1-k^*(\theta_2))}\right]-\frac{1}{\lambda(1-k^*(\theta_2))}\right\}$$

$$-\!-\frac{\theta_2}{\theta_1-\theta_2}\left\{(1-h^*(\theta_1))\sum_{m=1}^{\infty} f_m^*(\lambda(1-k^*(\theta_1)))\,[h^*(\theta_1)]^{m-1}\left[\frac{1}{\lambda(1-k^*(\theta_1))}\right]-\frac{1}{\lambda(1-k^*(\theta_1))}\right\}$$

$$\therefore\; E(T) = -\left.\frac{dl^*(s)}{ds}\right]_{s=0}$$

$$=\frac{\theta_1}{\theta_1-\theta_2}\left\{\frac{1}{\lambda(1-k^*(\theta_2))}-\frac{(1-h^*(\theta_2))}{\lambda(1-k^*(\theta_2))}\sum_{m=1}^{\infty} f_m^*(\lambda(1-k^*(\theta_2)))\,[h^*(\theta_2)]^{m-1}\right\} +$$

$$\frac{\theta_2}{\theta_1-\theta_2}\left\{\frac{1}{\lambda(1-k^*(\theta_1))}-\frac{(1-h^*(\theta_1))}{\lambda(1-k^*(\theta_1))}\sum_{m=1}^{\infty} f_m^*(\lambda(1-k^*(\theta_1)))\,[h^*(\theta_1)]^{m-1}\right\} \quad \text{\_\_\_\_ (8)}$$

Gurland [4] has obtained the expression for the c.d.f of the partial sum, $S_m = u_1 + u_2 + \ldots + u_m$ when the random variables u_i, $i = 1, 2, \ldots, m$ are exchangeable, exponentially distributed and are with constant correlation R.

Gurland has derived that the c.d.f of $S_m = u_1 + u_2 + \ldots + u_m$ is

$$F_m(x) = P(S_m \le x) = (1-R)\sum_{i=0}^{\infty} \frac{(mR)^i \quad \Psi(m+i, \frac{x}{b})}{(1-R+mR)^{i+1}(m+i-1)!}$$

where $\Psi(n,x) = \int_0^x e^{-x} x^{n-1} dx$ and $b = a(1-R)$, a being the parameter of the exponential distribution.

$$f_m^*(x) = \text{Laplace Transform of } f_m(x) = \frac{1}{(1+bs)^m\left(1+\frac{mRbs}{(1-R)(1+bs)}\right)} \qquad (9)$$

Noting that the interarrival times between decision making epochs are constantly correlated exponential variables with parameter a.

$$f_m^*\left(\lambda(1-k^*(\theta))\right) = \frac{(1-R)[1+b\lambda(1-k^*(\theta)]^{1-m}}{(1-R)+(1-k^*(\theta))[b\lambda-b\lambda R+mRb\lambda]}$$

$h(.)$ and $k(.)$ are exponentially distributed with parameters α, μ.

$$h^*(\theta_1) = \frac{\alpha}{\alpha+\theta_1} \qquad h^*(\theta_2) = \frac{\alpha}{\alpha+\theta_2}$$

$$k^*(\theta_1) = \frac{\mu}{\mu+\theta_1} \qquad k^*(\theta_2) = \frac{\mu}{\mu+\theta_2}$$

$$1-h^*(\theta_1) = \frac{\theta_1}{\theta_1+\alpha} \qquad 1-h^*(\theta_2) = \frac{\theta_2}{\theta_2+\alpha}$$

$$1-k^*(\theta_1) = \frac{\theta_1}{\theta_1+\mu} \qquad 1-k^*(\theta_2) = \frac{\theta_2}{\theta_2+\mu}$$

$$E(T) = \frac{\theta_1}{\theta_1-\theta_2}\left\{\frac{1}{\lambda\left(\frac{\theta_2}{\theta_2+\mu}\right)} - \frac{\frac{\theta_2}{\theta_2+\alpha}}{\lambda\left(\frac{\theta_2}{\theta_2+\mu}\right)}\sum_{m=1}^{\infty}\frac{(1-R)\left[1+b\lambda\left(\frac{\theta_2}{\theta_2+\mu}\right)\right]^{1-m}\left[\frac{\alpha}{\theta_2+\alpha}\right]^{m-1}}{(1-R)+\left(\frac{\theta_2}{\theta_2+\mu}\right)[b\lambda-b\lambda R+mRb\lambda]}\right\} +$$

