On Regular Pre-Semiclosed Sets In Bitopological Spaces

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Abstract

The generalized closed sets in point set topology have been found considerable interest among general topologists. Veerakumar[8] introduced and investigated pre-semi- closed sets and Anitha[1] introduced pgpr closed sets. P.Thangavelu[7] introduced and investigated the concept of regular pre-semiclosed sets in topological spaces. In this article the concept of regular pre-semiclosed sets and its relationships with other generalized sets are extended to bitopological spaces.

Keywords: (i,j) regular pre-semi closed, pairwise rps-continuous, pairwise pre rps-continuous, pairwise rps-irresolute, pairwise rps-closed, pairwise pre rps-closed ,RPSO-connected , pairwise RPSO-compact.

1.Introduction

Levine[4] introduced generalized closed (briefly g-closed) sets in topology. Researchers in topology studied several versions of generalized closed sets. In this paper the concept of regular pre-semiclosed (briefly rps-closed) set in bitopological spaces is introduced and their properties are investigated.

2.Preliminaries

Let (X,τ_1,τ_2) or simply X denotes a bitopological space. For any subset $A \subseteq X$, τ_i-int(A) and τ_i-cl(A) denote the interior and closure of a set A with respect to the topology τ_i, respectively. A^C denotes the complement of A in X unless explicitly stated.

We recall the following definitions.

Definition 2.1. A subset A of (X,τ_1,τ_2) is called

(i) (i,j)-semi-open [5] in (X,τ_1,τ_2) if there exists a τ_i -open set U with $U \subseteq A \subseteq \tau_j$ –cl(U).

(ii) (i,j) -pre-open [2] in (X,τ_1,τ_2) if there exists a τ_i -open set U with $A \subseteq U \subseteq \tau_j$ –cl(A).

(iii) (i,j) -semi-pre-open or (i,j) -β-open [3] in (X,τ_1,τ_2) if there exists an (i,j) -pre-open set U in (X,τ_1,τ_2) with $U \subseteq A \subseteq \tau_j$ –cl(U) that is if $A \subseteq \tau_j$-cl(τ_i-int(τ_j-clA)).

(iv) (i,j) -α-open [2], [6] in (X,τ_1,τ_2) if $A \subseteq \tau_i$-int (τ_j-cl(τ_i-intA)).

The complement of an (i,j) -semi-open set is (i,j) semi-closed. The (i,j) -pre-closed sets, (i,j) -semi-preclosed sets and (i,j) -α-closed sets will be analogously defined.

Theorem 2.2. The following results hold in a bitopological space.

(i) The union of an arbitrary collection of (i,j) -semi-open sets is (i,j) -semi-open;

(ii) The intersection of two (i,j) -semi-open sets is not (i,j) -semi-open;

(iii) The intersection of an arbitrary collection of (i,j)-semi-closed sets is (i,j)-semi-closed and

(iv) The union of two (i,j) -semi-closed sets is not (i,j)-semi-closed.

Jelic [2], Khedr et al. [3] and Sampath Kumar[6] respectively characterized (i,j) -pre-open sets, (i,j) semi-pre-open sets and (i,j) -α-open sets. The intersection of all (i,j) -semi-closed sub sets of (X,τ_1,τ_2) containing a subset A of X is the (i,j) -semi-closure of A, denoted by (i,j) -sclA. The union of all (i,j) –semiopen sets contained in A is called the (i,j) -semi-interior of A, denoted by (i,j) -sintA. The (i,j) -pre-closure, (i,j) semi-pre-closure, (i,j) -α-closure, (i,j) -pre-interior, (i,j) -semi-pre-interior and (i,j) -α-interior will be respectively denoted by (i,j) -pclA, (i,j) -spclA, (i,j) -αclA, (i,j) -pintA, (i,j) -spintA and (i,j) -αintA.

3. Regular pre-semiclosed sets in Bitopological Spaces

Definition 3.1: A subset A of a space(X,τ_1,τ_2) is called (i,j) regular pre-semi closed (briefly (i,j)-rps-closed) if τ_j-*spcl*A $\subseteq$U, whenever A$\subseteq$ U and U is τ_i-rg open in (X,τ_1,τ_2).

