PROPAGATION OF PLANE WAVES AT A LOOSELY BONDED INTERFACE BETWEEN VISCOUS LIQUID AND FLUID SATURATED POROUS SOLID HALF SPACE

Neelam Kumari

ABSTRACT

The reflection and transmission of plane waves from a loosely bonded interface between viscous liquid half space and a fluid saturated porous half space is presented. A longitudinal or a transverse wave impinges obliquely at the interface. Four relations among amplitudes of incidence, reflected and transmitted waves are obtained for a loosely bonded interface using suitable boundary conditions. The amplitude ratios have been computed numerically for a particular model and the results thus obtained are presented graphically. It is observed that the amplitude ratios depend not only on the angle of incidence of the incident wave, but also on material properties of the medium through which the waves traversed. Effect of bonding parameter is observed on amplitude ratios. A special cases with empty porous half space medium has been obtained from the present study.

I. INTRODUCTION

Porous medium is a subject of continued interest due to its importance in different branches of study like soil mechanics, material science, seismology, petroleum industry, geophysics, earth sciences, civil engineering and earthquake engineering etc. Many researchers made a lot of work on porous medium by taking different theories for the porous medium. Biot in 1941 proposed a general theory of three dimensional deformations of fluid saturated porous solids based on the work of Von Terzaghi (1923, 1925). Biot (1956a, 1956b, 1962) discussed the propagation of two dilatational waves

Assistant Professor, Department of Mathematics, Ch. Devi Lal University, Sirsa, 125055, India.
mail: neelamkumaricdlu@gmail.com

and one rotational elastic wave in fluid saturated porous solids. Many researchers worked by taking the Biot theory of Poroelasticity, e.g. Berryman (1980), Vashishth, Sharma and Gogna (1991), Kumar, Miglani and Garg (2002) and Sharma (2007) etc.

Biot's theory was based on the assumption of compressible constituents. But there are sufficient reasons for considering the fluid saturated porous constituents as incompressible. Bowen (1980) and de Boer and Ehlers (1990a, 1990b) developed a theory for incompressible fluid saturated porous medium based on the work of Fillunger model (1913). The assumption of incompressible constituents resembles the properties appearing in many porous media materials, which are of use in many branches of study stated above. Also, it avoids to the introduction of many complicated material parameters as in the Biot theory. Based on this theory, many researchers like de Boer & Didwania (2004), de Boer & Liu (1994), Tajuddin & Hussaini (2006) and Kumar et.al.(2011) Nath (2012), Nandal & Saini (2013), Kumari (2014) etc. studied some problems of wave propagation in fluid saturated porous media.

In the problems of wave propagation at the interface between two elastic half spaces, the contact between them is normally assumed to be welded. However, in certain situations, there are reasons for expecting that bonding is not complete. Murty in 1975 presented a theoretical model for reflection, transmission, and attenuation of elastic waves through a loosely bonded interface between two elastic solid half spaces by assuming that the interface behaves like a dislocation which preserves the continuity of stresses allowing a finite amount of slip. Vashisth and Gogna (1993), Kumar and Miglani (1996), Kumar and Singh (1997) etc. discussed the problems of reflection and transmission at the loosely bonded interface between two half spaces.

In the present study, to discuss the reflection and transmission of longitudinal and transverse waves at a loosely bonded interface between a viscous liquid half space and a fluid saturated porous half space, the porous media theory given by de Boer and Ehlers (1990) is used. The model considered is assumed to exist in the oceanic crust part of the earth and the propagation of wave through such a model will be of great use in the fields related to earth sciences. A special case of the problem is also solved by taking the fluid saturated porous half space to be empty porous solid. Amplitude ratios of various reflected and transmitted waves are computed for a particular model and the results are shown graphically to discuss them.

II. BASIC EQUATIONS

The equations governing the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence of body forces are given by de Boer and Ehlers (1990) as

$$\nabla.(\eta^S\dot{\mathbf{u}}^S + \eta^F\dot{\mathbf{u}}^F) = 0, \tag{1}$$

$$(\lambda^S + \mu^S)\nabla(\nabla.\mathbf{u}^S) + \mu^S\nabla^2\mathbf{u}^S - \eta^S\nabla p - \rho^S\ddot{\mathbf{u}}^S + S_v(\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S) = 0, \tag{2}$$

$$\eta^F\nabla p + \rho^F\ddot{\mathbf{u}}^F + S_v(\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S) = 0, \tag{3}$$

$$\mathbf{T}_E^S = 2\mu^S\mathbf{E}^S + \lambda^S(\mathbf{E}^S.\mathbf{I})\mathbf{I}, \tag{4}$$

$$\mathbf{E}^S = \frac{1}{2}(\text{grad}\,\mathbf{u}^S + \text{grad}^T\mathbf{u}^S), \tag{5}$$

where $\mathbf{u}^i, \dot{\mathbf{u}}^i, \ddot{\mathbf{u}}^i, \; i = F, S$ denote the displacement, velocity and acceleration of fluid and solid phases, respectively and p is the effective pore pressure of the incompressible pore fluid. ρ^S and $\rho^F$are the densities of the solid and fluid constituents, respectively. $\mathbf{T}_E^S$ is the effective stress in the solid phase and $\mathbf{E}^S$ is the linearized langrangian strain tensor. λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \tag{6}$$

In the case of isotropic permeability, the tensor S_v describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990) as

$$S_v = \frac{(\eta^F)^2 \gamma^{FR}}{K} I, \tag{7}$$

where γ^{FR} is the specific weight of the fluid and K is the Darcy's permeability coefficient of the porous medium and I stands for unit vector.