$$\frac{\theta_2}{\theta_1-\theta_2}\left\{\frac{1}{\lambda\left(\frac{\theta_1}{\theta_1+\mu}\right)} - \frac{\frac{\theta_1}{\theta_1+\alpha}}{\lambda\left(\frac{\theta_1}{\theta_1+\mu}\right)}\sum_{m=1}^{\infty}\frac{(1-R)\left[1+b\lambda\left(\frac{\theta_1}{\theta_1+\mu}\right)\right]^{1-m}\left[\frac{\alpha}{\theta_1+\alpha}\right]^{m-1}}{(1-R)+\left(\frac{\theta_1}{\theta_1+\mu}\right)[b\lambda-b\lambda R+mRb\lambda]}\right\}$$

$$= \frac{\theta_1}{\theta_1-\theta_2}\left\{\frac{\theta_2+\mu}{\lambda\theta_2} - \frac{(\theta_2+\mu)^2(1-R)}{\lambda(\theta_2+\alpha)}\sum_{m=1}^{\infty}\frac{\left[\frac{\theta_2+\mu}{\theta_2+\mu+b\lambda\theta_2}\right]^{m-1}\left[\frac{\alpha}{\theta_2+\alpha}\right]^{m-1}}{(1-R)(\theta_2+\mu)+b\lambda\theta_2[1-R+mR]}\right\} +$$

$$\frac{\theta_2}{\theta_1-\theta_2}\left\{\frac{\theta_1+\mu}{\lambda\theta_1} - \frac{(\theta_1+\mu)^2(1-R)}{\lambda(\theta_1+\alpha)}\sum_{m=1}^{\infty}\frac{\left[\frac{\theta_1+\mu}{\theta_1+\mu+b\lambda\theta_1}\right]^{m-1}\left[\frac{\alpha}{\theta_1+\alpha}\right]^{m-1}}{(1-R)(\theta_1+\mu)+b\lambda\theta_1[1-R+mR]}\right\} \qquad (10)$$

From this we can find the expectation of time to recruitment. As the series is infinite, it is tedious to find out. Therefore, we can also have the finite occasions of decision making and transfer of personnel as a special case.

Suppose two occasions of policy decisions and 'n' occasions of transfer of personnel are occurred.

Then putting $m = 2$ in equation (10) we get,

$$E(T) = \frac{\theta_1}{\theta_1 - \theta_2}\left[\frac{\theta_2 + \mu}{\lambda\theta_2} - \frac{(\theta_2 + \mu)^2(1-R)}{\lambda(\theta_2 + \alpha)}\left\{\frac{1}{(1-R)(\theta_2 + \mu) + b\lambda\theta_2} + \frac{(\theta_2 + \mu)\alpha}{(\theta_2 + \mu + b\lambda\theta_2)(\theta_2 + \alpha)[(\theta_2 + \mu)(1-R) + b\lambda\theta_2(1+R)]}\right\}\right]$$

$$+ \frac{\theta_2}{\theta_1 - \theta_2}\left[\frac{\theta_1 + \mu}{\lambda\theta_1} - \frac{(\theta_1 + \mu)^2(1-R)}{\lambda(\theta_1 + \alpha)}\left\{\frac{1}{(1-R)(\theta_1 + \mu) + b\lambda\theta_1} + \frac{(\theta_1 + \mu)\alpha}{(\theta_1 + \mu + b\lambda\theta_1)(\theta_1 + \alpha)[(\theta_1 + \mu)(1-R) + b\lambda\theta_1(1+R)]}\right\}\right]$$

NUMERICAL ILLUSTRATIONS

We discuss some numerical examples for the above case.

The parameters such as R, α and μ are taken in different input data values and keeping other values fixed. The expected time to recruitment is shown in Table - 1 to Table - 3 and Figure - 1 to Figure - 3 respectively.

