TRIPLE GENERATING FUNCTIONS OF JACOBI POLYNOMINALS OF THREE VARIABLES - II

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Abstract: The present paper is a study of triple generating functions of Jacobi polynomials of three variables.

Keywords: Jacobi polynomials of three variables, confluent hypergeometric functions, Triple generating functions.

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1. INTRODUCTION:

In our earlier paper [5] we obtained some triple generating functions of Jacobi polynomials of three variables. In the present paper we have obtained some another triple generating functions of Jacobi polynomials of three variables.

In our investigation we require the following definitions:
Jacobi polynomials of three variables are defined by [8]

$$p_n^{(\alpha_1,\beta_1;\alpha_2,\beta_2;\alpha_3,\beta_3)}(x,y,z)=\frac{(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_n}{(n!)^3}$$

$$\times F_A^{(3)}\left[-n,1+\alpha_1+\beta_1+n,1+\alpha_2+\beta_2+n,1+\alpha_3+\beta_3+n;1+\alpha_1,1+\alpha_2,\right.$$
$$\left.1+\alpha_3;\frac{1-x}{2},\frac{1-y}{2},\frac{1-z}{2}\right],\qquad (1.1)$$

where $F_A^{(3)}$ denotes one of the Lauricella's hypergeometric functions of three variables [7, p. 60] as follows:

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$$F_A^{(3)}[a,b_1,b_2,b_3;c_1,c_2,c_3;x,y,z] = \sum_{m_1,m_2,m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3}(b_1)_{m_1}(b_2)_{m_2}(b_3)_{m_3}}{(c_1)_{m_1}(c_2)_{m_2}(c_3)_{m_3}} \frac{x_1^{m_1}}{m_1!}\frac{x_2^{m_2}}{m_2!}\frac{x_3^{m_3}}{m_3!}. \quad (1.2)$$

The confluent hypergeometric functions of two variables are defined by [7]

$$\phi_1[\alpha,\beta;\gamma;x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(\alpha)_{m+n}(\beta)_m}{(\gamma)_{m+n}} \frac{x^m}{m!}\frac{y^n}{n!}, \qquad |x|<1, \quad |y|<\infty; \quad (1.3)$$

$$\phi_2[\beta,\beta';\gamma;x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(\beta)_m(\beta')_n}{(\gamma)_{m+n}} \frac{x^m}{m!}\frac{y^n}{n!}, \qquad |x|<\infty, \quad |y|<\infty; \quad (1.4)$$

$$\phi_3[\beta;\gamma;x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(\beta)_m}{(\gamma)_{m+n}} \frac{x^m}{m!}\frac{y^n}{n!}, \qquad |x|<\infty, \quad |y|<\infty; \quad (1.5)$$

$$\Psi_1[\alpha,\beta;\gamma,\gamma';x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(\alpha)_{m+n}(\beta)_m}{(\gamma)_m(\gamma')_n} \frac{x^m}{m!}\frac{y^n}{n!}, \qquad |x|<1, \quad |y|<\infty; \quad (1.6)$$

$$\Psi_2[\alpha;\gamma,\gamma';x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(\alpha)_{m+n}}{(\gamma)_m(\gamma')_n} \frac{x^m}{m!}\frac{y^n}{n!}, \qquad |x|<\infty, \quad |y|<\infty; \quad (1.7)$$

$$\Xi_1[\alpha,\alpha',\beta;\gamma;x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(\alpha)_m(\alpha')_n(\beta)_m}{(\gamma)_{m+n}} \frac{x^m}{m!}\frac{y^n}{n!}, \quad |x|<1, \quad |y|<\infty. \quad (1.8)$$

2. TRIPLE GENERATING FUNCTIONS:

The polynomials $P_n^{(\alpha_1,\beta_1;\alpha_2,\beta_2;\alpha_3,\beta_3)}(x,y,z)$ admits the following triple generating functions:

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n} \times P_n^{(\alpha_1+k,\beta_1-n;\alpha_2+m,\beta_2-m-n;\alpha_3,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\, {}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right] \Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_2\left[\mu,1+\alpha_2+\beta_2;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \quad (2.1)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\ \beta_1-n;\ \alpha_2,\ \beta_2-n;\ \alpha_3+m,\ \beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\ {}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\ \Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_2\left[\mu,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.2}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k(\mu)_m}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\ \beta_1-n;\ \alpha_2+k,\ \beta_2-n;\ \alpha_3+m,\ \beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\ {}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\ \Phi_1\left[1+\alpha_2+\beta_2,\lambda;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_2\left[\mu,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.3}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_m(1+\alpha_2+\beta_2)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\ \beta_1-k-n;\ \alpha_2+m,\ \beta_2-n;\ \alpha_3,\ \beta_3-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\ {}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_2+\beta_2,\mu;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.4}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_m(1+\alpha_3+\beta_3)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\ \beta_1-k-n;\ \alpha_2,\ \beta_2-n;\ \alpha_3+m,\ \beta_3-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\ {}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\mu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.5}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_m(1+\alpha_3+\beta_3)_m}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-k-n;\;\alpha_3+m,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\,\Phi_2\left[\lambda,1+\alpha_2+\beta_2;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\mu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \qquad (2.6)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-n;\;\alpha_2+m,\,\beta_2-m-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,v\right], \qquad (2.7)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \qquad (2.8)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_1\left[1+\alpha_2+\beta_2,\lambda;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \qquad (2.9)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m(1+\alpha_2+\beta_2)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2+m,\,\beta_2-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Phi_1\left[1+\alpha_2+\beta_2,\lambda;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.10}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m(1+\alpha_3+\beta_3)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\lambda;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.11}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m(1+\alpha_3+\beta_3)_m}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2+k,\,\beta_2-k-n;\,\alpha_3+m,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,u\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\lambda;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.12}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m(1+\alpha_2+\beta_2)_m}{k!m!(\delta)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-n;\,\alpha_2,\,\beta_2+m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_2+\beta_2,\mu;\delta,1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.13}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m(1+\alpha_3+\beta_3)_m}{k!m!(\delta)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\,\Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\mu;\delta,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \qquad (2.14)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k(\mu)_m(1+\alpha_3+\beta_3)_m}{k!m!(\delta)_m(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2+k,\,\beta_2-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\,\Phi_1\left[1+\alpha_2+\beta_2,\lambda;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\mu;\delta,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \qquad (2.15)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m(1+\alpha_2+\beta_2)_m}{k!m!(\delta)_k(1+\alpha_1)_n(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2+m,\,\beta_2-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\,\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\delta,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_2+\beta_2,\mu;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \qquad (2.16)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m(1+\alpha_3+\beta_3)_m}{k!m!(\delta)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\,\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\delta,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\mu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \qquad (2.17)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k(\mu)_m(1+\alpha_3+\beta_3)_m}{k!m!(\delta)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2,\,\beta_2+k-n;\,\alpha_3+m,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\,\Psi_1\left[1+\alpha_2+\beta_2,\lambda;\delta,1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\mu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.18}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(1+\alpha_2+\beta_2)_m}{k!m!(\mu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-n;\,\alpha_2,\,\beta_2+m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_2+\beta_2;\mu,1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.19}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_3+\beta_3;\mu,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.20}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_m(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2+k,\,\beta_2-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_1\left[1+\alpha_2+\beta_2,\lambda;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_3+\beta_3;\mu,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.21}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k(\lambda)_m(1+\alpha_2+\beta_2)_m}{k!m!(\mu)_k(1+\alpha_1)_n(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\;\alpha_2+m,\,\beta_2-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_2+\beta_2,\lambda;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.22}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k(\lambda)_m(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\lambda;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.23}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_2+\beta_2)_k(\lambda)_m(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2,\,\beta_2+k-n;\;\alpha_3+m,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Psi_2\left[1+\alpha_2+\beta_2;\mu,1+\alpha_2;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\lambda;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.24}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m(\nu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-n;\;\alpha_2+m,\,\beta_2-m-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Xi_1\left[\mu,1+\alpha_2+\beta_2,\nu;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.25}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m(\nu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_1\left[1+\alpha_1+\beta_1,\lambda;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Xi_1\left[\mu,1+\alpha_3+\beta_3,\nu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.26}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k(\mu)_m(\nu)_m}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2+k,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_1\left[1+\alpha_2+\beta_2,\lambda;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Xi_1\left[\mu,1+\