Assuming the displacement vector u^i $(i = F, S)$ as

$$u^i = (u^i, 0, w^i), \text{where} \quad i = F, S, \tag{8}$$

and hence equations (1)- (3) give the equations of motion for fluid saturated incompressible porous medium in the component form as

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{9}$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{10}$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{11}$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{12}$$

$$\eta^S \left[\frac{\partial^2 u^S}{\partial x \partial t} + \frac{\partial^2 w^S}{\partial z \partial t}\right] + \eta^F \left[\frac{\partial^2 u^F}{\partial x \partial t} + \frac{\partial^2 w^F}{\partial z \partial t}\right] = 0, \tag{13}$$

where

$$\theta^S = \frac{\partial (u^S)}{\partial x} + \frac{\partial (w^S)}{\partial z}. \tag{14}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \tag{15}$$

Using the Helmholtz decomposition of displacement vector, the displacement components u^i and w^i are related to the potential functions ϕ^i and ψ^i as

$$u^i = \frac{\partial \phi^i}{\partial x} + \frac{\partial \psi^i}{\partial z}, \quad w^i = \frac{\partial \phi^i}{\partial z} - \frac{\partial \psi^i}{\partial x}, \qquad i = F, S. \tag{16}$$

Using, (16) in equations (9) - (13), we obtain the following equations:

$$\nabla^2 \phi^S - \frac{1}{C^2}\frac{\partial^2 \phi^S}{\partial t^2} - \frac{S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\frac{\partial \phi^S}{\partial t} = 0, \tag{17}$$

$$\phi^F = -\frac{\eta^S}{\eta^F}\phi^S, \tag{18}$$

$$\mu^S \nabla^2 \psi^S - \rho^S \frac{\partial^2 \psi^S}{\partial t^2} + S_v\left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \tag{19}$$

$$\rho^F \frac{\partial^2 \psi^F}{\partial t^2} + S_v\left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \tag{20}$$

$$(\eta^F)^2 p - \eta^S \rho^F \frac{\partial^2 \phi^S}{\partial t^2} - S_v \frac{\partial \phi^S}{\partial t} = 0, \tag{21}$$

where

$$C = \sqrt{\frac{(\eta^F)^2(\lambda^S + 2\mu^S)}{(\eta^F)^2\rho^S + (\eta^S)^2\rho^F}}. \tag{22}$$

Also, the normal and tangential stresses in the solid phase takes the form,

$$t_{zz}{}^S = \lambda^S\left(\frac{\partial^2 \phi^S}{\partial x^2} + \frac{\partial^2 \phi^S}{\partial z^2}\right) + 2\mu^S\left(\frac{\partial^2 \phi^S}{\partial z^2} - \frac{\partial^2 \psi^S}{\partial x\,\partial z}\right), \tag{23}$$

$$t_{zx}{}^S = \mu^S\left(2\frac{\partial^2 \phi^S}{\partial x \partial z} + \frac{\partial^2 \psi^S}{\partial z^2} - \frac{\partial^2 \psi^S}{\partial x^2}\right). \tag{24}$$

The time harmonic solution of the system of equations (17)- (21) can be considered as

$$(\phi^S, \phi^F, \psi^S, \psi^F, p) = (\phi_1{}^S, \phi_1{}^F, \psi_1{}^S, \psi_1{}^F, p_1)\exp(i\omega t). \tag{25}$$

where ω is the complex circular frequency.

Making use of (25) in equations (17)-(21), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{C_1^2} - \frac{i\omega S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\right]\phi_1^S = 0, \tag{26}$$

$$[\mu^S\nabla^2 + \rho^S\omega^2 - i\omega S_v]\psi_1^S = -i\omega S_v\psi_1^F, \tag{27}$$

$$[-\omega^2\rho^F + i\omega S_v]\psi_1^F - i\omega S_v\psi_1^S = 0, \tag{28}$$

$$(\eta^F)^2 p_1 + \eta^S\rho^F\omega^2\phi_1^S - i\omega S_v\phi_1^S = 0, \tag{29}$$

$$\phi_1^F = -\frac{\eta^S}{\eta^F}\phi_1^S, \tag{30}$$

Equation (26) represents the propagation of a longitudinal wave with velocity V_1, give

$$V_1^2 = \frac{1}{G_1}, \tag{31}$$

where $$G_1 = \left[\frac{1}{C_1^2} - \frac{iS_v}{\omega(\lambda^S + 2\mu^S)(\eta^F)^2}\right]. \tag{32}$$