The class of all (i,j)-rps-closed sets in a bitopological space (X,τ_1,τ_2) is denoted by (i,j)-RPS-C(X,τ_1,τ_2)

Remark 3.2: The complement of (i,j)-rps-closed set is (i,j)-rps-open set.

Proposition 3.3 (i) Every τ_j--closed set is (i,j)-rps-closed.

(ii) Every τ_j-semi-pre-closed set is (i,j)-rps-closed.

(iii) Every (i,j)-pgpr-closed set is (i,j)-rps-closed.

(iv) Every τ_j-pre-closed set is (i,j)-rps-closed.

(v) Every τ_j-α-closed set is (i,j)-rps-closed.

(vi) Every τ_j-regular closed set is (i,j)-rps-closed.

The reverse implications are not true as shown in the examples:

Example 3.4: Consider the bitopological space (X,τ_1,τ_2) with $X=\{a,b,c,d\}$, $\tau_1=\{\phi,\{a\},X\}$ and $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},X\}$,then

(i) $\{a\}$ is (i,j)-rps-closed but not τ_j-closed.

(ii) $\{b,c,d\}$ is (i,j)-rps-closed but not τ_j-semi-pre-closed.

Example 3.5: Consider the bitopological space (X,τ_1,τ_2) with $X=\{a,b,c\}$, $\tau_1=\{\phi,\{a\},X\}$ and $\tau_2 = \{\phi,\{a\},\{c\},\{a,c\},X\}$,then $\{a\}$ and $\{c\}$ are(i,j)-rps-closed but not (i,j)-pgpr-closed,τ_j-pre-closed, τ_j-α-closed andτ_j-regular closed.

Proposition 3.6: (i) Every (i,j)-rps-closed set is (i,j)-pre-semi-closed.

(ii) Every (i,j)-rps-closed set is (i,j)-gspr-closed.

(iii) Every (i,j)-rps-closed set is (i,j)-gsp-closed.

Proof

(i) Let A be a (i,j)-rps-closed subset of a space (X,τ_1,τ_2). Let $A\subseteq U$ where U is τ_i-g-open. Since every τ_i-g-open set is τ_i-rg-open and since A is(i,j)-rps-closed , A is (i,j)-pre-semi-closed.

(ii) Let A be a (i,j)-rps-closed subset of a space (X,τ_1,τ_2) . Let $A\subseteq U$ and U is τ_i-regular-open. Since every τ_i-regular-open set is τ_i-rg open and since A is (i,j)-rps-closed, τ_j-$spcl A\subseteq$ U.Therefore A is (i,j)-gspr-closed.

(iii) Let A be a (i,j)-rps-closed subset of a space (X,τ_1,τ_2). Let $A\subseteq$ U and U is τ_i-open . Since every τ_i-open set is τ_i-g-open and since every τ_i-g-open set is τ_i-rg-open, τ_j-$spclA\subseteq$ U and hence A is (i,j)-gsp-closed.

The reverse implications are not true as shown in the examples:

Example 3.7 In Example 3.4, {a,b} is (i,j)-pre-semi-closed but not (i,j)-rps-closed.

In Example 3.5, {a,c} is (i,j)- gspr-closed and(i,j)-gsp-closed but not (i,j)-rps-closed.

The concept of (i,j)-gs-closed , (i,j)-g-closed, (i,j)-gp-closed, (i,j)-rg-closed, (i,j)-αg-closed, (i,j)-sg-closed, (i,j)-gα-closed, (i,j)-rwg-closed, (i,j)-wg-closed, (i,j)-gpr-closed sets are independent with the concept of (i,j)-rps-closed as shown in the following examples.

Example 3.8 Consider the bitopological space (X,τ_1,τ_2) with X={a,b,c}, $\tau_1=\{\phi,\{a\},X\}$ and $\tau_2=\{\phi,\{a,b\},X\}$,then {a,b} is (i,j)-gs-closed but not (i,j)-rps-closed and {a} is (i,j)-rps-closed but not (i,j)-gs-closed.

Example 3.9

In Example 3.5, {a} is (i,j)-rps-closed but not (i,j)-g-closed , (i,j)-gp-closed, (i,j)-αg-closed and (i,j)-wg-closed. Also {a,c} is (i,j)-g-closed, (i,j)-gp-closed, (i,j)-αg-closed and (i,j)-wg-closed but not(i,j)-rps-closed.

