An $MAP|PH|1$ Retrial Queue with Service Interruption and Orbital Search

A. Krishnamoorthy [a]

[a] *Department of Mathematics, Cochin University of Science and Technology, Kochi 682 022, India, Email:ak@cusat.ac.in*

Viswanath C. Narayanan [b]

[b] *Department of Mathematics, Govt. Engineering College, Trichur, India. Email:narayanan_viswanath@yahoo.com*

V. C. Joshua [c]

[c] *Department of Mathematics, C. M. S. College, Kottayam, India. Email:vcjoshua@yahoo.com*

T. G. Deepak [d]

[d] *Department of Mathematics, Indian Institute of Space Science and Technology, Thiruvanathapuram, India. Email:deepak@iist.ac.in*

Abstract

We consider a single server retrial queue in which the server is subject to service interruptions with auto repeat facilities and equipped with the mechanism of search of customers from the orbit. The customer whose service is interrupted rejoins the orbit with some known probability or leaves the system with the complementary probability. At the departure epoch, *i.e.*, the epoch at which the server becomes free either by a successful service completion or by a service interruption, the server goes in search of customers in the orbit with some probability or remains idle with the complementary probability. Search time is assumed to be negligible. Steady state analysis is performed. Several performance measures that help in efficient design of such systems, are computed. Some numerical illustrations are provided.

Key words: Service interruption; Orbital search; Retrial queues.
AMS subject classification: 60K20; 60K25; 68M20; 90B22
Received: November 30, 2008.
Research supported by NBHM (DAE) Research project. 48/5/2003/R&D II/3269.

1 Introduction

In the retrial queueing system customers (or calls or jobs) arriving to a busy system join a group of blocked customers called *orbit* and try to capture a free server after a random amount of time. Such systems occur in communication and computer networks, aircraft landing and take off, machine repair, inventory control and in several other fields. For a systematic account of the fundamental methods and results on this topic, we refer to the monographs by Falin and Templeton [11] and Artalejo and Gomez Correl [4], two extensive survey articles by Yang and Templeton [20] and Falin [10], and the bibliographical information in Artalejo [3].

Retrial queues considered by researchers so far have the characteristic that each service is preceded and followed by idle period, which is terminated either by the arrival of a customer from the orbit or by a primary customer. However, Artelajo et al. [5] considered a retrial queue in which, even without a waiting room, each service completion epoch need not necessarily be followed by an idle time. This is achieved as follows: Immediately after a service completion, the server selects a customer from the orbit with probability p or remains idle with probability $1-p$. If no search is made at a service completion epoch, as in the classical retrial queue, a competition takes place between primary (external) and secondary (orbital) customers for service. Thus if a search is made, a service is followed by another service. Else a service is followed by an idle time. The search time is assumed to be negligible. They considered an $M|G|1$ retrial queue with a search of customers from the orbit and derived several system characteristics. Also their results generalize the classical queue ($p = 1$) on one extream and the classical retrial queue ($p = 0$) on the other. Dudin et al. [9] generalized the result to a retrial queueing model in which primary customers arrive according to a batch Markovian arrival process (BMAP). Chakravarthy et al. [7] considered a multi-server retrial queue with search of customers from the orbit, in which primary customers arrive according to a Markovian arrival process (MAP). It may be recalled that Neuts and Ramalhoto [17] examined classical queue (without retrials) with search for customers immediately on completion of a service. Bocharov et al. [6] considered a single server queueing system with two independent Poisson flows of customers and a general service time distribution where the search for customers of different types is realized with non-pre-emptive priority service discipline.

Queueing system with service interruption has been introduced by Gaver [12] and White and Christie [19]. The latter considered the problem as a pre-emptive priority system. Different types of interruptions have been extensively studied by many researchers. Certain service interruption can be viewed as a special type of breakdown of the server in which the server is restricted instantaneously. See Aissani [1], Aissani and Artelejo [2] and references therein for queueing system with breakdown. Queues with service interruption also fall into category of queues with feedback (see Choi and Kulkarni [8]) and queues with disaster to the unit undergoing service (See Krishnamoorthy and Ushakumari [13]). Artalejo and Gomez-Corral [4] considered a retrial queueing system with two types of service interruptions. In their model, depending on the type of the interruption that the unit has encountered, it may rejoin the orbit for another attempt or leave the system forever.

In this paper, we consider a single server, retrial queue, in which the server is subject to service interruptions with auto repeat facilities, and equipped with the mechanism of search of customers from the orbit as described earlier. The customer whose service got interrupted rejoins the orbit with some known probability q or leaves the system with the complementary probability $1 - q$. At the departure epoch, (*i.e.*, the epoch at which the server becomes free either by a successful service completion or by a service interruption), the server goes in search of customers in the orbit with some known probability p, or remains idle with the complementary probability $1 - p$.

Several real life situation fit to our model. For example, consider the transmission of messages in facsimile networks having the autorepeat facilities. If the transmission is not successfully completed for some reason such as a power failure or transmission error, then the message leaves the server and joins the buffer with some known probability and leaves the system with complementary probability. Immediately after a successful transmission or an interruption, instead of staying idle, the server goes in search of customers in the buffer with a known probability. By the introduction of the 'search mechanism' the idle time of the server is considerably reduced thereby attaining the optimum utilization of the service facility.

