Almost Unbiased Estimator of Finite Population Mean In Survey Sampling

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Abstract

For estimating the population mean $\overline{Y}$ of the variable under study y, an almost unbiased estimator has been proposed. The expression for the bias and mean square error of the proposed estimator has been obtained and it has been shown that proposed estimator is unbiased as well as equally efficient with the regression estimator. The comparison of proposed estimator with the regression estimator has also been made upto second order of approximation w.r.t. their biases and mean square errors. The results have also been illustrated numerically.

Key words : Regression estimator, bias, unbiased estimator, variance, mean square error

Mathematics Subject Classification : **62Dxx**

1. Introduction

When variable under study y and auxiliary variable x are highly correlated to each other then regression, ratio and ratio type estimators are generally used in survey sampling. More attention has been given to the ratio estimator because of its easy computation and applicability for general sampling designs. The ratio estimator provides an efficient estimate of the population mean if the regression of y on x is linear and the regression line passes through origin. It is well known that regression estimator is always efficient than other estimators when relationship between y and x is linear.

In literature, various authors viz Nath [5], Rao [7], Royal and Cumberland [8], Deng and Wu [1, 2] etc. have significant contribution for estimation of parameters in the use of regression method of estimation.

In the present paper, we have proposed a new estimator which is almost unbiased. The expressions for bias and mean square error have been obtained. The theoretical results have also been verified numerically by taking some empirical populations from the literature.

2. Notations and Results

Suppose a simple random sample of size n is drawn from a finite population of size N without replacement and observation on variable y and x are taken. The sample selected is divided randomly into k sub-samples of size m each such that n = mk.

Let Y_i and X_i be values of the variables y and x on the ith unit of the populations. The corresponding small letters denote the values in the sample. Suppose $y_{i'j}$ and $x_{i'j}$ denote the value on i' th unit in sample of size n' (=mk-m), obtained after omitting the jth subsample from the main sample. Then

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i \qquad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{y}_j = \frac{1}{n'}\sum_{i'=1}^{n'} y_{i'j} \;;\; j = 1, 2, \ldots, k \qquad \bar{x}_j = \frac{1}{n'}\sum_{i'=1}^{n'} x_{i'j} \;;\; j = 1, 2, \ldots, k$$

$$\mu_{\alpha.\beta} = (N-1)^{-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^{\alpha}(X_i - \bar{X})^{\beta}$$

$$S_X^{\,2} = (N-1)^{-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$$

$$S_{XY} = (N-1)^{-1}\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})$$

$$s_x^{\,2} = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$s_{xy} = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$\rho = \frac{\mu_{11}}{\sqrt{\mu_{20}\mu_{02}}} \qquad \beta = \frac{\mu_{11}}{\mu_{02}} \qquad t = k\left[1 - \frac{n'}{N}\right]$$

Defining

$$\varepsilon_1 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, \qquad \varepsilon_{1j} = \frac{\bar{y}_j-\bar{Y}}{\bar{Y}}; \qquad j = 1, 2, \ldots, k$$

$$\varepsilon_2 = \frac{\bar{x}-\bar{X}}{\bar{X}}, \qquad \varepsilon_{2j} = \frac{\bar{x}_j-\bar{X}}{\bar{X}}; \qquad j = 1, 2, \ldots, k$$

$$\varepsilon_3 = \frac{s_{xy}-S_{XY}}{S_{XY}}, \qquad \varepsilon_{3j} = \frac{s_{xyj}-S_{XY}}{S_{XY}}; \qquad j = 1, 2, \ldots, k$$

$$\varepsilon_4 = \frac{s_x^{\,2}-S_X^{\,2}}{S_X^{\,2}}, \qquad \varepsilon_{4j} = \frac{s_{xj}^{\,2}-S_X^{\,2}}{S_X^{\,2}}; \qquad j = 1, 2, \ldots, k$$