Example 3.10 Consider the bitopological space (X,τ_1,τ_2) with $X=\{a,b,c\}$, $\tau_1 =\{\phi,\{a\},\{c\},\{a,c\},X\}$ and $\tau_2 = \{\phi,\{a\},X\}$,then $\{c\}$ is (i,j)-rps-closed but not(i,j)-rg-closed and $\{a,b\}$ is (i,j)-rps-closed but not (i,j)-gα-closed .Also $\{a,c\}$ is (i,j)-rg-closed and (i,j)-gα-closed but not (i,j)-rps-closed.

Example 3.11 In Example 3.10 $\{a,b\}$ is (i,j)-rps-closed but not(i,j)-sg-closed. Consider the bitopological space (X,τ_1,τ_2) with $X=\{a,b,c\}$, $\tau_1 =\{\phi,\{c\},\{a,b\},X\}$ and $\tau_2 = \{\phi,\{a\},\{a,c\},X\}$,then $\{a,c\}$ is (i,j)-sg-closed but not (i,j)-rps-closed.

Example 3.12 In Example 3.5 $\{a,c\}$ is (i,j)-rwg-closed but not(i,j)-rps-closed.

In Example 3.10 $\{a,c\}$ is (i,j)-rps-closed but not(i,j)-rwg-closed.

4. Pairwise rps-continuous function

Definition 4.1 A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(a) pairwise semi-pre-continuous if $f^{-1}(U)$ is (i,j)-semi pre closed in X, for each σ_j -closed set U in Y .

(b) pairwise pgpr -continuous if $f^{-1}(U)$ is (i,j)- pgpr closed in X, for each σ_j -closed set U in Y.

(c) pairwise pre -continuous if $f^{-1}(U)$ is (i,j)-pre closed in X, for each σ_j -closed set U in Y.

(d) pairwise α -continuous if $f^{-1}(U)$ is (i,j)-α closed in X, for each σ_j -closed set U in Y.

(e) pairwise pre-semi-continuous if $f^{-1}(U)$ is (i,j)-pre-semi closed in X, for each σ_j -closed set U in Y.

(f) pairwise gspr -continuous if $f^{-1}(U)$ is (i,j)-gspr closed in X, for each σ_j -closed set U in Y.

(g) pairwise gsp continuous if $f^{-1}(U)$ is (i,j)-gsp closed in X, for each σ_j -closed set U in Y.

Definition 4.2 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise rps-continuous if $f^{-1}(U)$ is (i,j)-rps closed in X, for each σ_j -closed set U in Y.

Theorem 4.3 Every pairwise continuous function is pairwise rps-continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be pairwise continuous. Let U be a σ_j -closed set in Y . Then $f^{-1}(U)$ is τ_j-closed in X. Since every τ_j -closed set is (i,j)-rps closed , we have f is pairwise rps-continuous.

The converse of the above theorem need not be true in general. The next example supports our claim.

Example4.4: Consider an identity function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ with X=Y={a,b,c}, $\tau_1 = \{\phi,\{a\},X\}$, $\tau_2 = \{\phi,\{a\},\{c\},\{a,c\},X\}$, $\sigma_1 = \{\phi,\{a\},\{c\},\{a,c\},Y\}$ and $\sigma_2 = \{\phi,\{a,b\},Y\}$.Then {c} is pairwise rps-continuous but not pairwise continuous.

Since every τ_j-semi-pre-closed,(i,j)-pgpr-closed,τ_j-pre-closed,τ_j-α-closed sets are (i,j)-rps-closed, we have every pairwise semi-pre continuous, pairwise pgpr continuous , pairwise pre continuous, pairwise α-continuous mappings are pairwise rps-continuous. But the converse of the above need not be true.

Example4.5: Consider an identity function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ with X=Y={a,b,c,d}, $\tau_1 = \{\phi,\{a\},X\}$, $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},X\}$,$\sigma_1 = \{\phi,\{a\},\{c\},\{a,c\},Y\}$ and $\sigma_2 = \{\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,b,c\},Y\}$ then {b,c,d} is pairwise rps-continuous but not pairwise semi-pre continuous.