From equations (27) and (28), we get

$$\left[\nabla^2 + \frac{\omega^2}{V_2^2}\right]\psi_1^S = 0. \tag{33}$$

Equation (33) corresponds to the propagation of a transverse wave with velocity V_2, g

$$V_2^2 = \frac{1}{G_2},$$

where

$$G_2 = \left\{\frac{\rho^S}{\mu^S} - \frac{iS_v}{\mu^S\omega} - \frac{S_v^2}{\mu^S(-\rho^S\omega^2 + i\omega S_v)}\right\}. \tag{34}$$

Further, following Fehler (1982) the equations of motion for a viscous liquid medium in terms of the potential functions ϕ and ψ corresponding to longitudinal and transverse waves are as

$$k^*\nabla^2\phi^* + \frac{4}{3}\eta\frac{\partial}{\partial t}\nabla^2\phi^* = \rho^*\frac{\partial^2\phi^*}{\partial t^2}, \qquad \eta\frac{\partial}{\partial t}\nabla^2\psi^* = \rho^*\frac{\partial^2\psi^*}{\partial t^2}, \tag{35}$$

where k^* is the bulk modulus, ρ^* is the fluid density, η is the fluid viscosity. The components of displacements and stresses are given by

$$u^* = \frac{\partial\phi^*}{\partial x} - \frac{\partial\psi^*}{\partial z}, \qquad w^* = \frac{\partial\phi^*}{\partial z} + \frac{\partial\psi^*}{\partial x}, \qquad t_{zx}^* = \eta\frac{\partial}{\partial t}\left[2\frac{\partial^2\phi^*}{\partial x\partial z} + \frac{\partial^2\psi^*}{\partial x^2} - \frac{\partial^2\psi^*}{\partial z^2}\right], \tag{36}$$

$$t_{zz}^* = \left[k^* - \frac{2}{3}\eta\frac{\partial}{\partial t}\right]\left[\frac{\partial^2\phi^*}{\partial x^2} + \frac{\partial^2\phi^*}{\partial z^2}\right] + 2\eta\frac{\partial}{\partial t}\left[\frac{\partial^2\psi^*}{\partial x\partial z} - \frac{\partial^2\phi^*}{\partial z^2}\right]. \tag{37}$$

III. FORMULATION OF THE PROBLEM.

A viscous liquid half space medium M_2 lying over an incompressible fluid saturated porous half space medium M_1 is considered. Taking the Cartesian coordinate system in such a way that the interface between the two half-spaces represents the xy-plane and z-axis is pointed vertically downward in the lower half space medium M_1, so the medium M_1 is represented as $z \geq 0$ and medium M_2 as $z \leq 0$. A plane wave (P or SV-wave) propagates through the half space medium M_1 and incident at the plane interface z=0, making an angle θ_0 with normal to the surface. Corresponding to each incident wave (P or SV-wave), we get reflected P and SV- waves in the medium M_1 and transmitted P and SV-waves in medium M_2. The problem considered is a two dimensional problem with $\frac{\partial}{\partial y} \equiv 0$

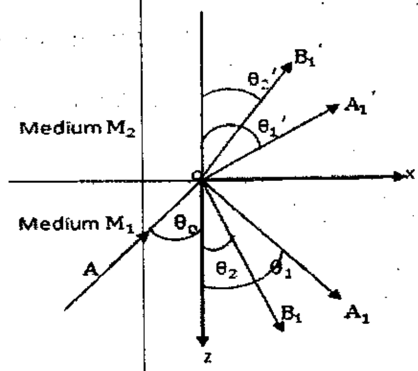


Fig.1 Geometry of the problem

IV. SOLUTION OF THE PROBLEM

The potential function solution of the equations (17)-(21) can be taken as

$$\{\phi^s,\phi^F,p\}=\{1,m_1,m_2\}[A_{01}\exp\{ik_1(x\sin\theta_0+z\cos\theta_0)+i\omega_1 t\}+A_1\exp\{ik_1(x\sin\theta_1+z\cos\theta_1)+i\omega_1 t\}], \tag{38}$$

$$\{\psi^s,\psi^F\}=\{1,m_3\}[B_{01}\exp\{ik_2(x\sin\theta_0-z\cos\theta_0)+i\omega_2 t\}+B_1\exp\{ik_2(x\sin\theta_2+z\cos\theta_2)+i\omega_2 t\}], \tag{39}$$

where

$$m_1=-\frac{\eta^s}{\eta^F},\quad m_2=-\left[\frac{\eta^s\omega_1{}^2\rho^F-i\omega_1 S_v}{(\eta^F)^2}\right],\quad m_3=\frac{i\omega_2 S_v}{i\omega_2 S_v-\omega_2{}^2\rho^F} \tag{40}$$

A_{01} and B_{01} are the amplitudes of the incident P and SV-waves, respectively, and A_1, B_1 are the amplitudes of the reflected P and SV-waves, respectively.