alpha_3+\beta_3,\nu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right]. \tag{2.27}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(\nu)_m(1+\alpha_2+\beta_2)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2+m,\,\beta_2-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_2+\beta_2,\nu;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.28}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(\nu)_m(1+\alpha_3+\beta_3)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\nu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.29}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(\nu)_m(1+\alpha_3+\beta_3)_m}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-k-n;\;\alpha_3+m,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Xi_1\left[\lambda,1+\alpha_2+\beta_2,\mu;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_1\left[1+\alpha_3+\beta_3,\nu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.30}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2+m,\,\beta_2-m-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,v\right], \tag{2.31}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \tag{2.32}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-k-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_2\left[\lambda,1+\alpha_2+\beta_2;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \tag{2.33}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2+m,\,\beta_2-m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Phi_2\left[\lambda,1+\alpha_2+\beta_2;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.34}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Phi_2\left[\lambda,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.35}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2+k,\,\beta_2-k-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,u\right]$$
$$\times\Phi_2\left[\lambda,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.36}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_m(1+\alpha_2+\beta_2)_m}{k!m!(\delta)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2,\,\beta_2+m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_2+\beta_2,\mu;\delta,1+\alpha_2;v,\frac{1}{2}(y-1)t\right]. \tag{2.37}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_m(1+\alpha_3+\beta_3)_m}{k!m!(\delta)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\mu;\delta,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.38}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_m(1+\alpha_3+\beta_3)_m}{k!m!(\delta)_m(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2+k,\,\beta_2-k-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_2\left[\lambda,1+\alpha_2+\beta_2;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\mu;\delta,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.39}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m}{k!m!(\delta)_k(1+\alpha_1)_n(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2+m,\,\beta_2-m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\delta,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_2\left[\mu,1+\alpha_2+\beta_2;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.40}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m}{k!m!(\delta)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\delta,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_2\left[\mu,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.41}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k(\mu)_m}{k!m!(\delta)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2,\,\beta_2+k-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Psi_1\left[1+\alpha_2+\beta_2,\lambda;\delta,1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_2\left[\mu,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.42}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_m}{k!m!(\mu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2+m-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_2+\beta_2;\mu,1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.43}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3,\,\beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_3+\beta_3;\mu,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.44}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_m(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-k-n;\;\alpha_3,\,\beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_2\left[\lambda,1+\alpha_2+\beta_2;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_3+\beta_3;\mu,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.45}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k(\lambda)_m}{k!m!(\mu)_k(1+\alpha_1)_n(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2+m,\,\beta_2-m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_2\left[\lambda,1+\alpha_2+\beta_2;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.46}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k(\lambda)_m}{k!m!(\mu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_2\left[\lambda,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.47}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_2+\beta_2)_k(\lambda)_m}{k!m!(\mu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2,\,\beta_2+k-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Psi_2\left[1+\alpha_2+\beta_2;\mu,1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_2\left[\lambda,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.48}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_m(\nu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2+m,\,\beta_2-m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Xi_1\left[\mu,1+\alpha_2+\beta_2,\nu;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.49}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_m(\nu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_2\left[\lambda,1+\alpha_1+\beta_1;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Xi_1\left[\mu,1+\alpha_3+\beta_3,\nu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.50}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_m(\nu)_m}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-k-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_2\left[\lambda,1+\alpha_2+\beta_2;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Xi_1\left[\mu,1+\alpha_3+\beta_3,\nu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.51}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(\nu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2+m,\,\beta_2-m-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_2\left[\nu,1+\alpha_2+\beta_2;1+\alpha_2;u,\frac{1}{2}(y-1)t\right], \tag{2.52}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(\nu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_2\left[\nu,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.53}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(\nu)_m}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-k-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Xi_1\left[\lambda,1+\alpha_2+\beta_2,\mu;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_2\left[\nu,1+\alpha_3+\beta_3;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.54}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m(1+\alpha_2+\beta_2)_m}{k!m!(\mu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2+m-n;\;\alpha_3,\,\beta_3+n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Psi_1\left[1+\alpha_2+\beta_2,\lambda;\mu,1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.55}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3,\,\beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\lambda;\mu,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.56}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_m(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-k-n;\;\alpha_3,\,\beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,u\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\lambda;\mu,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.57}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k}{k!m!