Example 4.6: Consider an identity function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ with $X=Y=\{a,b,c\}$, $\tau_1 =\{\phi,\{a\},X\}$, $\tau_2 = \{\phi,\{a\},\{c\},\{a,c\},X\}$, $\sigma_1 =\{\phi,\{a,c\},Y\}$ and $\sigma_2=\{\phi,\{b\},\{a,b\},\{b,c\},Y\}$, then $\{a\}$ and $\{c\}$ are pairwise rps-continuous but not pairwise pgpr continuous, pairwise pre continuous and pairwise α-continuous.

Since every (i,j)-rps-closed set is τ_j-pre-semi-closed , (i,j)-gspr-closed , (i,j)-gsp-closed , we have every pairwise rps-continuous mapping is pairwise pre-semi continuous, pairwise gspr continuous, pairwise gsp continuous. But the converse of the above need not be true.

Example 4.7: Consider an identity function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ with $X=Y=\{a,b,c,d\}$, $\tau_1 =\{\phi,\{a\},X\}$, $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},X\}$, $\sigma_1 =\{\phi,\{a,c\},Y\}$ and $\sigma_2=\{\phi,\{c\},\{a,c\},\{b,c\},\{a,b,c\},Y\}$, then $\{b\}$ is pairwise pre-semi continuous but not pairwise rps continuous.

Example 4.8 Consider an identity function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ with $X=Y=\{a,b,c\}$, $\tau_1 =\{\phi,\{a\},X\}$,$\tau_2 = \{\phi,\{a\},\{c\},\{a,c\},X\}$,$\sigma_1 =\{\phi,\{a,c\},Y\}$ and $\sigma_2=\{\phi,\{b\},Y\}$ then $\{a,c\}$ is pairwise gspr continuous and pairwise gsp continuous but not pairwise rps-continuous.

Theorem 4.9 The following are equivalent: For a function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$

(a) f is pairwise rps continuous.

(b) $f^{-1}(U)$ is (i,j) rps open for each σ_j-open set U in Y, $i \neq j$, $i, j = 1, 2$.

Proof. (a) $\Rightarrow$ (b): Suppose that f is pairwise rps continuous. Let A be σ_j –open in Y . Then A^C is σ_j -closed in Y . Since f is pairwise rps-continuous, we have $f^{-1}(A^C)$ is (i,j) rps closed in X, $i \neq j$ and $i, j = 1,2$. Consequently, $f^{-1}(A)$ is (i,j) rps open in X.

(b) $\Rightarrow$ (a) Suppose that $f^{-1}(U)$ is (i,j) rps open for each σ_i -open set U in Y ,i $\neq$j and i, j = 1,2. Let V be σ_j -closed in Y . Then V^C is σ_j -open in Y . Therefore, by our assumption, $f^{-1}(V^C)$ is(i,j) rps open in X, i $\neq$j and i, j = 1,2. Hence $f^{-1}(V)$ is (i,j) rps closed in X. This completes the proof.

Definition 4.10: A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is

(a) pairwise Pre semi-irresolute if $f^{-1}(U)$ is (i,j)- Pre semi closed in X for each (i,j)-Pre semi closed set U in Y , i$\neq$j and i, j = 1,2.

(b) pairwise gspr-irresolute if $f^{-1}(U)$ is (i,j)-gspr closed in X for each (i,j)- gspr closed set U in Y , i $\neq$j and i, j = 1,2.

(c) pairwise gsp-irresolute if $f^{-1}(U)$ is (i,j)-gsp closed in X for each (i,j)-gsp closed set U in Y , i $\neq$j and i, j = 1,2.

Definition 4.11: A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is pairwise rps-irresolute if $f^{-1}(U)$ is (i,j)-rps closed in X for each (i,j)- rps closed set U in Y , i $\neq$j and i, j = 1,2.

Concerning the composition of functions, we have the following.

Theorem 4.12: Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ be two functions. Then

(a) If f and g are pairwise rps-irresolute, then gof is pairwise rps-irresolute.