Following Kumar & Tomar (2001), the solution of the system of equation (35) is taken in the form

$$\phi^* = A_1{}^*\exp\left\{-i\left(K^{*2}-k_1{}^{*2}\sin^2\theta_1{}'\right)^{\frac{1}{2}}z\right\}\exp\left\{i\left(\omega_1{}^*t+k_1{}^*x\sin\theta_1{}'\right)\right\}, \tag{41}$$

$$\psi^* = B_1^* \exp\left\{-i\left(\frac{i\omega}{\nu} + k_2^{*2}\sin^2\theta_2'\right)^{\frac{1}{2}} z\right\}\exp\{i(\omega_2^* t + k_2^* x \sin\theta_2')\}, \tag{42}$$

where A_1^* and B_1^* are the amplitudes of transmitted P- and SV-waves.

Also

$$K^* = \frac{\omega}{c^*}\left(1 + \frac{4}{3}\frac{i\omega\nu}{c^{*2}}\right)^{-1/2}, \quad c^{*2} = \frac{k^*}{\rho^*}, \quad \nu = \frac{\eta}{\rho^*}, \tag{43}$$

k_1^* and k_2^* are the wave numbers of transmitted P- and SV-waves, respectively.

V. BOUNDARY CONDITIONS

The appropriate boundary conditions at the interface z=0 are the continuity of displacements and stresses. Mathematically, these boundary conditions can be expressed as:

$$t_{zz}^S - p = t_{zz}^*, \quad t_{zx}^S = t_{zx}^*, \quad w^s = w^*, \quad t_{zx}^* = (u^s - u^*) \tag{44}$$

where $K_t = ik\mu\tau$ and $\tau = \gamma/(1-\gamma)\sin\theta_0$

γ is bonding constant. $0 \le \gamma \le 1$. $\gamma = 0$ corresponds to smooth surface and $\gamma = 1$ corresponds to a welded interface

Snell's law:

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_1'}{V_1'} = \frac{\sin\theta_2'}{V_2'}, \tag{45}$$

where

$$V_1^* = \left[\frac{k^*}{\rho^*}\left(1 + \frac{4}{3}\frac{i\omega\eta}{k^*}\right)\right]^{1/2}, \quad V_2^* = \left[\frac{i\omega\eta}{\rho^*}\right]^{\frac{1}{2}}. \tag{46}$$

Also

$$k_1V_1 = k_2V_2 = k_1^*V_1^* = k_2^*V_2^* = \omega \text{ at } z = 0, \tag{47}$$

For incident longitudinal P-wave, we take

$$V_0 = V_1, \quad \theta_0 = \theta_1, \tag{48}$$

For incident transverse SV-wave, we have

$$V_0 = V_2, \quad \theta_0 = \theta_2, \tag{49}$$

For incident longitudinal wave at the interface z=0, we put $B_{01} = 0$ in equation (39) and for incident transverse wave, we put $A_{01} = 0$ in equation (38). Now substitute the expressions of potentials given by (38)-(39) in equations (16) and then in (23)-(24), and also substitute (41)-(42) in (36)-(37). Then using all these in boundary conditions (44), we get a system of four non homogeneous equations which can be written as

$$\sum_{j=1}^{4} a_{ij} Z_j = Y_i, \quad (i = 1,2,3,4), \tag{50}$$

where

$$Z_1 = \frac{A_1}{A}, \quad Z_2 = \frac{A_2}{A}, \quad Z_3 = \frac{A_1^-}{A}, \quad Z_4 = \frac{B_1^-}{A}, \tag{51}$$

where A is the amplitude of incident wave.

Coefficients a_{ij} in non dimensional form are obtained as

$$a_{11} = \frac{\lambda^S}{\mu^S} + 2\cos^2\theta_1 + \frac{m_2}{\mu^S k_1^{\,2}}, \qquad a_{12} = -2\sin\theta_2\cos\theta_2 \frac{k_2^{\,2}}{k_1^{\,2}},$$

$$a_{13} = \frac{1}{\mu^S k_1^{\,2}}\left[K^{*2}\left(\frac{8}{3} i\,\eta\,\omega_1^{\,*} - k^*\right) - 2i\,\eta\,\omega_1^{\,*} k_1^{\,*2} \sin^2\theta_1'\right]$$

$$a_{14} = 2i\eta\,\omega_2^{\,*}\, k_2^{\,*} \sin\theta_2' \left(\frac{i\omega}{v} + k_2^{\,*2} \sin^2\theta_2'\right)^{1/2}$$

$$a_{21} = 2\sin\theta_1\cos\theta_1, \qquad a_{22} = \frac{k_2^{\,2}}{k_1^{\,2}}(\cos^2\theta_2 - \sin^2\theta_2),$$

