ECCENTRIC CONNECTIVITY INDEX OF GENERALIZED COMPLEMENTARY PRISMS

[1]S. Arockiaraj and [2]Vijaya Kumari

Abstract

The eccentric connectivity index of a graph G is defined as $\xi^c(G) = \sum_{v\in V(G)} deg(v)ecc(v)$,where $ecc(v)$ is the eccentricity of a vertex v in G .In this paper we have obtained some bounds for the complimentary prism $G\overline{G}$, the generalized complementary prism $G_{m+n}, G_{m,n}, G^p_{m,m}, G^c_{m,m}$, the Cartesian product $K_m \times C_n, K_m \times P_n, K_m \times K_m$ and the cycles identifying at a vertex.

Keywords. eccentricity, radius, diameter, complementary prism, Cartesian product.

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*Research Scholar of Karpagam University, Coimbatore, Tamilnadu, INDIA

[1]Department of Mathematics, Mepco Schlenk Engineering College, Sivakasi - 626 005, Tamilnadu, INDIA. e-mail : psarockiaraj@gmail.com

[2]Department of Mathematics, Vijaya College, Mulky - 574 154, Karnataka, INDIA. e-mail : vijayasullal@gmail.com

1 Introduction

Throughout this paper, all graphs we considered are simple and connected. For a vertex $v \in V(G)$, $deg(v)$ denotes the degree of v. $\delta(G)$ and $\Delta(G)$ represent the minimum and maximum degree of G respectively. For vertices $u, v \in V(G)$, the distance $d(u, v)$ is defined as the length of the shortest path between u and v in G. The eccentricity $\xi(v)$ of a vertex v is the maximum among the distances from v to remaining vertices. The radius $r(G)$ of graph is the minimum eccentricity of the vertices of G, while the diameter $d(G)$ of a graph is the maximum eccentricity of the vertices of G. The total eccentricity of the graph G, denoted by $\xi(G)$ is defined as the sum of eccentricities of all the vertices of graph G. That is, $\xi(G) = \sum_{v \in V(G)} ecc(v)$. The eccentric connectivity index of G denoted by $\xi^c(G)$, is defined as $\xi^c(G) = \sum_{v \in V(G)} deg(v) ecc(v)$.

Kathiresan and Arockiaraj introduced some generalization of complementary prisms and studied the Wiener index of those generalized complementary prisms [7].

Let G and H be any two graphs on p_1 and p_2 vertices, respectively and let R and S be subsets of $V(G) = \{u_1, u_2, \ldots, u_{p_1}\}$ and $V(H) = \{v_1, v_2, \ldots, v_{p_2}\}$ respectively. The *complementary product* $G(R)\Box H(S)$ has the vertex set $\{(u_i, v_j) : 1 \le i \le p_1, 1 \le j \le p_2\}$ and (u_i, v_j) and (u_h, v_k) are adjacent in $G(R)\Box H(S)$

(i) if $i = h, u_i \in R$ and $v_j v_k \in E(H)$, or if $i = h, u_i \notin R$ and $v_j v_k \notin E(H)$ or

(ii) if $j = k, v_j \in S$ and $u_i u_h \in E(G)$, or if $j = k$, $v_j \notin S$ and $u_i u_h \notin E(G)$.

In other words, $G(R)\Box H(S)$ is the graph formed by replacing each vertex $u_i \in R$ of G by a copy of H, each vertex $u_i \notin R$ of G by a copy of $\overline{H}$, each vertex $v_j \in S$ of H by a copy of G and each vertex $v_j \notin S$ of H by a copy of $\overline{G}$. If $R = V(G)$ (respectively, $S = V(H)$), the complementary product can be written as $G\Box H(S)$ (respectively, $G(R)\Box H$). The *complementary prism* $G\overline{G}$ obtained from G is $G\Box K_2(S)$ with $|S| = 1$. That is, $G\overline{G}$ has a copy of G and a copy of $\overline{G}$ with a matching between the corresponding vertices [7].