Then

$$E(\varepsilon_1) = E(\varepsilon_2) = E(\varepsilon_3) = E(\varepsilon_4) = 0$$

$$E(\varepsilon_{1j}) = E(\varepsilon_{2j}) = E(\varepsilon_{3j}) = E(\varepsilon_{4j}) = 0; \ \forall j$$

Upto second order of approximation

$$E\left[(\bar{x}-\bar{X})(s_{xy}-S_{XY})\right] = \frac{(N-n)}{nN}\mu_{12}$$

$$E\left[(\bar{x}-\bar{X})(s_x^{\,2}-S_X^{\,2})\right] = \frac{(N-n)}{nN}\mu_{03}$$

$$E\left[(\bar{x}-\bar{X})s_x^{\,4}\right] = \frac{(N-n)}{n^2(N-1)(N-2)}\left[(N-2n)\mu_{05} + \left\{2N(n-1) - \frac{2N(n-1)(4N-5n-2)}{n(N-3)}\right\}\mu_{02}\mu_{03}\right]$$

$$E\left[(\bar{x}-\bar{X})s_x^{\,6}\right] = \frac{3N(N-n)(n-1)}{n^3(N-1)(N-2)(N-3)}\left[(N-2n+1)\mu_{05}\mu_{02} + (N-n-1)\mu_{04}\mu_{03} + \left\{n(n-2) - \frac{N(n-2)(7N-8n-4)}{n(N-4)}\right\}\mu_{03}\mu_{02}^{\,2}\right]$$

$$E\left[(\bar{x}-\bar{X})^2 s_x^{\,2}\right] = \frac{(N-n)}{n^2(N-1)(N-2)}\left[(N-2n)\mu_{04} + \left\{N(n-1) - \frac{3N(n-1)(N-n-1)}{n(N-3)}\right\}\mu_{02}^{\,2}\right]$$

$$E\left[(\bar{x}-\bar{X})^2 s_x^{\,4}\right] = \frac{N(N-n)(n-1)}{n^3(N-1)(N-2)(N-3)}\left[(3N-5n+1)\mu_{04}\mu_{02} + 2(N-n-1)\mu_{03}^{\,2} + N\left\{(n-2) - \frac{6(n-2)(N-n-1)}{n(N-4)}\right\}\mu_{02}^{\,3}\right]$$

$$E\left[(\bar{y}-\bar{Y})^2 s_x^{\,2}\right] = \frac{N(N-n)}{n^2(N-1)(N-2)(N-3)}\left\{(N-2n+1)\mu_{22} + (Nn-2n-N+1)\mu_{02}\mu_{20} - 2(N-n-1)\mu_{11}^{\,2}\right\}$$

$$E\left[(\bar{x}-\bar{X})(\bar{y}-\bar{Y})s_{xy}\right] = \frac{N(N-n)}{n^2(N-1)(N-2)(N-3)}\left\{(N-2n+1)\mu_{22} + (N-n-1)\mu_{02}\mu_{20} + (n-2)(N-1)\mu_{11}^{\,2}\right\}$$

3. Proposed Estimators of Population Mean $\bar{Y}$

When information on population mean $\bar{X}$ of variable x is available in advance then we have proposed an estimator of population mean $\bar{Y}$ under the sampling design defined in section 2 as:

$$\hat{\bar{Y}}_{HG} = t\bar{y}_{lr} + \frac{(1-t)}{k}\sum_{j=1}^{k}\bar{y}_{lrj} \tag{3.1}$$

Where

$$\bar{y}_{lr} = \bar{y} + \frac{s_{xy}}{s_x^2}\left(\bar{x} - \bar{X}\right) \quad \text{and}$$

$$\bar{y}_{lrj} = \bar{y}_j + \frac{s_{xyj}}{s_{xj}^2}\left(\bar{x}_j - \bar{X}\right) \qquad ; \qquad j = 1, 2, \ldots, k$$

are regression estimators based on main sample of size n and sample of size n'obtained by omitting jth subsample from a main sample.