$$a_{23} = \frac{1}{\mu^S k_1^{\,2}}\left[2i\,\eta\,\omega_1^{*}\,k_1^{*}\sin\theta_1'\left(K^{*2} - k_1^{*2}\sin^2\theta_1'\right)^{1/2}\right], \quad a_{24} = \frac{1}{\mu^S k_1^{\,2}}\left[i\,\eta\,\omega_2^{*}\left(\frac{i\omega}{v}\right)\right]$$

$$a_{31} = \cos\theta_1, \quad a_{32} = -\frac{k_2}{k_1}\sin\theta_2, \quad a_{33} = \frac{1}{k_1}\left(K^{*2} - k_1^{*2}\sin^2\theta_1'\right)^{1/2}, \quad a_{34} = -\frac{k_2^{*}}{k_1}\sin\theta_2'$$

$$a_{41} = k_t\, i\sin\theta_1, \quad a_{42} = \frac{i\,k_t\,k_2}{k_1}\cos\theta_2,$$

$$a_{43} = -\frac{i\,k_t\,k_1^{*}}{k_1}\sin\theta_1' - \frac{1}{k_1}\left[2i\,\eta\,\omega_1^{*}\,k_1^{*}\sin\theta_1'\left(K^{*2} - k_1^{*2}\sin^2\theta_1'\right)^{1/2}\right],$$

$$a_{44} = -\frac{i\,k_t}{k_1}\left(\frac{i\omega}{v} + k_2^{*2}\sin^2\theta_2'\right)^{\frac{1}{2}} - \frac{1}{k_1}\left[i\,\eta\,\omega_2^{*}\left(\frac{i\omega}{v}\right)\right]. \qquad (52)$$

For incident longitudinal wave, we have

$$A = A_{01}, Y_1 = -a_{11}, Y_2 = a_{21}, Y_3 = a_{31}, Y_4 = -a_{41}, \qquad (53)$$

For incident transverse wave, we have

$$A = B_{01}, Y_1 = a_{12}, Y_2 = -a_{22}, Y_3 = -a_{32}, Y_4 = a_{42}, \qquad (54)$$

VI. SPECIAL CASE

If pores are absent or gas is filled in the pores then ρ^F is very small as compared to ρ^S and can be neglected, so the relation (22) gives us

$$C = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. \qquad (55)$$

and the coefficients a_{11} in (52) changes to

$$a_{11} = \frac{\lambda^S}{\mu^S} + 2\cos^2\theta_1, \qquad (56)$$

and the remaining coefficients in (52) remain same. In this situation the problem reduces to the problem of viscous liquid half space over empty porous solid half space.

VII. NUMERICAL RESULTS AND DISCUSSION

The theoretical results obtained above indicate that the amplitude ratios Z_i $(i = 1,2,3)$ depend on the angle of incidence of incident wave and material properties. The behaviour of various amplitude ratios are observed through numerical computations by considering a particular model, for which the numerical values of the parameters are taken as under:-

In medium M_1, the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu [10] as

$$\eta^S = 0.67, \quad \eta^F = 0.33, \quad \rho^S = 1.34\ \mathrm{Mg/m^3}, \quad \rho^F = 0.33\ \mathrm{Mg/m^3}, \quad \lambda^S = 5.5833\ \mathrm{MN/m^2},$$

$$K^F = 0.01\mathrm{m/s}, \quad \gamma^{FR} = 10.00\mathrm{KN/m^2}, \quad \mu^S = 8.3750\mathrm{N/m^2}, \quad \omega^* = 10/\mathrm{s},$$

In medium M_2,

$$k^* = 0.119 \times 10^{11}\ \mathrm{dyne/cm^2}, \qquad \rho^* = 1.01\mathrm{g/cm^3}, \eta = 0.0014\ \text{poise}$$

Using MATLAB, a computer programme has been developed and modulus of amplitude ratios $|Z_i|$, $(i = 1,2,3,4,)$ for various reflected and transmitted waves have been computed. $|Z_1|$ and $|Z_2|$ represent the modulus of amplitude ratios for reflected P and reflected SV-wave respectively. Also, $|Z_2|$ and $|Z_4|$ represent the modulus of amplitude ratios for transmitted P and transmitted SV-wave respectively. Dashed dotted line represents the variations of the amplitude ratios for bonding constant $\gamma = 0$, dotted line correspond $\gamma = 0.25$, dashed line for $\gamma = 0.5$, solid line for $\gamma = 0.75$ and bold dotted line for $\gamma = 1$ in all the figures (2)-(17) w.r.t. angle of incidence of the incident P or SV-wave. The variations in all the figures are shown for the range $0^0 \le \theta \le 90^0$.