(b) If f is pairwise rps-irresolute and g is pairwise rps-continuous, then gof is pairwise rps-continuous.(c) If f is pairwise Pre semi-irresolute and g is pairwise rps-continuous, then gof is pairwise Pre semi -continuous.

(d) If f is pairwise gspr-irresolute and g is pairwise rps-continuous, then gof is pairwise gspr -continuous.

(e) If f is pairwise gsp-irresolute and g is pairwise rps-continuous, then gof is pairwise gsp-continuous.

(f) If f is pairwise rps-continuous and g is pairwise continuous, then gof is pairwise rps-continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be pairwise rps-irresolute. Let U be (i,j) -rps closed set in Z, , $i \neq j$ and i, j = 1,2. Since g is pairwise rps-irresolute, $g^{-1}(U)$ is (i,j) -rps closed in Y . Since f is pairwise rps-irresolute, $(gof)^{-1} = f^{-1}[g^{-1}(U)]$ is (i,j) -rps closed in X. Therefore, gof is pairwise rps-irresolute.

The proofs of (b)-(f) are similar.

But the composition of two pairwise rps-continuous functions is not a pairwise rps-continuous function in general as shown in the following example.

Example 4.13 Consider a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be identity functions with X=Y=Z={a,b,c}, $\tau_1 = \{\phi, \{a\}, X\}$,$\tau_2 = \{\phi, \{a\}, \{c\}, \{a,c\}, X\}$, $\sigma_1 = \{\phi, \{a\}, \{a,c\}, Y\}$ and $\sigma_2 = \{\phi, \{b\}, \{a,c\}, Y\}$, $\mu_1 = \{\phi, \{a\}, \{a,b\} \{a,c\}, Z\}$ and $\mu_2 = \{\phi, \{b\}, Z\}$ then for a μ_2-closed set {a,c}in Z, $(gof)^{-1}(\{a, c\}) = \{a, c\}$ is not (i,j) -rps closed in X. Hence gof is not pairwise rps-continuous.

Definition 4.14: A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise pre rps-continuous if $f^{-1}(U)$ is (i,j) -rps closed in X for each σ_j-semi closed set U in Y , $i \neq j$ and i, j = 1,2.

Obviously every pairwise pre rps-continuous function is pairwise rps-continuous.

But it is not reversible. It is shown in the following example.

Example 4.15 Consider a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be an identity function with X=Y={a,b,c}, $\tau_1 = \{\phi, \{a\}, \{c\}, \{a,c\}, X\}$,$\tau_2 = \{\phi, \{a\}, X\}$,$\sigma_1 = \{\phi, \{b\}, Y\}$ and $\sigma_2 = \{\phi, \{a\}, \{c\}, \{a,c\}, Y\}$ then f is pairwise rps-continuous but not pairwise pre rps-continuous ,since the σ_2 -semi closed set {a} in Y is not (i,j)-rps closed in X.

Definition 4.16 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise rps-closed if $f(U)$ is (i,j) -rps closed for each τ_j -closed set U in X, $i \neq j$ and $i, j = 1,2$.

Definition 4.17 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise pre rps-closed if $f(U)$ is (i,j)-rps closed for each τ_j -semi closed set U in X, $i \neq j$ and $i, j = 1,2$.

5. Pairwise RPSO-connected space

Definition 5.1 A bitopological space (X, τ_1, τ_2) is pairwise RPSO-connected if X cannot be expressed as the union of two nonempty disjoint sets A and B such that $[A \cap \tau_1\text{-rpscl}(B)] \cup [\tau_2\text{-rpscl}(A) \cap B] = \phi$.

Suppose X can be so expressed then X is called pairwise RPSO-disconnected and we write $X = A\backslash B$ and call this pairwise RPSO-separation of X.

Theorem 5.2 The following conditions are equivalent for any bitopological space.

(a) X is pairwise RPSO-connected.

(b) X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1 –rps open and B is τ_2-rps open.

(c) X contains no nonempty proper subset which is both τ_1 –rps open and τ_2-rps closed.

