Blood Flow in Human Body

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Abstract

Blood being an important constituent of human body have been studied in various ways. Here, its flow have been considered in two phase model of blood flow through narrow tube with viscosity near the walls, blood volumetric flow equation is counted and the shear stress at the walls have been evaluated . Analytical models for flow of blood have been utilized to study the flow in different conditions. A non-Newtonian model has been undertaken by using Cauchy stress tensor to study blood flow and by associating momentum and constitutive equation a new form is being formed. A one dimensional view has been taken by dealing with viscosity and shear rate in the bases of viscometric data which is being carried to find dimensionless momentum equation.

Pulsatile nature of blood has been taken which occurs due to pressure by heart beating and by considering various limitations and wave equation has been evaluated.

Keywords: Plasma, Hematocrit, Blood Coagulation, Blood Rheology

AMS Subject Classification (2000): **76D17, 76D05**

1. Introduction

Human blood is a suspension of formed elements in liquid called plasma. Plasma is made of mainly water (91%) and contains traces of inorganic and organic salts. About 7% by weight of plasma is protein. The formed element consists of red blood cells and platelets. Red cells are about 97% of the total cell volume and its concentration in human blood called hematocrit is normally about 42.45% by volume. RBC's are heavier than plasma, so in non flowing blood RBC's tends to settle. Biconcave disc shaped RBC's prime function is to transport O_2 and CO_2 throughout body and buffering the blood so as to regulate pH. Hemoglobin is present to extent of more than 33 wt% inside the RBC's and act as reversible binding for O_2 and CO_2. White blood cells (WBC's) are of several types and somewhat larger than RBC's. WBC's produces antibodies or direct engulf and digest the bacteria. Platelets are small flat cells of $3\mu m$ diameter and plays important role in blood coagulation and clotting.

2. Blood Rheology

Blood is a complex fluid, containing a fair number of relatively small molecules and a large variety of macromolecules. Rheological behaviour of blood that is, its deformation and flow behaviour, depends on apparent viscosity on flow rate, temperature, etc. Blood viscosity can be measured by using one of three common viscometers.

(a) The capillary tube type,

(b) The concentric cylinder type, and

(c) The cone and plate type.

The coefficient of viscosity μ for Newtonian fluids is given by Newton's law as

$$\tau = \mu\left(\frac{dr}{dt}\right), \qquad (2.1)$$

where τ = shear stress on fluid.

$\frac{dr}{dt}$ = strain rate (velocity gradient) and

μ = coefficient of viscosity.

For Newtonian fluids the value of μ is independent of τ or $\left(\frac{dr}{dt}\right)$ and for non-Newtonian fluids μ can vary with τ or $\left(\frac{dr}{dt}\right)$.

Poiseuille's Flow:

The flow of fluids through a straight circular tube is known as Poiseuille flow [13]. Poiseuille law is applicable for steady laminar flow of a Newtonian fluid through a rigid circular tube and is given as,

$$Q = \pi \frac{\Delta p}{L} . \frac{R}{8\mu}, \qquad (2.2)$$

where Q = flow rate through the tube,

Δp = pressure drop across the tube,

L = Length of the tube,

R = radius of the tube, and

μ = coefficient of viscosity of the fluid.

This law is applicable for experimental determination of μ of an Newtonian fluid, and is not applicable for non-Newtonian fluids.

Blood Viscosity described by Casson's Equation:

If we compute the shear strain rate of the fluid at the tube wall, using the formulas obtained for a Newtonian fluid, we have

$$\left.\frac{du}{dr}\right]_{r=a} = \frac{1}{2}\frac{a}{\mu}\frac{dp}{dx},$$

or,

$$\left.\frac{du}{dr}\right]_{r=a} = -\frac{4u_m}{a}, \qquad (2.3)$$

where u_m is the mean velocity.

3. Two-phase model for flow of blood in narrow tubes with increased effective viscosity near the wall

Nair et al [14] used two phase model for blood in modeling transport of oxygen in the arterioles. Seshadri and Jaffrin [15] modeled the outer layer as cell depleted having a lower hematocrit than in core. Pries et al [12, 16, and 17] derived empirical relationships of the relative apparent viscosity and mean tube hematocrit as parametric functions of tube diameter and discharge hematocrit from in vitro [16, 17] and in vivo [12] data. Diamiano [18] has presented a semi-empirical model for the blood flow in glycocalyx - lined micro vessels greater than 20μm in diameter.