In $G\overline{G}$, we have an edge $v\overline{v}$ for each vertex v in G. The authors consider this edge as K_2 or $K_{1,1}$ or P_2. By taking m copies of G and n copies of $\overline{G}$, they generalize the complementary prism as a graph $G\Box H(S)$, where $H = K_{m+n}$ (or

$K_{m,n}$) and S is a subset of $V(H)$ having m vertices and $H = C_{2m}$ (or P_{2m}) whose vertex set is $\{v_1, v_2, \ldots, v_{2m}\}$ and $S = \{v_1, v_3, \ldots, v_{2m-1}\}$ [7].

Motivated by these works, we have obtained the bounds of eccentric connectivity index for the complimentary prism $G\overline{G}$, the generalized complimentary prism G_{m+n},$G_{m,n}$,$G^p_{m,m}$,$G^c_{m,m}$, the Cartesian product $K_m \times C_n$, $K_m \times P_n$, $K_m \times K_m$ and the cycles identifying at a vertex.

Theorem 1.1. [7] *For the complementary prism $G\overline{G}, r(G\overline{G}) = 2$ and*

$$d(G\overline{G}) = \begin{cases} 2 & \text{if } d(G) = d(\overline{G}) = 2 \\ 3 & \text{otherwise.} \end{cases}$$

Theorem 1.2. [7] *For any connected graph G with $p \geq 2$,*

$$d(G_{m+n}) = \begin{cases} 2 & \text{if } d(G) = d(\overline{G}) = 2 \text{ and } m = n = 1 \\ 3 & \text{otherwise.} \end{cases}$$

Theorem 1.3. [7] *For any connected graph G with $p \geq 2$,*

$$d(G_{m,n}) = \begin{cases} 2 & \text{if } d(G) = d(\overline{G}) = 2 \text{ and } m = n = 1 \\ 3 & \text{otherwise.} \end{cases}$$

Theorem 1.4. [7] *For any connected graph G with $p \geq 2$,*

$$d(G^p_{m,m}) = \begin{cases} 2m & \text{if } m > 1 \\ 2 & \text{if } m = 1 \text{ and } d(G) = d(\overline{G}) = 2 \\ 3 & \text{otherwise.} \end{cases}$$

Theorem 1.5. [7] *For any connected graph G with $p \geq 2$ $d(G^c_{m,m}) = 2r + 1$ if $m = 2r \geq 2$ and r is a positive integer.*

2 Main Results

Theorem 2.1. *For any connected graph $G \notin F_{22}$ on p vertices, $2p(p+1) \leq \xi^c(G\overline{G}) \leq 3p(p+1)$. When $G \in F_{22}$, $\xi^c(G\overline{G})$ =$2p(p+1)$.*

Proof. For any connected graph G with $G \notin F_{22}$ of p vertices by Theorem 1.1, $r(G\overline{G}) = 2$ and $d(G\overline{G}) = 3$. So for any vertex v in $G\overline{G}$, $2 \leq ecc(v) \leq 3$.

$$\begin{aligned}
\text{Now, } \xi^c(G\overline{G}) &= \sum_{v\in V(G\overline{G})} deg(v)ecc(v)\\
&\geq 2 \sum_{v\in V(G\overline{G})} deg(v)\\
&\geq 4\left(\binom{p}{2}+p\right)\\
&\geq 4\left[\frac{p(p-1)}{2}+p\right]\\
&\geq 2p(p+1).\\
\text{Also, } \xi^c(G\overline{G}) &= \sum_{v\in V(G\overline{G})} deg(v)ecc(v)\\
&\leq 3 \sum_{v\in V(G\overline{G})} deg(v)\\
&\leq 3\left(\binom{p}{2}+p\right)\\
&\leq 3\left[\frac{p(p-1)}{2}+p\right]\\
&\leq 3p(p+1).
\end{aligned}$$

Hence $2p(p+1) \leq \xi^c(G\overline{G}) \leq 3p(p+1)$. When $G \in F_{22}$, $r(G\overline{G}) = 2$ and $d(G\overline{G}) = 2$ and hence $\xi^c(G\overline{G}) = 2p(p+1)$. □

Theorem 2.2. *For any connected graph G with $m \neq 1$ or $n \neq 1$, $2(m-n)q + np(p-1) + (m+n)^2 - (m+n) \leq \xi^c(G_{(m+n)}) \leq 3(m-n)q + \left(\frac{3np(p-1)}{2} + (m+n)^2 - (m+n)\right)$.*