Proof. (a) $\Rightarrow$ (b) : Assume that X is pairwise RPSO-connected. Suppose that X can be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1 –rps open and B is τ_2-rps open. Then $A \cap B = \phi$. Consequently $A \subseteq B^C$. Then τ_2-rps $cl(A) \subseteq \tau_2$-rps $cl(B^C) = B^C$. Therefore, τ_2-rps $cl(A) \cap B = \phi$. Similarlywe can prove $A \cap \tau_1$ –rps $cl(B) = \phi$. Hence $[A \cap \tau_1$ –rps $cl(B)] \cup [\tau_2$-rps $cl(A) \cap B] = \phi$. This is a contradiction to the fact that X is pairwise RPSO-connected.

Therefore,X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1 –rps open and B is τ_2-rps open.

(b)$\Rightarrow$ (c) : Suppose that X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1 –rps open and B is τ_2-rps open. Suppose that X contains a nonempty proper subset A which is both τ_1 –rps open andτ_2-rps closed. Then $X = A \cup A^C$ where A is τ_1–rps open, A^C is τ_2-rps open and A, A^C are disjoint. This is the contradiction to our assumption. Therefore, X contains no nonempty proper subset which is both τ_1–rps open and τ_2-rps closed.

(c)$\Rightarrow$ (a) : Suppose that X contains no nonempty proper subset which is both τ_1–rps open and τ_2-rps closed. Suppose that X is pairwise RPSO-disconnected. Then X can be expressed as the union of two nonempty disjoint sets A and B such that $[A \cap \tau_1\text{–rps } cl(B)] \cup [\tau_2\text{-rps } cl(A) \cap B] = \phi$. Since $A \cap B = \phi$, we have $A = B^C$ and $B = A^C$. Since τ_2-rps $cl(A) \cap B = \phi$, we have τ_2-rps $cl(A) \subseteq B^C$. Hence τ_2-rps $cl(A) \subseteq A$. Therefore, A is τ_2-rps closed. Similarly, B is τ_1–rps closed. Since $A = B^C$, A is τ_1–rps open. Therefore, there exists a nonempty proper set A which is both τ_1–rps open and τ_2-rps closed. This is the contradiction to our assumption.Therefore, X is pairwise RPSO-connected.

Theorem 5.3 If A is pairwise RPSO-connected subset of a bitopological space (X, τ_1, τ_2) which has the pairwise RPSO-separation $X = C\backslash D$, then $A \subseteq C$ or $A \subseteq D$.

Proof. Suppose that (X, τ_1, τ_2) has the pairwise RPSO-separation $X = C\backslash D$. Then $X = C \cup D$ where C and D are two nonempty disjoint sets such that $[C \cap \tau_1\text{–rps } cl(D)] \cup [\tau_2\text{-rps } cl(C) \cap D] = \phi$. Since $C \cap D = \phi$, we have $C = D^C$ and $D = C^C$. Now, $[(C \cap A) \cap \tau_1\text{–rps } cl(D \cap A)] \cup [\tau_2\text{-rps } cl(C \cap A) \cap (D \cap A)] \subseteq [C \cap \tau_1\text{–rps } cl(D)] \cup [\tau_2\text{-rpscl}(C) \cap D] = \phi$. Hence $A = (C \cap A)\backslash(D \cap A)$ is pairwise RPSO-separation of A.Since A is pairwise RPSO-connected, we have either $(C \cap A) = \phi$ or $(D \cap A) = \phi$. Consequently, $A \subseteq C^C$ or $A \subseteq D^C$. Therefore, $A \subseteq C$ or $A \subseteq D$.

Theorem 5.4 If A is pairwise RPSO-connected and $A \subseteq B \subseteq \tau_1$–rps cl(A) $\cap \tau_2$-rps cl(A) then B is pairwise RPSO-connected.

Proof. Suppose that B is not pairwise RPSO -connected. Then $B = C \cup D$ where C and D are two nonempty disjoint sets such that $[C \cap \tau_1$–rps cl(D)] $\cup$ [τ_2-rps cl(C) $\cap D] = \phi$. Since A is pairwise RPSO-connected, we have $A \subseteq C$ or $A \subseteq D$. Suppose $A \subseteq C$. Then $D \subseteq D \cap B \subseteq D \cap \tau_2$-rps cl(A) $\subseteq D \cap \tau_2$-rps cl(C) $= \phi$. Therefore, $\phi \subseteq D \subseteq \phi$. Consequently, $D = \phi$. Similarly, we can prove $C = \phi$ if $A \subseteq D$ {by Theorem 5.3}.This is the contradiction to the fact that C and D are nonempty. Therefore, B is pairwise RPSO-connected.