Incident P-wave

Figures (2)-(5) represent the variations of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of incident P-wave. The behaviour of all these distribution curves for reflected P-wave and for transmitted P-wave is opposite. For reflected SV–wave and transmitted SV-wave, the behaviour of all curves is also same i.e. Increasing from normal incidence to maximum value and then decreasing from maximum value to grazing incidence. Figures (6)-(9) show the variations of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of incident P-wave in special case. The effect of fluid filled in the pores of fluid saturated porous medium is clear by comparing the maximum values of corresponding amplitude ratio in figures (2)-(5) and (6)-(9). For reflected SV–wave and transmitted SV-wave, the behaviour of all curves is same. Also in the figures (6)-(9), the values corresponding to bonding parameter $\gamma = 1$ i.e., for welded interface are large in comparison to other interface parameters except reflected P-wave.

Incident SV-wave

Figures (10)-(17) show the variations of the amplitude ratios for reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of the incident SV-wave. The behaviour of all these curves in figures (10)and(12) is same i.e. Increasing from normal incidence to maximum value and then decreasing from maximum value to grazing incidence.. In the figures (11)-(13), the pattern of all these curves is almost same. Also in the figures (10)-(13), the values corresponding to bonding parameter $\gamma = 1$, i.e., for welded interface are large in comparison to other interface parameters. The behaviour of all these curves in figures (14)and(16) is same i.e. Increasing from normal incidence to maximum value and then decreasing from maximum value to grazing incidence.. In the figures (15)-(17), the pattern of all these curves is same. Also in the figures (10)-(13), the values corresponding to bonding parameter $\gamma = 1$ i.e., for welded interface are large in comparison to other interface parameters except transmitted P-wave .The effect of fluid filled in the pores of fluid saturated porous medium is clear by comparing the maximum values of corresponding amplitude ratio in figures (10)-(13) and (14)-(17).

VIII. CONCLUSION

Reflection and transmission phenomenon of incident elastic waves at a loosely interface between viscous liquid half space and fluid saturated porous half space has been studied when P-wave or SV-wave is incident. It is observed that the amplitudes ratios of various reflected and transmitted waves depend on the angle of incidence of the incident wave and material properties. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on amplitudes ratios. Effect of bonding parameter is observed on amplitude ratios.