Proof. When either $m \neq 1$ or $n \neq 1$, by Theorem 1.2, $r(G_{m+n}) = 2$ and $d(G_{m+n}) = 3$. So, for any vertex $v \in V(G_{m+n}), 2 \leq ecc(v) \leq 3$. The number of edges in G_{m+n} is

$$\begin{aligned}
|E(G_{m+n})| &= mq + n\left(\binom{p}{2} - q\right) + \binom{m+n}{2}\\
&= (m-n)q + \frac{1}{2}[np^2 - np + (m+n)^2 - (m+n)].
\end{aligned}$$

$$\text{Therefore, } \xi^c(G_{m+n}) = \sum_{v\in V(G_{m+n})} deg(v)ecc(v)$$

$$\geq 2 \sum_{v \in V(G_{m+n})} deg(v)$$

$$\geq 2(m-n)q + [np^2 - np + (m+n)^2 - (m+n)]$$

$$\text{Also, } \xi^c(G_{m+n}) = \sum_{v \in V(G_{m+n})} deg(v)ecc(v)$$

$$\leq 3 \sum_{v \in V(G_{m+n})} deg(v)$$

$$\leq 3(m-n)q + \frac{3}{2}[np^2 - np + (m+n)^2 - (m+n)]$$

Hence the result follows □

Theorem 2.3. *For any connected graph G with $m > 1$, $n > 1$ and $G \notin F_{22}$, $2(m-n)q + np(p-1) + mn \leq \xi^c(G_{m,n}) \leq 3(m-n)q + \frac{3}{2}[np(p-1) + mn]$.*

Proof. Since $m > 1$, $n > 1$ and $G \notin F_{22}$, by Theorem 1.3, $d(G_{m,n}) = 3$. This implies that $2 \leq ecc(v) \leq 3$. If G has q edges, then $\overline{G}$ has $\binom{p}{2} - q$ edges.

$$\text{So } \mid E(G_{m,n}) \mid = mq + n\left(\binom{p}{2} - q\right) + mn$$

$$= (m-n)q + \frac{n}{2}[p^2 - p + 2m].$$

$$\text{Hence, } \xi^c(G_{m,n}) = \sum_{v \in V(G_{m,n})} deg(v)ecc(v)$$

$$\geq 2 \sum_{v \in V(G_{(m,n)})} deg(v)$$

$$\geq 2(m-n)q + np(p-1) + mn.$$

$$\text{Also, } \xi^c(G_{m,n}) = \sum_{v \in V(G_{(m,n)})} deg(v)ecc(v)$$

$$\leq 3 \sum_{v \in V(G_{(m,n)})} deg(v)$$

$$\leq 3(m-n)q + \frac{3}{2}[np(p-1) + mn].$$

Hence the result follows. □

Theorem 2.4. *For any connected graph G of p vertices with $m > 1$, $\frac{mp}{2}(mp + 3m - 2) \leq \xi^c(G^p_{m,m}) \leq mp(mp + 3m - 2)$.*

Proof. For $m > 1$, by Theorem 1.4, $r(G^p_{m,m}) = m$ and $d(G^p_{m,m}) = 2m$. This implies that $m \leq ecc(v) \leq 2m$. If G has q edges, then $\overline{G}$ has $\binom{p}{2} - q$ edges.

$$\begin{aligned}
\text{So } \mid E(G_{m,m}) \mid &= mq + m\left(\binom{p}{2} - q\right) + p(2m-1)\\
&= \frac{mp^2 - mp}{2} + 2mp - p\\
&= \frac{p}{2}[mp + 3m - 2].\\
\text{Now, } \xi^c(G^p_{m,m}) &= \sum_{v\in G^p_{m,m}} deg(v)ecc(v)\\
&\leq 2m \sum_{v\in G^p_{m,m}} deg(v)\\
&\leq mp[mp + 3m - 2].\\
\text{Also, } \xi^c(G^p_{m,m}) &= \sum_{v\in G^p_{m,m}} deg(v)ecc(v)\\
&\geq m \sum_{v\in G^p_{m,m}} deg(v)\\
&\geq \frac{mp}{2}(mp + 3m - 2)
\end{aligned}$$

Thus the result follows. □

Theorem 2.5. *For any connected graph G with p vertices and even integer $m \geq 2$, $\xi^c(G^c_{m,m}) = \left(\frac{m+1}{2}\right)[mp^2 + 3mp]$.*