(\mu)_k(1+\alpha_1)_n(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\;\alpha_2+m,\,\beta_2-m-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,v\right], \tag{2.58}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k}{k!m!(\mu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \tag{2.59}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k}{k!m!(\mu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2,\,\beta_2+k-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Psi_1\left[1+\alpha_2+\beta_2,\lambda;\mu,1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \tag{2.60}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_2+\beta_2)_m}{k!m!(\lambda)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2+m-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Psi_2\left[1+\alpha_2+\beta_2;\lambda,1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.61}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_3+\beta_3)_m}{k!m!(\lambda)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Psi_2\left[1+\alpha_3+\beta_3;\lambda,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \qquad (2.62)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_3+\beta_3)_m}{k!m!(\lambda)_m(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2+k,\,\beta_2-k-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,u\right]$$
$$\times\Psi_2\left[1+\alpha_3+\beta_3;\lambda,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \qquad (2.63)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k}{k!m!(\lambda)_k(1+\alpha_1)_n(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2+m,\,\beta_2-m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\lambda,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,v\right], \qquad (2.64)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k}{k!m!(\lambda)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\lambda,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \qquad (2.65)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_2+\beta_2)_k}{k!m!(\lambda)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2,\,\beta_2+k-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Psi_2\left[1+\alpha_2+\beta_2;\lambda,1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \tag{2.66}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m(\mu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2+m,\,\beta_2-m-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Xi_1\left[\lambda,1+\alpha_2+\beta_2,\mu;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.67}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m(\mu)_m}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Phi_3\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t,u\right]$$
$$\times\Xi_1\left[\lambda,1+\alpha_3+\beta_3,\mu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.68}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_m(\mu)_m}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-k-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,u\right]$$
$$\times\Xi_1\left[\lambda,1+\alpha_3+\beta_3,\mu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.69}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2+m,\,\beta_2-m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t,v\right], \qquad (2.70)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k}{k!m!(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \qquad (2.71)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k}{k!m!(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2+k,\,\beta_2-k-n;\,\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Xi_1\left[\lambda,1+\alpha_2+\beta_2,\mu;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Phi_3\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t,v\right], \qquad (2.72)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(1+\alpha_2+\beta_2)_m}{k!m!(\mu)_k(\delta)_m(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2,\,\beta_2+m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^k v^m t^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_2+\beta_2;\delta,1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \qquad (2.73)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_k(\delta)_m(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_3+\beta_3;\delta,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.74}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_k(\delta)_m(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\,\alpha_2,\,\beta_2+k-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Psi_1\left[1+\alpha_2+\beta_2,\lambda;\mu,1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_3+\beta_3;\delta,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.75}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k(\lambda)_m(1+\alpha_2+\beta_2)_m}{k!m!(\mu)_k(\delta)_m(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2,\,\beta_2+m-n;\,\alpha_3,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_2+\beta_2,\lambda;\delta,1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.76}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k(\lambda)_m(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_k(\delta)_m(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\,\alpha_2,\,\beta_2-n;\,\alpha_3,\,\beta_3+m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\mu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\lambda;\delta,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.77}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_2+\beta_2)_k(\lambda)_m(1+\alpha_3+\beta_3)_m}{k!m!(\mu)_k(\delta)_m(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2,\,\beta_2+k-n;\;\alpha_3,\,\beta_3+m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Psi_2\left[1+\alpha_2+\beta_2;\mu,1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\lambda;\delta,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.78}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m(\delta)_m}{k!m!(\nu)_k(1+\alpha_1)_n(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\;\alpha_2+m,\,\beta_2-m-n;\;\alpha_3,\,\beta_3-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\nu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Xi_1\left[\mu,1+\alpha_2+\beta_2,\delta;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.79}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_1+\beta_1)_k(\mu)_m(\delta)_m}{k!m!(\nu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Psi_1\left[1+\alpha_1+\beta_1,\lambda;\nu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Xi_1\left[\mu,1+\alpha_3+\beta_3,\delta;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.80}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(1+\alpha_2+\beta_2)_k(\mu)_m(\delta)_m}{k!m!(\nu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2,\,\beta_2+k-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Psi_1\left[1+\alpha_2+\beta_2,\lambda;\nu,1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Xi_1\left[\mu,1+\alpha_3+\beta_3,\delta;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.81}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(\delta)_m(1+\alpha_2+\beta_2)_m}{k!m!(\nu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\ \beta_1-k-n;\ \alpha_2,\ \beta_2+m-n;\ \alpha_3,\ \beta_3-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_2+\beta_2,\delta;\nu,1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \qquad (2.82)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(\delta)_m(1+\alpha_3+\beta_3)_m}{k!m!(\nu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\ \beta_1-k-n;\ \alpha_2,\ \beta_2-n;\ \alpha_3,\ \beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\delta;\nu,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \qquad (2.83)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(\delta)_m(1+\alpha_3+\beta_3)_m}{k!m!(\nu)_m(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\ \beta_1-n;\ \alpha_2+k,\ \beta_2-k-n;\ \alpha_3,\ \beta_3+m-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Xi_1\left[\lambda,1+\alpha_2+\beta_2,\mu;1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Psi_1\left[1+\alpha_3+\beta_3,\delta;\nu,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \qquad (2.84)$$