To find the bias and mean square errors of proposed estimator upto second order of approximation, we expand $\hat{\bar{Y}}_{HG}$ in terms of $\varepsilon's$ upto fourth degree of approximation.

$$\hat{\bar{Y}}_{HG} = \bar{Y}\left[1 + t\left\{\varepsilon_1 - \beta\frac{\bar{X}}{\bar{Y}}\left(\varepsilon_2 - \varepsilon_2\varepsilon_4 + \varepsilon_2\varepsilon_3 + \varepsilon_2\varepsilon_4^2 - \varepsilon_2\varepsilon_3\varepsilon_4 - \varepsilon_2\varepsilon_4^3 + \varepsilon_2\varepsilon_3\varepsilon_4^2\right)\right\} + \right.$$

$$\left.\frac{1-t}{k}\sum_{j=1}^{k}\left\{\varepsilon_{1j} - \beta\frac{\bar{X}}{\bar{Y}}\left(\varepsilon_{2j} - \varepsilon_{2j}\varepsilon_{4j} + \varepsilon_{2j}\varepsilon_{3j} + \varepsilon_{2j}\varepsilon_{4j}^2 - \varepsilon_{2j}\varepsilon_{3j}\varepsilon_{4j} - \varepsilon_{2j}\varepsilon_{4j}^3 + \varepsilon_{2j}\varepsilon_{3j}\varepsilon_{4j}^2\right)\right\}\right] \tag{3.2}$$

Taking expectation on both sides of (3.2) and by using the results of section 2, we get

$$E\left(\hat{\bar{Y}}_{HG}\right) = \bar{Y} + 3\beta\left[A\left\{\frac{tf_1}{n} + \frac{(1-t)f_1'}{n'}\right\} + \left\{\frac{tD}{n^2} + \frac{(1-t)D'}{n'^2}\right\}\right]$$

$$= \bar{Y} + \frac{3\beta}{n^2}\left\{tD - \frac{k^2 D'(t-1)}{(k-1)^2}\right\}$$

Which implies that

$$Bias\left(\hat{\bar{Y}}_{HG}\right) = E\left(\hat{\bar{Y}}_{HG}\right) - \bar{Y}$$

$$= \frac{3\beta}{n^2}\left\{tD - \frac{k^2 D'(t-1)}{(k-1)^2}\right\}$$

Where

$$f_1 = \frac{N-n}{N} \qquad f_2 = \frac{(N-n)(N-2n)}{N(N-2)}$$

$$f_3 = \frac{(N-1)(N-n)(4N-5n-2)}{N(N-2)(N-3)} \qquad f_4 = \frac{(N-1)(N-n)(N-n-1)}{N(N-2)(N-3)}$$

$$f_1' = \frac{N-n'}{N} \qquad f_2' = \frac{(N-n')(N-2n')}{N(N-2)}$$

$$f_3' = \frac{(N-1)(N-n')(4N-5n'-2)}{N(N-2)(N-3)} \qquad f_4' = \frac{(N-1)(N-n')(N-n'-1)}{N(N-2)(N-3)}$$

$$A = \frac{\mu_{12}}{S_{XY}} - \frac{\mu_{03}}{S_X^{\,2}} \qquad B = \frac{\mu_{14}}{S_{XY}} - \frac{\mu_{05}}{S_X^{\,2}}$$

$$C = \frac{\mu_{13}}{S_{XY}} - \frac{\mu_{04}}{S_X^{\,2}} \qquad D = \frac{f_2 B}{S_X^{\,2}} - 2f_3 A - \frac{f_4 C}{S_X^{\,4}}$$

$$D' = \frac{f_2' B}{S_X^{\,2}} - 2f_3' A - \frac{f_4' C}{S_X^{\,4}}$$

Theorem 3.1

Upto second order of approximation, the bias of the proposed estimator $\hat{\bar{Y}}_{HG}$ is given by:

$$Bias\left(\hat{\bar{Y}}_{HG}\right) = \frac{3\beta}{n^2}\left\{tD - \frac{k^2 D'(t-1)}{(k-1)^2}\right\} \tag{3.3}$$