Mathematical model:

Consider a two layer model for blood flow within a cylindrical tube of

R = radius of vessel

r_h = radius of central core

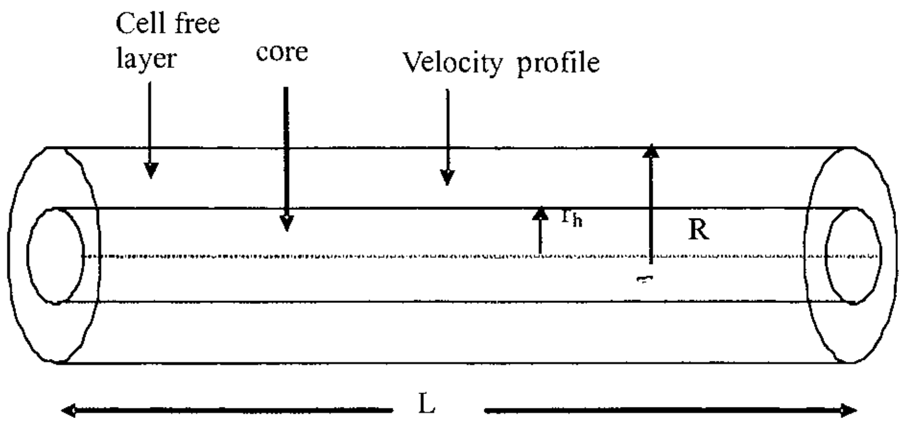
μ_c = effective viscosity

H_c = suspension erythrocyte of uniform hematocrit

μ_0 = effective viscosity

μ_0 is higher than true plasma viscosity, μ_{pl}.

Due of high enough shear rates fluid in each region can be regarded as Newtonian.



Schematic diagram of the model.

Conservation equations and boundary conditions:

Equation of motion for incompressible steady fully developed flow in a tube is

$$-\frac{\partial p}{\partial z}+\frac{\mu_c}{r}\frac{d}{dr}\left(r\frac{\partial u_c}{\partial r}\right)=0, \quad 0 \le r \le r_h, \tag{3.1}$$

for the central core with red blood cells and

$$-\frac{\partial p}{\partial z}+\frac{\mu_0}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_0}{\partial r}\right)=0, \quad r_h < r < R, \tag{3.2}$$

for the cell free layer, u_c and u_0 are respectively the velocities in the core and the plasma layer in the axial direction, p the hydraulic pressure and r, z represents the radial and axial directions in the tube.

Boundary Condition for the flow:

(a) The velocity gradient vanishes along the axis of tube, due to symmetry

$$\frac{\partial u_c}{\partial r} = 0 \quad \text{at} \quad r = 0. \tag{3.3}$$

(b) Assuming no slip condition at the wall.

$$u_0 = 0 \quad \text{at} \quad r = R. \tag{3.4}$$

(c) The velocity and shear stress are continuous at the interface of plasma and the core

$$\text{(i)} \quad \mu_c\big|_{r=r_h} = \dot{u}_0\big|_{r=r_h} \tag{3.5}$$

$$\text{(ii)} \quad \mu_c \frac{\partial u_c}{\partial r}\bigg|_{r=r_h} = \mu_0 \frac{\partial u_0}{\partial r}\bigg|_{r=r_1}. \tag{3.6}$$

Solution of equation (3.1) and (3.2), subject to boundary conditions (3.3) - (3.6) is given by

$$u_c(\xi) = \frac{PR^2}{4\mu_0}[1 - \lambda^2 + \frac{\mu_0}{\mu_c}(\lambda^2 - \xi^2)], \quad 0 \le \xi \le \lambda,$$

$$u_0(\xi) = \frac{PR^2}{4\mu_0}[1 - \xi^2], \; \lambda \le \xi \le 1, \tag{3.7}$$

Where $\xi = \frac{r}{R}$, $\lambda = \frac{r_h}{R}$ and $P = \frac{\Delta p}{L}$,

where Δp represents pressure drop along the length L of the tube.

Velocity in the core can be expressed as

$$u_c(\xi) = u_{max}(1 - B\xi^2), \tag{3.8}$$

Where

$$u_{max} = \frac{PR^2}{4\mu_0}\{1 - \lambda^2(1 - \frac{\mu_0}{\mu_c})\}, \tag{3.9}$$

$$B = \frac{\mu_0 / \mu_c}{1-\lambda^2(1-\frac{\mu_0}{\mu_c})}, \quad (3.10)$$

B indicates bluntness of velocity profile and represents deviation from parabolic flow.