Proof. For $m \geq 2$, by Theorem 1.5, $r(G^c_{m,m}) = d(G^c_{m,m}) = 2m + 1$.

$$\begin{aligned}
\text{Also, } \mid E(G_{m,m}) \mid &= mq + m\left(\binom{p}{2} - q\right) + 2mp\\
&= mq + m[\frac{p(p-1)}{2} - q] + 2mp\\
&= \frac{mp^2 - mp}{2} + 2mp\\
&= \frac{1}{2}[mp^2 - mp + 4mp]\\
&= mp^2 + 3mp.\\
\text{Therefore, } \xi^c(G^c_{m,m}) &= \sum_{v\in V(G^c_{m,m})} deg(v)ecc(v)
\end{aligned}$$

$$= (m+1) \sum_{v \in G^c_{m,m}} deg(v)$$
$$= \frac{(m+1)}{2}[mp^2 + 3mp].$$

□

Theorem 2.6. *For any $m, n \geq 1$,*

$$\xi^c(K_m \times P_n) = \begin{cases} \frac{3m^2n^2+2m^2n+3mn^2-6mn-m^2-m}{4}, & \text{if } n \text{ is odd} \\ \frac{3m^2n^2+2m^2n+3mn^2-6mn}{4}, & \text{if } n \text{ is even.} \end{cases}$$

Proof. Let $v_{i,j}$, $1 \leq i \leq m$, $1 \leq j \leq n$ be the vertices of $K_m \times P_n$ where the induced subgraph of $\{v_{i,j} : 1 \leq i \leq m\}$ is the j^{th} copy of K_m and the induced subgraph of $\{v_{i,j} : 1 \leq j \leq n\}$ is the i^{th} copy of P_n. In this graph,

$$ecc(v_{i,j}) = \begin{cases} n-j+1, & 1 \leq j \leq \lceil \frac{n}{2} \rceil \\ ecc(v_{i,n-j+1}), & \lceil \frac{n}{2} \rceil + 1 \leq j \leq n \text{ for all } 1 \leq i \leq m. \end{cases}$$

$$deg(v_{i,1}) = m = deg(v_{i,n}), \text{ for all } 1 \leq i \leq m \text{ and}$$
$$deg(v_{i,j}) = m+1, 2 \leq j \leq n-1 \text{ and } 1 \leq i \leq m.$$

$$\xi^c(K_m \times P_n) = \sum_{v \in V(K_m \times P_n)} deg(v)ecc(v)$$
$$= 2\sum_{i=1}^{m} deg(v_{i,1})ecc(v_{i,1}) + \sum_{i=1}^{m}\sum_{j=2}^{n-1} deg(v_{i,j})ecc(v_{i,j})$$
$$= 2m\sum_{i=1}^{m} ecc(v_{i,1}) + (m+1)\sum_{i=1}^{m}\sum_{j=2}^{n-1} ecc(v_{i,j})$$
$$= 2m^2n + (m+1)\sum_{i=1}^{m}\sum_{j=2}^{n-1} ecc(v_{i,j}).$$

Case 1. n is odd.

In this case,

$$\sum_{i=1}^{m}\sum_{j=2}^{n-1} ecc(v_{i,j}) = 2\sum_{i=1}^{m}\sum_{j=2}^{\frac{n-1}{2}} ecc(v_{i,j}) + \sum_{i=1}^{m} ecc(v_{i,\frac{n+1}{2}})$$
$$= 2\sum_{i=1}^{m}\sum_{j=2}^{\frac{n-1}{2}}(n-j+1) + \sum_{i=1}^{m}\frac{n+1}{2}$$

$$= 2\sum_{i=1}^{m}\left[\sum_{j=2}^{\frac{n-1}{2}}(n+1) - \sum_{j=2}^{\frac{n-1}{2}} j\right] + m\left(\frac{n+1}{2}\right)$$
$$= 2m\left[\left(\frac{n-3}{2}\right)(n+1) - \frac{(\frac{n-1}{2})(\frac{n+1}{2})+1}{2}\right] + m\left(\frac{n+1}{2}\right)$$
$$= \frac{m}{4}[3n^2 - 6n - 1].$$

Therefore,$\xi^c(K_m \times P_n) = 2m^2n + (m+1)\frac{m}{4}(3n^2 - 6n - 1)$
$$= \frac{3m^2n^2 + 2m^2n + 3mn^2 - 6mn - m^2 - m}{4}.$$

Case 2. n is even.