When N>>n, then

$$Bias\left(\hat{\bar{Y}}_{HG}\right) = -\frac{3\beta k}{n^2(k-1)}\left\{\frac{B}{S_X^{\,2}} - 2A - \frac{C}{S_X^{\,4}}\right\} \tag{3.4}$$

Theorem 3.2

Upto the terms of O(n^{-2}) the variance of the proposed estimator $\hat{\bar{Y}}_{HG}$ is given by:

$$V(\hat{\bar{Y}}_{HG}) = \frac{1}{n}\left\{f_1\mu_{20}\left(1-\rho^2\right)\right\} - \frac{1}{n^2}\left\{t(t-2)\Delta - \frac{k^2(t-1)^2}{(k-1)^2}\Delta'\right\} \tag{3.5}$$

Where

$$\Delta = \frac{(N-n)\{4(N-2n)(N-3)+(N-1)(3N-5n+1)\}\beta^2}{(N-1)(N-2)(N-3)}\left(\frac{\mu_{13}}{S_{XY}}-\frac{\mu_{04}}{S_X^{\ 2}}\right)$$

$$-\frac{2(N-n)(N-1)(N-n-1)\beta^2}{(N-1)(N-2)(N-3)}\left(\frac{\mu_{12}^{\ 2}}{S_{XY}^{\ 2}}+\frac{3\mu_{03}^{\ 2}}{S_X^{\ 4}}+\frac{4\mu_{03}\mu_{12}}{S_X^{\ 2}S_{XY}}\right)$$

$$-\frac{4(N-1)(N-n)(N-n-1)\beta}{(N-1)(N-2)(N-3)}\frac{\mu_{12}}{S_X^{\ 2}}\left(\frac{\mu_{03}}{S_X^{\ 2}}-\frac{\mu_{12}}{S_{XY}}\right)+\frac{4(N-n)(N-2n)\beta}{N(N-2)}\left(\frac{\mu_{22}}{S_{XY}}-\frac{\mu_{13}}{S_X^{\ 2}}\right)$$

and Δ' is simply obtained by replacing n by n' in Δ

For N>>n

$$V(\hat{\bar{Y}}_{HG})=\frac{1}{n}\{\mu_{20}(1-\rho^2)\}+\frac{1}{n^2}\{2k\Delta_1\} \qquad (3.6)$$

Here

$$\Delta_1=5\beta^2\left(\frac{\mu_{04}}{S_X^{\ 2}}-\frac{\mu_{13}}{S_{XY}}\right)-2\beta^2\left(\frac{\mu_{12}^{\ 2}}{S_{XY}^{\ 2}}+\frac{3\mu_{03}^{\ 2}}{S_X^{\ 4}}+\frac{4\mu_{03}\mu_{12}}{S_X^{\ 2}S_{XY}}\right)-4\beta\frac{\mu_{12}}{S_X^{\ 2}}\left(\frac{\mu_{03}}{S_X^{\ 2}}-\frac{\mu_{12}}{S_{XY}}\right)+4\beta\left(\frac{\mu_{22}}{S_{XY}}-\frac{\mu_{13}}{S_X^{\ 2}}\right)$$

Proof

Variance of estimator $\hat{\bar{Y}}_{HG}$ is obtained as:

$$V\left(\hat{\bar{Y}}_{HG}\right)=V\left\{t\bar{y}_{lr}+\frac{(1-t)}{k}\sum_{j=1}^{k}\bar{y}_{lrj}\right\}$$

$$=t^2V(\bar{y}_{lr})+\frac{(1-t)^2}{k^2}V(\sum_{j=1}^{k}\bar{y}_{lrj})+\frac{2t(1-t)}{k}Cov(\bar{y}_{lr},\sum_{j=1}^{k}\bar{y}_{lrj}) \qquad (3.7)$$