The following table gives the values of R & $E(T)$, when $\lambda = 1.2$, $\alpha = 1$, $b = 0.24$, $\mu = 2$, $\theta_1 = 0.2$ & $\theta_2 = 0.1$

R	$E(T)$
− 0.6	9.7248
− 0.4	11.7309
− 0.2	14.3792
0	18.0547
0.2	23.5263
0.4	32.5865
0.6	50.6027

Table – 1.

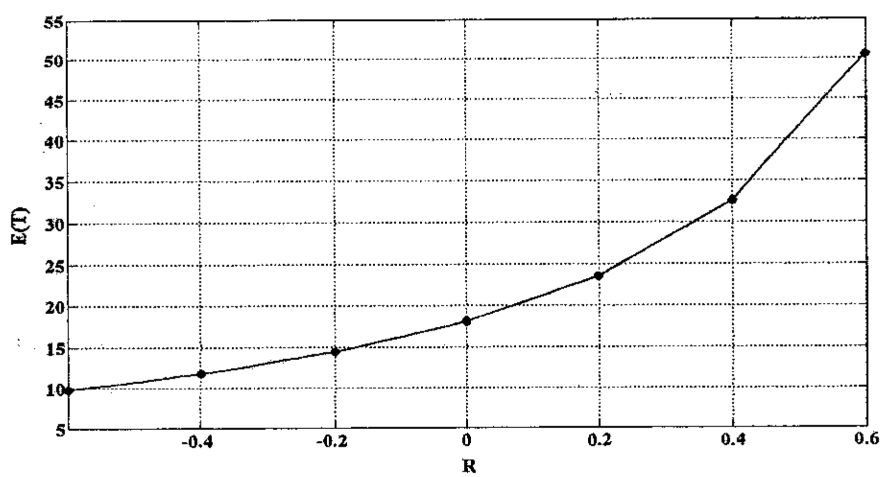

Fig – 1.

If the value of R, which is the parameter of the correlation between the inter-decision time increases and input data values $\lambda = 1.2$, $\alpha = 1$, $b = 0.24$, $\mu = 2$, $\theta_1 = 0.2$ & $\theta_2 = 0.1$ are fixed, the expected time to recruitment increases, as shown in Table - 1 and Figure - 1 respectively.

The following table gives the values of μ & $E(T)$, when $\lambda = 2.2$, $\alpha = 1$, $b = 1$, $R = 0.6$, $\theta_1 = 0.2$ & $\theta_2 = 0.1$

μ	$E(T)$
1	31.7330
2	32.7256
3	33.5559
4	34.1807
5	34.5705
6	34.7091

Table – 2.

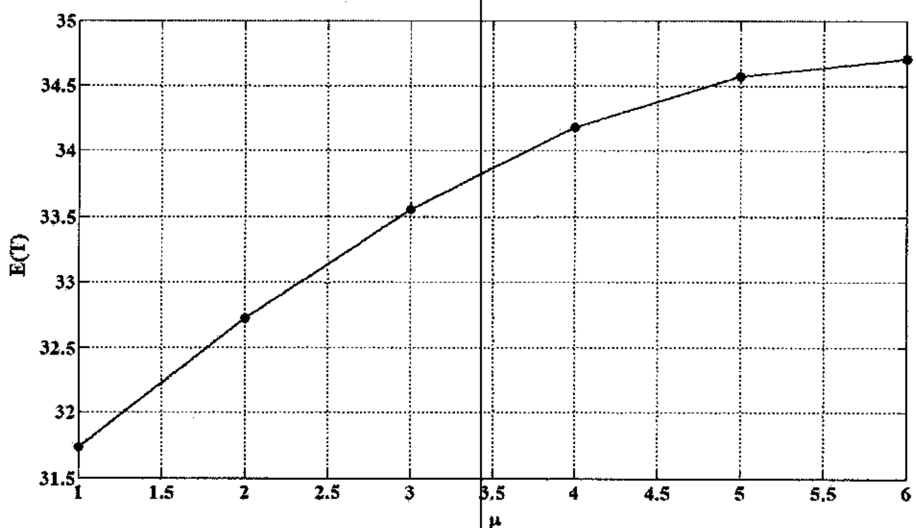

Fig. -2 : Expected time to recruitment for variations in μ

In table - 2, as a value of μ which is the parameter of the random variable representing the amount of depletion at every epoch of transfer of personnel increases, then it is seen that $E(T)$ which is the expected time to recruitment increases, which is also depicted in figure 2. This is due to the fact that Y is distributed as exponential with parameter μ and $E(Y) = \frac{1}{\mu}$. As μ increases $E(Y)$ namely the average amount of depletion decreases and hence $E(T)$ namely time to recruitment increases.