In this case,

$$\sum_{i=1}^{m}\sum_{j=2}^{n-1} ecc(v_{i,j}) = 2\sum_{i=1}^{m}\sum_{j=2}^{\frac{n}{2}} ecc(v_{i,j})$$
$$= 2\sum_{i=1}^{m}\sum_{j=2}^{\frac{n}{2}}(n-j+1)$$
$$= 2\sum_{i=1}^{m}\left[\sum_{j=2}^{\frac{n}{2}}(n+1) - \sum_{j=2}^{\frac{n}{2}} j\right]$$
$$= 2m\left[\left(\frac{n}{2}-1\right)(n+1) - \frac{\frac{n}{2}(\frac{n}{2}+1)}{2} + 1\right]$$
$$= \frac{m}{4}[3n^2 - 6n].$$

Therefore, $\xi^c(K_m \times P_n) = 2m^2n + (m+1)\left[\frac{m}{4}(3n^2 - 6n)\right]$
$$= \frac{8m^2n + (m^2+m)(3n^2 - 6n)}{4}$$
$$= \frac{3m^2n^2 + 2m^2n + 3mn^2 - 6mn}{4}.$$

□

Theorem 2.7. *For any $m \geq 1$ and $n \geq 3$, $\xi^c(K_m \times C_n) = mn(m+1)(\lfloor \frac{n}{2} \rfloor + 1)$.*

Proof. In $K_m \times C_n$, $ecc(v) = \lfloor \frac{n}{2} \rfloor + 1$ and $deg(v) = m+1$, for all $v \in V(K_m \times C_n)$.

$$\begin{aligned} \text{Hence, } \xi^c(K_m \times C_n) &= \sum_{v \in V(K_m \times C_n)} deg(v)ecc(v) \\ &= (m+1) \sum_{v \in V(K_m \times C_n)} \left(\left\lfloor \frac{n}{2} \right\rfloor + 1 \right) \\ &= mn(m+1) \left(\left\lfloor \frac{n}{2} \right\rfloor + 1 \right). \end{aligned}$$

□

Theorem 2.8. $\xi^c(K_m \times K_n) = 2mn(m+n-2)$ *for any* $m, n \geq 2$.

Proof. Since $K_m \times K_n \in F_{22}$ and each vertex is of degree $m+n-2$, the result follows. □

Theorem 2.9. *Let* G *be the graph obtained by identifying a vertex of the cycles* C_m *and* C_n. *Then*

$$\xi^c(G) = \begin{cases} m^2 + n^2 + mn - m - n - 1 & \text{if } m \text{ and } n \text{ are odd} \\ m^2 + n^2 + mn - m & \text{if } m \text{ is odd and } n \text{ is even} \\ m^2 + n^2 + mn - n & \text{if } m \text{ is even and } n \text{ is odd} \\ m^2 + n^2 + mn & \text{if } m \text{ and } n \text{ is even.} \end{cases}$$

Proof. Let $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_n$ be the vertices of the cycles C_m and C_n respectively and u_1 and v_1 are identified as a single vertex in G. Assume that $m \leq n$. In G,

$$ecc(u_i) = \begin{cases} \lfloor \frac{n}{2} \rfloor + i - 1, & 1 \leq i \leq \lfloor \frac{m}{2} \rfloor + 1 \\ ecc(u_{m-i}), & \lfloor \frac{m}{2} \rfloor + 2 \leq i \leq m \end{cases} \quad \text{and}$$

$$ecc(v_i) = \begin{cases} \lfloor \frac{n}{2} \rfloor, & 2 \leq i \leq \lfloor \frac{n}{2} \rfloor - \lfloor \frac{m}{2} \rfloor + 1 \\ i + \lfloor \frac{m}{2} \rfloor - 1, & \lfloor \frac{n}{2} \rfloor - \lfloor \frac{m}{2} \rfloor + 2 \leq i \leq \lfloor \frac{n}{2} \rfloor + 1 \\ ecc(v_{n-i}), & \lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n. \end{cases}$$

$$\text{Also, } deg(u_i) = \begin{cases} 4, & i = 1 \\ 2, & 2 \leq i \leq m \end{cases} \quad \text{and}$$

$$deg(v_i) = 2, \; 2 \leq i \leq n$$

$$\text{Therefore, } \xi^c(G) = \sum_{v\in V(G)} deg(v)ecc(v)$$
$$= 4ecc(u_1) + 2\sum_{i=2}^{m} ecc(u_i) + 2\sum_{i=2}^{n} ecc(v_i).$$

Case 1. m and n are odd.