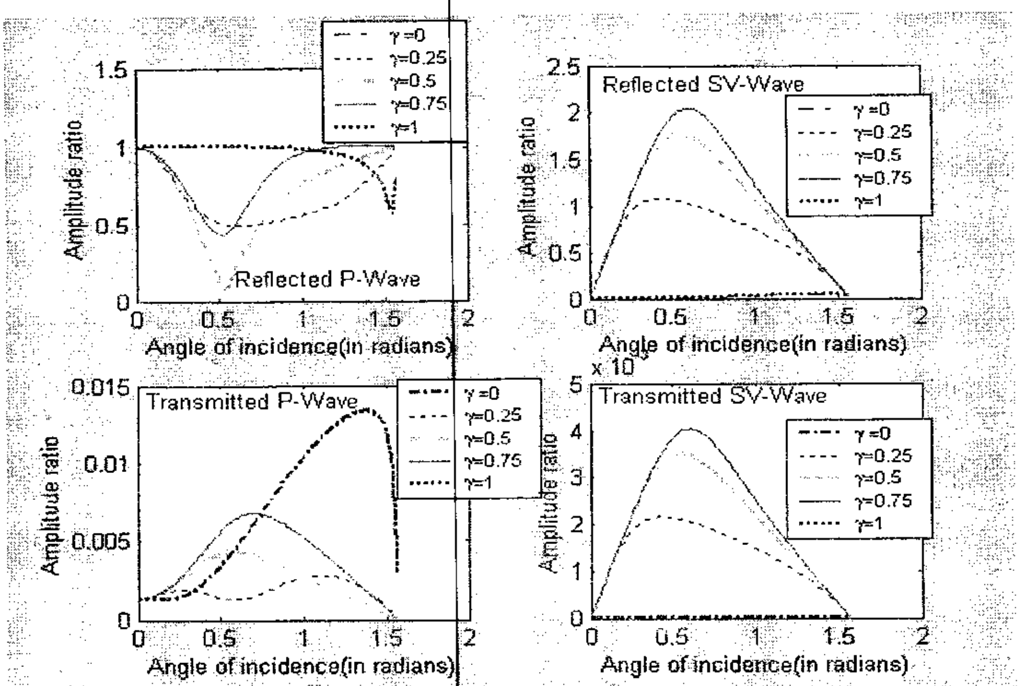
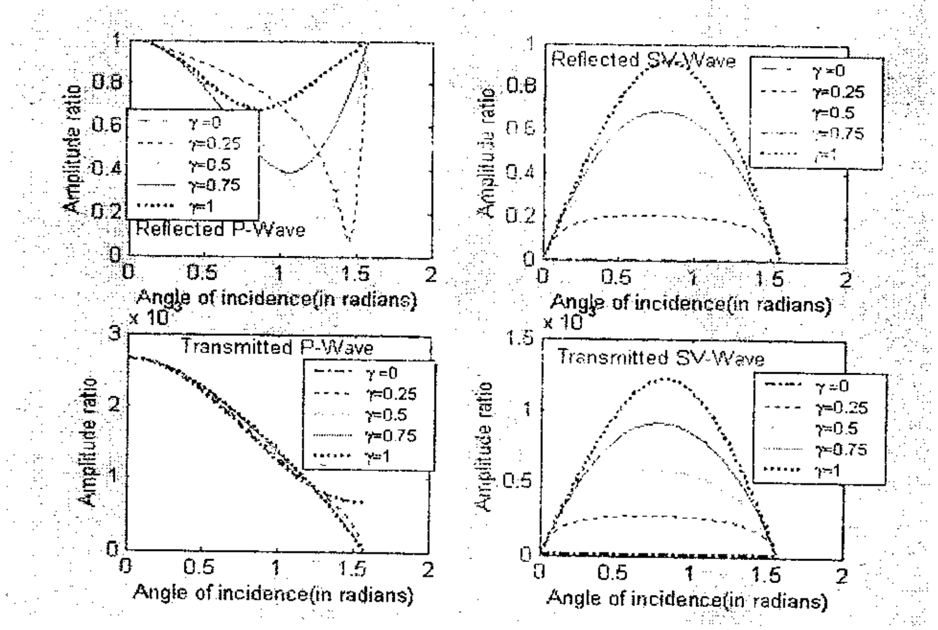
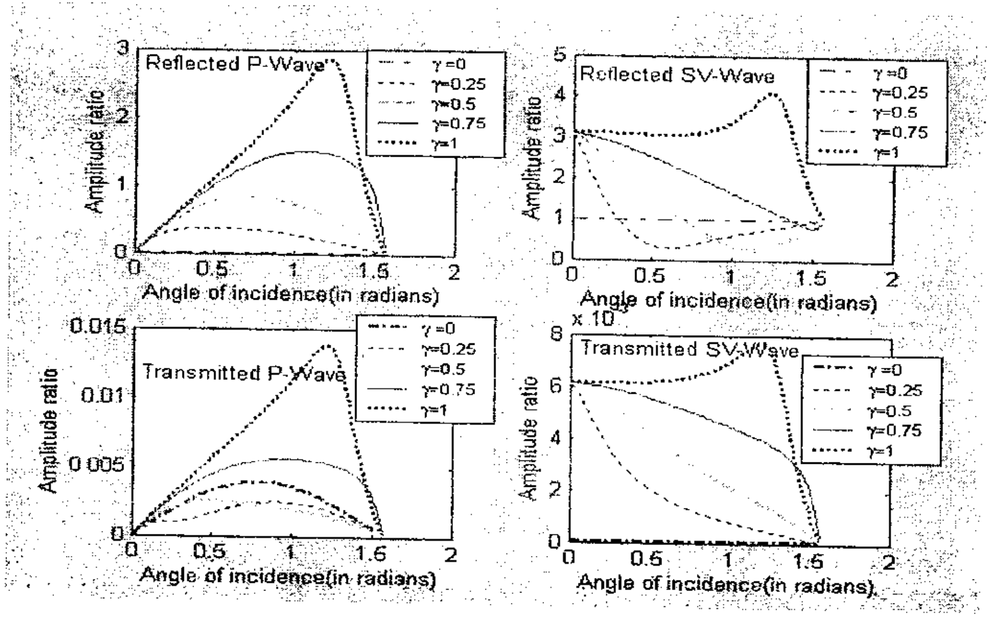


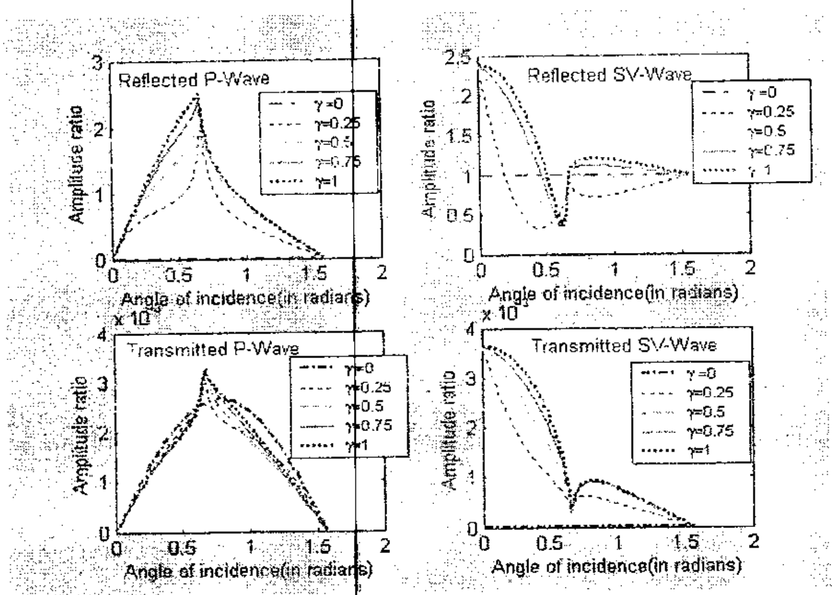
Figure 2-5. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave.