$$\sum_{k=0}^{\infty}\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k(\lambda)_m(\mu)_m}{k!m!(\nu)_k(1+\alpha_1)_n(1+\alpha_2)_{m+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\ \beta_1+k-n;\ \alpha_2+m,\ \beta_2-m-n;\ \alpha_3,\ \beta_3-n)}(x,y,z)u^k v^m t^n$$
$$=e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\nu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Xi_1\left[\lambda,1+\alpha_2+\beta_2,\mu;1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \qquad (2.85)$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_1+\beta_1)_k(\lambda)_m(\mu)_m}{k!m!(\nu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1+k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Psi_2\left[1+\alpha_1+\beta_1;\nu,1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Xi_1\left[\lambda,1+\alpha_3+\beta_3,\mu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.86}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(1+\alpha_2+\beta_2)_k(\lambda)_m(\mu)_m}{k!m!(\nu)_k(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_{m+n}}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2,\,\beta_2+k-n;\;\alpha_3+m,\,\beta_3-m-n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_1+\beta_1;1+\alpha_1;\frac{1}{2}(x-1)t\right]\Psi_2\left[1+\alpha_2+\beta_2;\nu,1+\alpha_2;u,\frac{1}{2}(y-1)t\right]$$
$$\times\Xi_1\left[\lambda,1+\alpha_3+\beta_3,\mu;1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.87}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(1+\alpha_2+\beta_2)_m}{k!m!(\nu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2+m-n;\;\alpha_3,\,\beta_3+n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_3+\beta_3;1+\alpha_3;\frac{1}{2}(z-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_2+\beta_2;\nu,1+\alpha_2;v,\frac{1}{2}(y-1)t\right], \tag{2.88}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(1+\alpha_3+\beta_3)_m}{k!m!(\nu)_m(1+\alpha_1)_{k+n}(1+\alpha_2)_n(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1+k,\,\beta_1-k-n;\;\alpha_2,\,\beta_2-n;\;\alpha_3,\,\beta_3+m+n)}(x,y,z)u^kv^mt^n$$
$$= e^t\,{}_1F_1\left[1+\alpha_2+\beta_2;1+\alpha_2;\frac{1}{2}(y-1)t\right]\Xi_1\left[\lambda,1+\alpha_1+\beta_1,\mu;1+\alpha_1;u,\frac{1}{2}(x-1)t\right]$$
$$\times\Psi_2\left[1+\alpha_3+\beta_3;\nu,1+\alpha_3;v,\frac{1}{2}(z-1)t\right], \tag{2.89}$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(n!)^2(\lambda)_k(\mu)_k(1+\alpha_3+\beta_3)_m}{k!m!(\nu)_m(1+\alpha_1)_n(1+\alpha_2)_{k+n}(1+\alpha_3)_n}$$
$$\times P_n^{(\alpha_1,\,\beta_1-n;\;\alpha_2+k,\,\beta_2-k-n;\;\alpha_3,\,\beta_3+m+n)}(x,y,z)u^kv^mt^n$$