In this case,

$$\xi^c(G) = 4\left(\frac{n-1}{2}\right) + 4\sum_{i=2}^{\frac{m+1}{2}}\left(\frac{n-3}{2}+i\right) + 4\sum_{i=2}^{\frac{n-m}{2}+1}\left(\frac{n-1}{2}\right)$$
$$+ 4\sum_{i=\frac{n-m}{2}+2}^{\frac{n+1}{2}}\left(\frac{m-3}{2}+i\right)$$
$$= 4\left(\frac{n-1}{2}\right) + 4\left(\frac{m-1}{2}\right)\left(\frac{n-3}{2}\right) + 4\frac{(\frac{m+1}{2})(\frac{m+3}{2})}{2} - 4$$
$$+ 4\left(\frac{n-m}{2}\right)\left(\frac{n-1}{2}\right) + 4\left(\frac{m-1}{2}\right)\left(\frac{m-3}{2}\right)$$
$$+ 4\left[\frac{(\frac{n+1}{2})(\frac{n+3}{2})}{2} - \frac{(\frac{n-m}{2}+1)(\frac{n-m}{2}+2)}{2}\right]$$
$$= m^2 + n^2 + mn - m - n - 1.$$

Case 2. m is odd and n is even.

In this case,

$$\xi^c(G) = 4\left(\frac{n}{2}\right) + 4\sum_{i=2}^{\frac{m+1}{2}}\left(\frac{n-2}{2}+i\right) + 4\sum_{i=2}^{\frac{n-m+3}{2}}\left(\frac{n}{2}\right)$$
$$+ 4\sum_{i=\frac{n-m+5}{2}}^{\frac{n}{2}}\left[\left(\frac{m-3}{2}\right)+i\right] + 2\left(\frac{m+n-1}{2}\right)$$
$$= 2n + 4\left(\frac{m-1}{2}\right)\left(\frac{n-2}{2}\right) + 4\frac{(\frac{m+1}{2})(\frac{m+3}{2})}{2} - 4 + 4\left(\frac{n-m+1}{2}\right)\left(\frac{n}{2}\right)$$
$$+ 4\left(\frac{m-3}{2}\right)\left(\frac{m-3}{2}\right) + 4\left[\frac{(\frac{n}{2})(\frac{n+2}{2})}{2} - \frac{(\frac{n-m+3}{2})(\frac{n-m+5}{2})}{2}\right] + (n+m-1)$$
$$= m^2 + n^2 + mn - m.$$

Case 3. m is even and n is odd.

In this case,

$$\xi^c(G) = 4\left(\frac{n-1}{2}\right) + 4\sum_{i=2}^{\frac{m}{2}}\left(\left(\frac{n-3}{2}\right)+i\right) + 2\left(\frac{m+n-1}{2}\right) + 4\sum_{i=2}^{\frac{n-m+1}{2}}\left(\frac{n-1}{2}\right)$$
$$+ 4\sum_{i=\frac{n-m+2}{2}}^{\frac{n+1}{2}}\left(\frac{m-2}{2}+i\right)$$
$$= m^2 + n^2 + mn - n.$$

Case 4. m is even and n is odd.

In this case,

$$\xi^c(G) = 4\left(\frac{n}{2}\right) + 4\sum_{i=2}^{\frac{m}{2}}\left[\left(\frac{n-2}{2}\right)+i\right] + 2\left(\frac{m+n}{2}\right) + 4\sum_{i=2}^{\frac{n-m+2}{2}}\left(\frac{n}{2}\right)$$
$$+ 4\sum_{i=\frac{n-m+4}{2}}^{\frac{n}{2}}\left(\frac{m-2}{2}+i\right) + 2\left(\frac{m+n}{2}\right)$$
$$= m^2 + n^2 + mn.$$

□

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