Theorem 5.5 The union of any family of pairwise RPSO -connected sets having a nonempty intersection is pairwise RPSO -connected.

Proof. Let I be an index set and $i \in I$. Let $A = \cup A_i$ where each A_i is pairwise RPSO-connected with $\cap A_i \neq \phi$. Suppose that A is not pairwise RPSO-connected. Then $A = C \cup D$, where C and D are two nonempty disjoint sets such that $[C \cap \tau_1$–rps cl(D)] $\cup$ [τ_2-rps cl(C) $\cap D] = \phi$. Since A_i is pairwise RPSO-connected and $A_i \subseteq A$, we have $A_i \subseteq C$ or $A_i \subseteq D$. Therefore, $\cup(A_i) \subseteq C$ or $\cup(A_i) \subseteq D$. Hence, $A \subseteq C$ or $A \subseteq D$. Since $\cap A_i \neq \phi$, we have $x \in \cap A_i$. Therefore, $x \in A_i$ for all i. Consequently, $x \in A$. Therefore, $x \in C$ or $x \in D$. Suppose $x \in C$. Since $C \cap D = \phi$, we have $x \notin D$. Therefore, $A \not\subset D$. Therefore, $A \subseteq C$. Therefore, A is not pairwise RPSO-connected. This shows that A is pairwise RPSO-connected.

Theorem 5.6 Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise continuous bijective and pairwise pre semi closed. Then inverse image of a σ_i -rps closed set in Y is τ_i -rps closed set in X.

Theorem 5.7 Let f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise continuous bijective and pairwise pre semi closed function. Then the image of a pairwise RPSO-connected space under f is pairwise RPSO-connected.

Proof. Let f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be pairwise continuous surjection and pairwise pre semi closed. Let X is pairwise RPSO-connected. Suppose that Yis pairwise RPSO-disconnected. Then $Y = A \cup B$ where A is σ_1 rps open and B is σ_2 rps open in Y . Since f is pairwise continuous and pairwise pre semi closed, we have $f^{-1}(A)$ is τ_1 rps open and $f^{-1}(B)$ is τ_2 rps open in X. Also X $= f^{-1}(A) \cup f^{-1}(B)$, $f^{-1}(A)$ and $f^{-1}(B)$ are two nonempty disjoint sets. Then X is pairwise RPSO-disconnected. This is the contradiction to the fact that X is pairwise RPSO-connected. Therefore, Y is pairwise RPSO-connected.

6. Pairwise RPSO-compact space

Definition 6.1 A nonempty collection $\zeta = \{A_i, i \in I$, an index set$\}$ is called a pairwise rps-open cover of a bitopological space (X, τ_1, τ_2) if $X = \cup A_i$ and $\zeta \subseteq \tau_1$-RPSO$(X, \tau_1, \tau_2) \cup \tau_2$ -RPSO(X, τ_1, τ_2) and ζ contains at least one member of τ_1-RPSO(X, τ_1, τ_2) and one member of τ_2 -RPSO(X, τ_1, τ_2).

Definition 6.2 A bitopological space (X, τ_1, τ_2) is pairwise RPSO-compact if every pairwise rps-open cover of X has a finite subcover.

Definition 6.3 A set A of a bitopological space (X, τ_1, τ_2) is pairwise RPSO-compact relative to X if every pairwise rps-open cover of B has a finite subcover as a subspace.

Theorem 6.4 Every pairwise rps-compact space is pairwise compact.

Proof. Let (X, τ_1, τ_2) be pairwise RPSO-compact. Let $\zeta = \{A_i, i \in I$, an index set$\}$be a pairwise open cover of X. Then $X = \cup A_i$ and $\zeta \subseteq \tau_1 \cup \tau_2$ and ζ containsat least one member of τ_1 and one member of τ_2. Since every τ_i-open set is τ_i -rps open, we have $X = \cup A_i$ and $\zeta \subseteq \tau_1$-RPSO$(X, \tau_1, \tau_2) \cup \tau_2$ -RPSO(X, τ_1, τ_2) and ζ contains at least one member ofτ_1-RPSO(X, τ_1, τ_2) and one member of τ_2 -RPSO(X, τ_1, τ_2). Therefore, ζ is the pairwise rps-open cover of X. Since X is pairwise RPSO-compact, we have ζ has the finite subcover. Therefore, X is pairwise compact.