$$= e^t\, {}_1F_1\left[1+\alpha_1+\beta_1; 1+\alpha_1; \frac{1}{2}(x-1)t\right] \Xi_1\left[\lambda, 1+\alpha_2+\beta_2, \mu; 1+\alpha_2; u, \frac{1}{2}(y-1)t\right]$$
$$\times \Psi_2\left[1+\alpha_3+\beta_3; \nu, 1+\alpha_3; v, \frac{1}{2}(z-1)t\right]. \qquad (2.90)$$

PROOF OF (2.1): Starting with left-side of (2.1) and using the definition (1.1) of Jacobi polynomials of three variables, one gets

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{n}\sum_{s=0}^{n-r}\sum_{l=0}^{n-r-s} \frac{((\lambda)_k(1+\alpha_1+\beta_1)_{k+r}(\mu)_m(1+\alpha_2+\beta_2)_s(1+\alpha_3+\beta_3)_l}{k!m!r!s!l!(n-r-s-l)!(1+\alpha_1)_{k+r}(1+\alpha_2)_{m+s}(1+\alpha_3)_l}$$
$$\times \left(\frac{x-1}{2}\right)^r \left(\frac{y-1}{2}\right)^s \left(\frac{z-1}{2}\right)^l u^k v^m t^n$$

$$= \sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\sum_{l=0}^{\infty} \frac{(1+\alpha_1+\beta_1)_{k+r}(\lambda)_k(\mu)_m(1+\alpha_2+\beta_2)_s(1+\alpha_3+\beta_3)_l}{k!m!n!r!s!l!(1+\alpha_1)_{k+r}(1+\alpha_2)_{m+s}(1+\alpha_3)_l}$$
$$\times \left(\frac{x-1}{2}\right)^r \left(\frac{y-1}{2}\right)^s \left(\frac{z-1}{2}\right)^l u^k v^m t^{n+r+s+l}$$

$$= e^t \sum_{l=0}^{\infty} \frac{(1+\alpha_3+\beta_3)_l}{l!(1+\alpha_3)_l}\left(\frac{1}{2}(z-1)t\right)^l \sum_{k=0}^{\infty}\sum_{r=0}^{\infty} \frac{(1+\alpha_1+\beta_1)_{k+r}(\lambda)_k}{k!r!(1+\alpha_1)_{k+r}} u^k \left(\frac{1}{2}(x-1)t\right)^r$$
$$\times \sum_{m=0}^{\infty}\sum_{s=0}^{\infty} \frac{(\mu)_m(1+\alpha_2+\beta_2)_s}{m!s!(1+\alpha_2)_{m+s}} v^m \left(\frac{1}{2}(y-1)t\right)^s$$

$$= e^t\, {}_1F_1\left[1+\alpha_3+\beta_3; 1+\alpha_3; \frac{1}{2}(z-1)t\right] \Phi_1\left[1+\alpha_1+\beta_1, \lambda; 1+\alpha_1; u, \frac{1}{2}(x-1)t\right]$$
$$\times \Phi_2\left[\mu, 1+\alpha_2+\beta_2; 1+\alpha_2; v, \frac{1}{2}(y-1)t\right]$$

which proves (2.1).

The proof of results (2.2) to (2.90) are similar to that of (2.1).

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