Using the results of section 2 and after some algebraic simplifications, $V(\bar{y}_{lr})$, $V(\sum_{j=1}^{k}\bar{y}_{lrj})$ and $Cov(\bar{y}_{lr},\sum_{j=1}^{k}\bar{y}_{lrj})$ are obtained upto terms of O(n⁻²) are

$$V(\bar{y}_{lr})=\frac{f_1}{n}\{\mu_{20}(1-\rho^2)\}+\frac{1}{n^2}\{\Delta\} \qquad (3.8)$$

$$V(\sum_{j=1}^{k}\bar{y}_{lrj})=k^2\left\{\frac{f_1}{n}\{\mu_{20}(1-\rho^2)\}+\frac{1}{n'^2}\{\Delta'\}\right\} \qquad (3.9)$$

$$Cov(\bar{y}_{lr},\sum_{j=1}^{k}\bar{y}_{lrj})=k\left[\frac{f_1}{n}\{\mu_{20}(1-\rho^2)\}+\frac{1}{n^2}\{\Delta\}\right] \qquad (3.10)$$

Using the results of (3.8), (3.9) & (3.10) in (3.7), we get

$$V(\hat{\bar{Y}}_{HG})=\frac{1}{n}\{f_1\mu_{20}(1-\rho^2)\}-\frac{1}{n^2}\left\{t(t-2)\Delta-\frac{k^2(t-1)^2}{(k-1)^2}\Delta'\right\}$$

Hence the proof

To verify the results, we simply explain $V(\sum_{j=1}^{k}\bar{y}_{lrj})$ given in (3.9) as:

$$V(\sum_{j=1}^{k}\bar{y}_{lrj})=\sum_{j=1}^{K}V(\bar{y}_{lrj})+\sum_{j=1,j\neq j'}^{k}\sum_{j'=1}^{k}Cov(\bar{y}_{lrj},\bar{y}_{lrj'}) \qquad (3.11)$$

Using the results of section 2 and after simplification, we get

$$V(\sum_{j=1}^{k}\bar{y}_{lrj})=\frac{f_1'}{n'}\{\mu_{20}(1-\rho^2)\}+\frac{1}{n'^2}\{\Delta'\} \qquad (3.12)$$

$$Cov(\bar{y}_{lrj},\bar{y}_{lrj'})=\frac{1}{k-1}\left\{k\left(\frac{1}{n}-\frac{1}{N}\right)-\left(\frac{1}{n'}-\frac{1}{N}\right)\right\}\mu_{20}(1-\rho^2)+\frac{1}{n'^2}\Delta' \qquad (3.13)$$

Substitute the results from (3.12) and (3.13) in (3.11) and after some simplification, we get

$$V(\sum_{j=1}^{k}\bar{y}_{lrj})=k^2\left\{\frac{f_1}{n}\{\mu_{20}(1-\rho^2)\}+\frac{1}{n'^2}\{\Delta'\}\right\}$$

Remark 3.1.1

From the theorem (3.1), we see that the bias of the proposed estimator is of $O(n^{-2})$ so its contribution to mean square error will be of $O(n^{-4})$. Hence upto terms of $O(n^{-2})$;

$$V(\hat{\bar{Y}}_{HG})=MSE(\hat{\bar{Y}}_{HG})$$

4. Comparison

For comparing the proposed estimators with the linear regression estimator under the same sampling design, we first write the expressions of bias and mean square error of the regression estimator upto terms of $O(n^{-2})$ as:

$$Bias(\bar{y}_{lr})=3\beta\left\{\frac{f_1A}{n}+\frac{D}{n^2}\right\} \qquad (4.1)$$

$$MSE(\bar{y}_{lr})=\frac{f_1}{n}\{\mu_{20}(1-\rho^2)\}+\frac{1}{n^2}\{\Delta\} \qquad (4.2)$$

For N>>n

$$Bias(\bar{y}_{lr}) = 3\beta\left\{\frac{A}{n} + \frac{D_1}{n^2}\right\} \tag{4.3}$$

$$MSE(\bar{y}_{lr}) = \frac{1}{n}\left\{\mu_{20}\left(1-\rho^2\right)\right\} + \frac{1}{n^2}\left\{\Delta_1\right\} \tag{4.4}$$