But the converse of the above theorem need not be true in general.

Theorem 6.5 Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise continuous, bijective and pairwise pre semi closed. Then the image of a pairwise RPSO-compact space under f is pairwise RPSO - compact.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be pairwise continuous surjection and pairwise pre semi closed. Let X be pairwise RPSO -compact. Let $\zeta = \{A_i, i \in I$, an index set$\}$ be a pairwise rps-open cover of Y . Then $Y = \cup A_i$ and $\zeta \subseteq \sigma_1$ -RPSO$(Y, \sigma_1, \sigma_2) \cup \sigma_2$ -RPSO(Y, σ_1, σ_2) and ζ contains at least one member of σ_1 -RPSO(Y, σ_1, σ_2) and one member of σ_2 -RPSO(Y, σ_1, σ_2).

Therefore, $X = f^{-1}[\cup(A_i)] = \cup f^{-1}(A_i)$ and $f^{-1}(\zeta) \subseteq \tau_1$-RPSO$(X, \tau_1, \tau_2) \cup \tau_2$ -RPSO(X, τ_1, τ_2) and $f^{-1}(\zeta)$ contains at least one member of τ_1-RPSO(X, τ_1, τ_2) and one member of τ_2 -RPSO(X, τ_1, τ_2). Therefore, $f^{-1}(\zeta)$ is the pairwise rps-open cover of X. Since X is pairwise RPSO-compact, we have $X = \cup f^{-1}(A_i)$, $i = 1$ to n. $\Rightarrow Y = f(X) = \cup(A_i)$, $i = 1$ to n.Hence, ζ has the finite subcover. Therefore, Y is pairwise RPSO-compact.

Theorem 6.6 If Y is τ_1- rps closed subset of a pairwise RPSO-compact space (X, τ_1, τ_2), then Y is τ_2-RPSO compact.

Proof. Let (X, τ_1, τ_2) be a pairwise RPSO-compact space. Let $\zeta = \{A_i, i \in I$, an index set$\}$ be a τ_2-rps open cover of Y . Since Y is τ_1- rps closed subset, Y^C is τ_1- rps open. Also $\zeta \cup Y^C = Y^C \cup \{A_i$, $i \in I$, an index set$\}$ be a pairwise rps-open cover of X. Since X is pairwise RPSO-compact, $X = Y^C \cup A_1 \cup \ldots \cup A_n$. Hence $Y = A_1 \cup \ldots \cup A_n$. Therefore, Y is τ_2-RPSO compact.

Since every τ_1-closed set is τ_1-rps closed, we have the following.

Theorem 6.7 If Y is τ_1-closed subset of a pairwise RPSO-compact space (X, τ_1, τ_2), then Y is τ_2-RPSO compact.

Theorem 6.8 If (X, τ_1) and (X, τ_2) are Hausdorff and (X, τ_1, τ_2) is pairwise RPSO-compact, then $\tau_1 = \tau_2$.

Proof. Let (X, τ_1) and (X, τ_2) be Hausdorff and (X, τ_1, τ_2) is pairwise RPSO-compact. Since every pairwise RPSO - compact space is pairwise compact, we have (X, τ_1) and (X, τ_2) are Hausdorff and (X, τ_1, τ_2) is pairwise compact. Let F be τ_1-closed in X. Then F^C is τ_1- open in X. Let $\zeta = \{A_i, i \in I$, an index set$\}$ be the τ_2-open cover for X. Therefore, $\zeta \cup F^C$ is the pairwise open cover for X. Since X is pairwise compact, $X = F^C \cup A1 \cup \ldots \cup An$. Hence $F = A1 \cup \ldots \cup An$. Hence F is τ_2-compact. Since (X, τ_2) is Hausdorf, we have F is τ_2-closed. Similarly, every τ_2 closed set is τ_1-closed. Therefore, $\tau_1 = \tau_2$.

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