The following table gives the values of α & $E(T)$, when $\lambda = 1.2$, $R = 0.6$, $b = 0.24$, $\mu = 1$, $\theta_1 = 0.2$ & $\theta_2 = 0.1$

α	$E(T)$
1	53.5675
2	54.9736
3	55.4829
4	55.7460
5	55.9067
6	56.0151

Table – 3.

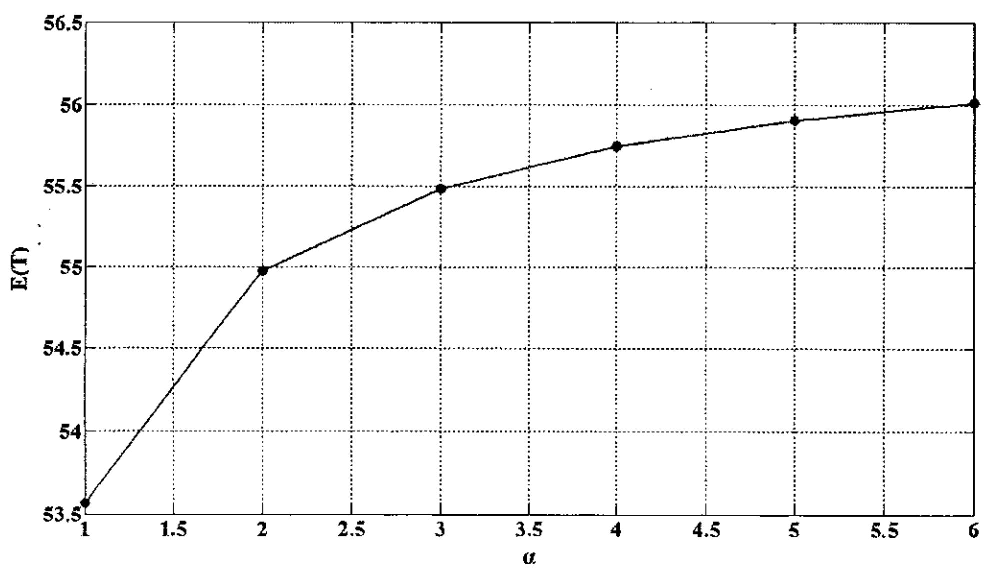

Fig. - 3 : Expected time to recruitment for variations in α

From Table - 3 it is observed that, if a value of α , which is the parameter of the random variable representing the amount of depletion at every epoch of decision increases, the expected time to recruitment increases, which is also depicted in Figure - 2.

CONCLUSION

The influence of the parameters as R, α and μ on the performance measure is analyzed numerically. If the value of R, which is the parameter of the correlation between the inter-decision time increases, the expected time to recruitment increases as shown in Table - 1 and Figure - 1 respectively. In Table - 2, as a value of μ which is the parameter of the random variable representing the amount of depletion at every epoch of transfer of personnel increases then it is seen that $E(T)$ which is the expected time to recruitment increases, which is also depicted in Figure - 2. This is due to the fact that Y is distributed as exponential with parameter μ and $E(Y)=\frac{1}{\mu}$. As μ increases $E(Y)$ namely the average amount of depletion decreases and hence $E(T)$ namely expected time to recruitment increases. From Table – 3, it is observed that, if a value of α , which is the parameter of the random variable representing the amount of depletion at every epoch of decision increases, the expected time to recruitment increases, which is also depicted in Figure – 3.

The best advantage of application of random process is that any real world situation is often conceptualized as a mathematical model and therefore the optimal solution can be derived by using standard techniques. The formulation of appropriate policies which might be profitable for the organization is achieved by the use of stochastic models in manpower planning. For the development of human resource management, the transformation of real life situations into mathematical model and identification of those areas are to be analyzed.

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