Where

$$D_1 = \frac{B}{S_X^{\,2}} - 2A - \frac{C}{S_X^{\,4}}$$

From (3.4) and (4.3), we see that upto first order of approximation the proposed estimator is unbiased but linear regression estimator is biased. So upto second order of approximation, we have

$$\left|\frac{Bias(\hat{\bar{Y}}_{HG})}{Bias(\bar{y}_{lr})}\right| = \left|\frac{kD_1}{(k-1)(nA + D_1)}\right| < 1 \tag{4.5}$$

From (4.5), we see that upto second order of approximation bias of the proposed estimator will also be smaller than the linear regression estimator.

From (3.3) & (4.4):

$$\mathrm{MSE}\left(\hat{\bar{Y}}_{HG}\right) = MSE_1(\bar{y}_{lr}) + \frac{1}{\mathrm{n}^2}\left\{\Delta_1(2k-1)\right\} \tag{4.6}$$

The second term on the R.H.S. of (4.6) is negligible. It will tend to zero as sample size becomes large.

5. An Empirical Study:

To have the rough idea of the results obtained numerically, we have taken 4 natural populations considered in literature. The values of the parameters involved for each population are given in table1, Tables 2 & 3 give the biases and efficiencies of the estimators respectively.

Table 1: Descriptions of populations

Popln. No.	Source	N	y	x	ρ_{yx}	C_x	C_y
1	Murthy (1967) p. 127	128	Cultivated area (acre)	Area (acre)	0.82	0.62	0.57
2	Murthy (1967) p. 127	128	No. of cultivators	No. of persons	0.81	0.60	0.67
3	Murthy (1967) p. 127	128	No. of Cultivators	Area(Acre)	0.66	0.62	0.67
4	Murthy (1967) p. 178	108	area under winter paddy (ac)	geographical area (ac)	0.79	0.69	0.78

Table 2: Biases of the estimators upto terms of o(n^{-2})

Popln. No.	Sample Size	No. of Groups	$\frac{\lvert Bias \rvert}{3\beta}$		Relative Bias	
			$\bar{y}_{lr}$	$\hat{\bar{Y}}_{HG}$	$\bar{y}_{lr}$	$\hat{\bar{Y}}_{HG}$
1	90	2	4.09	0.20	100	4.89
1	90	3	4.09	0.15	100	3.67
1	90	5	4.09	0.12	100	2.93
2	90	2	3.27	0.60	100	18.35
2	90	3	3.27	0.45	100	13.76
3	90	2	1.08	0.79	100	73.15
3	90	3	1.08	0.59	100	54.63
3	90	5	1.08	0.49	100	45.37
4	60	2	0.12	0.09	100	75
4	60	3	0.12	0.07	100	58.33
4	60	4	0.12	0.06	100	50

Table 3: Efficiencies of estimators upto terms of $O(n^{-2})$ w.r.t. mean per unit estimator ($\bar{y}$)

Popln. No.	Sample Size	No. of Groups	Relative Efficiencies		
			$\bar{y}$	$\bar{y}_{lr}$	$\hat{\bar{Y}}_{HG}$
1	90	2	100	194	143
1	90	3	100	194	113
1	90	5	100	194	80
2	90	2	100	162	111
2	90	3	100	162	105
3	90	2	100	166	156
3	90	3	100	166	147
3	90	5	100	166	132
4	60	2	100	186	143
4	60	3	100	186	116
4	60	4	100	186	101

6. Conclusion

From tables 2 and 3, we see that the proposed estimator is almost unbiased as compared to linear regression estimator and equally efficient with it. From table 3, we also note that the performance of the proposed estimator is very good when sample is divided into utmost three sub-samples. Hence we conclude that proposed estimator can be recommended for use as compared to linear regression estimator for getting good results.

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