Volumetric flow of blood is given by

$$Q = 2\pi R^2 \int u_c(\xi)\xi d\xi + 2\pi R^2 \int_\lambda^1 u_0(\xi) d\xi . \quad (3.11)$$

Overall mass balance of the cells in the tube is defined by

$$Q = \frac{\pi P R^4}{8\mu_0}\left[\frac{\mu_0}{\mu_c}\lambda^4 + 1 - \lambda^4\right], \quad (3.12)$$

in which H_D is discharge hematocrit and

$$h(\xi) = \begin{cases} H_C, & 0 \le \xi \le \lambda \\ 0, & \lambda < \xi \le 1. \end{cases} \quad (3.13)$$

On solving (3.11) and (3.12) using equation (3.13) we get,

$$Q = \frac{\pi P R^4}{8\mu_0}\left[\frac{\mu_0}{\mu_c}\lambda^4 + 1 - \lambda^4\right], \quad (3.14)$$

$$QH_D = \frac{\pi P R^4 H_c}{8\mu_0}\left[\frac{\mu_0}{\mu_c}\lambda^4 + 2\lambda^2(1-\lambda^2)\right]. \quad (3.15)$$

Rewriting

$$Q = \frac{\pi P R^4}{8\mu_{pp}}, \quad (3.16)$$

where apparent total tube flow viscosity (μ_{pp}) is

$$\mu_{app} = \mu_0[\frac{\mu_0}{\mu_c}\lambda^4 + 1 - \lambda^4]^{-1}. \quad (3.17)$$

The tube hematocrit H_T is defined by

$$H_T = 2\int_0^1 h(\xi)\xi d\xi . \qquad (3.18)$$

The shear stress at the wall is obtained as

$$T_W = \frac{PR}{2} = \frac{4\mu_{app}\overline{U}}{R}, \qquad (3.19)$$

where $\overline{U}$ is average velocity of the blood.

Pries et al. [18] developed a parametric description of the reduction of the tube hematocrit relative to discharge hematocrit on the basis of broad literature data on human red blood cells suspension flows through glass tubes with different diameters. Pries et al. [19] have used their earlier form of H_T with H_D in vitro

$$\frac{H_T}{H_D} = H_D + (1 - H_D)(1 + 1.7e^{-0.35D} - 0.6e^{-0.01D}), \qquad (3.20)$$

where D is diameter of the glass tube in microns. The coefficient in the first exponent was 0.415 in (3.20) instead of 0.35. Pries et al. [19] have derived an empirical equation for the variation of relative apparent blood viscosity (μ_{rel}) with H_D and D,

$$\mu_{rel} = \frac{\mu_{app}}{\mu_{pl}} = 0.1 + (\mu_{rel,0.45}{}^{-1})\frac{(1 - H_D)^C - 1}{(1 - 0.45)^C - 1}, \qquad (3.21)$$

where

$$\mu_{rel,0.45} = 220e^{-1.3D} + 3.2 - 2.44e^{-0.06D^{0.645}} . \qquad (3.22)$$

$$C = (0.8 + e^{-0.0753D})(-1 + \frac{1}{1 + 10^{-11}D^{12}}) + \frac{1}{1 + 10^{-11}D^{12}} \qquad (3.23)$$

Due to geometrical roughness of the interface between the core and cell free plasma layer and occasional presence of red blood cells near the wall, the energy dissipation in plasma layer results in an effective viscosity that may be larger than the plasma viscosity. This normalized effective viscosity, $\beta = \mu_0/\mu_{pl}$ may depends in the thickness of the cell free layer and the core hematocrit.

4. Results and Discussion

Using equation (3.22) for tube hematocrit as a function of discharge and tube diameter, we found that the value of β become less than one for large value of tube diameter.

5. Conclusions

A two phase model for the flow of blood in narrow tubes consists of central core of suspended erythrocytes and a cell-free layer surrounding the core. Assumption was made that the viscosity in the cell free layer may differ from that the plasma as a result of dissipation of energy near the wall from the core due to the roughness of the surface between core and the cell-free plasma layer. Consistent system of non-linear equations is solved numerically to estimate β, λ, H_C.

The thickness of the cell free layer computed from the model is in good agreement with experimental observations. Model proposed is physically consistent and works for the tubes of diameter in the range $20 \leq D \leq 300\mu m.$, but for small tubes the dimension of red blood cell becomes comparable to the tube diameter, thus the continuum approach breakdown.

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