Figs. 6-9. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of empty porous solids.



Figs. 10-13. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of



Figs. 14-17. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave in case of empty porous solids.

IX. REFERENCES

[1] Berryman, J. G., *Biot's theory of elastic waves in fluid-saturated porous media*, Acoustical Society of America, 68(51), 52-52, 1980.

[2] Biot, M. A., *General theory of three dimensional consolidations*, *J. Appl. Phys.*, 12, 155 – 161, 1941.

[3] Biot, M. A. , *General solution of the equations of elasticity and consolidation for a porous material*, J. Appl. Mech., 23, 91-96,1956a.

[4] Biot, M. A., *Theory of deformation of a porous viscoelastic anisotropic solid*, J. Appl. Phys., 27, 459-46, 1956b.

[5] Biot, M. A., *Generalized theory of acoustic propagation in porous dissipative media*, J. Acoust. Soc. Am., 34, 1254-1264,1962.

[6] Bowen, R.M., *Incompressible porous media models by use of the theory of mixtures*, International Journal of Engineering Science, 18(9), 1129-1148, 1980.

[7] de Boer, and Didwania, A. K., *Two phase flow and capillarity phenomenon in porous solid- A Continuum Thermomechanical Approach*, Transport in Porous Media (TIPM), 56, 137-170, 2004.

[8] de Boer, R. and Ehlers, W., *The development of the concept of effective stress*, Acta Mechanica A 83, 77-92,1990.

[9] de Boer, R. and. Ehlers, W ,*Uplift, friction and capillarity-three fundamental effects for liquid- saturated porous solids*, Int. J, Solid Structures B, 26,43-57,1990.

[10] de Boer, R., Ehlers, W. and Liu, Z., *One dimensional transient wave propagation in fluid saturated incompressible porous media*, Arch. App. Mech. 63, 59-72, 1993.

[11] de Boer, R. and Liu, Z., *Plane waves in a semi-infinite fluid saturated porous medium*, Transport in Porous Media, 16 (2), 147-173, 1994.

[12] Fillunger, P.; *Der Auftrieb in Talsperren. Osterr., Wochenschrift fur den offentl. Baudienst*, 19, 532 - 556, 1913.

[13] Kumar, R and Hundal, B.S., *Surface wave propagation in a fluid-saturated incompressible porous medium*, Sadhana, 155-166, 32(3), 2007.

[14] Kumar, R., Miglani, A. and Grag, N.R. , *Response of an anisotropic liquid-saturated porous medium due to two dimensional sources*, Proc. Indian Acad. Sci. (Earth Planet Sci.),111(2), 143-151, 2002.

[15] Kumar,R.,Miglani,A. and Kumar,S., *Reflection and Transmission of plane waves between two different fluid saturated porous half spaces*, Bull. Pol. Ac., Tech., 227-234, 59(2), 2011.

[16] Kumar,R. and Singh,B., Reflection and transmission of elastic waves at a loosely bonded interface between an elastic and micropolar elastic solid, Indian Journal of Pure and Applied Mathematics,28,1133-1153,1997.

[17] Kumar, R. and Tomar, S. K. ,*Reflection and transmission of elastic waves at viscous liquid /at a liquid micropolar solid interface.*

[18] Kumari, N., *Wave propagation at a loosely bonded interface between two fluids saturated incompressible porous solids*, International Journal of Statistika and Mathematika,9(3),108-114,2014.

[19] Nath, C.N., *On the parametric model of loose bonding between poroelastic half spaces*, Journal of Vibration and Control,18(9),1261-1274,2012.

[20] Nandal, J.S., and Saini, T.N., *Reflection and refraction at an imperfectly bonded interface between poroelastic solid and cracked elastic solid*, Journal of Seismology,17,239-253,2013.

[21] Sharma, M.D., *Wave propagation in a general anisotropic medium: Biot's theories and homogenisation theory*, Journal of Earth System Science, 116(4), 357-367, 2007.

[22] Tajuddin, M. and Hussaini, S.J., *Reflection of plane waves at boundaries of a liquid filled poroelastic half-space*, J. Applied Geophysics 58,59-86,2006.

[23] Vashisth, A.K., and Gogna, M.L.,The effect of loose boundaries on wave propagation in a porous solid :reflection and refraction of seismic waves across a plane interface, international Journal of Solid Structure,30,2485-2499,1993.

[24] Vashisth, A.K., and Gupta, V., *Reflection and transmission of plane waves incident at the interface between an elastic basement and a sedimentary layer underlying a homogeneous ocean*, International Journal of Scientific & Engineering Research,4(11),2013.

[25] Vashishth, A.K. , Sharma, M.D. and Gogna, M.L., *Reflection and transmission of elastic waves a loosely bonded interface between an elastic solid and liquid saturated porous solid*,Geophys.J.Int.,105,601-617,1991.