This paper is presented as follows. Section 2 provides the mathematical formulation of the problem. In section 3 we address the stability of the system and obtain the long run system state distribution. Some performance measures of significance, are provided in section 4a. and in section 4b these are illustrated numerically.

2 The mathematical model

We consider a single server retrial queueing system to which primary customers arrive according to a Markovian arrival process (MAP) with representation (D_0, D_1) of order n. A detailed description of the MAP is given in the next paragraph. The time between successive retrials are assumed to be exponential with rate $j\mu$, when there are j customers in the orbit. (*i.e.*, the classical retrial policy). The service is interrupted at an exponential rate σ. The interrupted customer goes back to the orbit with a known probability q or leaves the system with the probability $1-q$. At the epoch when the server becomes free, either by a service completion or by a service interruption, it goes for search of customers in the orbit with probability p or remains idle with probability $1-p$. The search time is assured to be negligible. The service time is assumed to follow a phase-type distribution (PH-distribution) with representation (β, S) of order m.

Markovian arrival process (MAP) was introduced in Lucantoni [15]. It can be described as follows. Consider a continuous time Markovian chain $\{X_t : t \geq 0\}$ on a finite state space, say $\{1, 2, \ldots, n\}$. We call this the directing process of the MAP. The sojourn time of the chain $\{X_t\}$ in state i is exponentially distributed with parameter λ_i. At the end of the sojourn time the chain jumps to state j and it triggers an arrival with probability $P_1(i,j)$ and no arrival with probability $P_0(i,j)$; $i, j \in \{1, 2, \ldots, n\}$. We assume $P_0(i,i) = 0$ and $\sum_{k=0}^{1}\sum_{j=1}^{n} P_k(i,j) = 1$.

Write $D_k = (\lambda_i P_k(i,j))$, $k = 0, 1$ so that D_0 and D_1 are matrices of order n. The pair (D_0, D_1) is called the representation of the MAP.

PH-distributions and PH-renewal processes were introduced by Neuts [16]. They have gained wide spread attention in the area of Stochastic modelling, particularly in queueing theory. A phase-type distribution on $[0, \infty)$ of order m is defined as the absorption time distribution in a finite state Markov process with m transient states and one absorbing state:

Consider a Markov chain on the state space $\{0, 1, 2, \ldots, m\}$ with infinitesimal generator $\begin{bmatrix} S & S^0 \\ 0 & 0 \end{bmatrix}$ where the $m \times m$ matrix S satisfies $S_{ii} < 0$, for $1 \leq i \leq m$, and $S_{ij} \geq 0$ for $i \neq j$. Also, $Se + S^0 = 0$ and the initial probability vector is given by (β, β_0), with $\beta e + \beta_0 = 1$, where e is a column vector of 1's of order m. It can be shown that the states $\{1, 2, \ldots, m\}$ are transient if and only if the matrix S is non-singular. Also, the probability distribution $H(\cdot)$ of the time until absorption in the state 0, starting from any of the transient states $1, 2, \ldots, m$ corresponding to the initial probability vector (β_0, β) (usually, we assume that $\beta_0 = 0$), is given by

$$H(x) = 1 - \beta \exp(Sx)e, \quad \text{for } x \geq 0.$$

Our model can be studied as a level dependent quasi-birth and death process (LDQBD) with the state space given by $\epsilon = \{j, \bar{j};\ j \geq 0\}$ where $j =$

$\{(j,0,l); j \geq 0,\ 1 \leq l \leq n\}$ and

$$\bar{j} = \left\{(j,1,k,l);\ j \geq 0,\ 1 \leq k \leq m,\ 1 \leq l \leq n\right\}.$$

The states j (≥ 0) correspond to the idle server with j customers in the orbit and the arrival phase is l whereas the states $\bar{j} \geq 0$, correspond to the busy server with the service process in the phase k, j customers in the orbit and phase of the arrival process l. Then, the infinitesimal generator Q of the LDQBD, with the state space E, is given by

$$Q = \begin{bmatrix} B_0 & A_0 & 0 & 0 & 0 & 0 & \cdots \\ A_{21} & A_{11} & A_0 & 0 & 0 & 0 & \cdots \\ 0 & A_{22} & A_{12} & A_0 & 0 & 0 & \cdots \\ 0 & 0 & A_{23} & A_{13} & A_0 & 0 & \cdots \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

where

$$B_0 = \begin{bmatrix} D_0 & \beta \otimes D_1 \\ (\sigma(1-q)e + S^0) \otimes I & (S - \sigma I + \sigma pqe\beta) \oplus I + I \otimes D_0 \end{bmatrix};$$

$$A_0 = \begin{bmatrix} 0 & 0 \\ \sigma q(1-p)e \otimes I & I \otimes D_1 \end{bmatrix};$$

$$A_{2i} = \begin{bmatrix} 0 & i\mu\beta \otimes I \\ 0 & (pS^0\beta + \sigma p(1-q)e\beta) \otimes I \end{bmatrix};\ i \geq 1;$$

and for $i \geq 1$,

$$A_{1i} = \begin{bmatrix} D_0 - i\mu I & \beta \otimes D_1 \\ [(1-p)S^0 + \sigma(1-p)(1-q)e] \otimes I & (S - \sigma I + \sigma pqe\beta) \otimes I + I \otimes D_0 \end{bmatrix}.$$

3 Stationary Distribution

Let $x = (x(0), x(1), x(2) \dots)$ be the stationary probability vector of the above level dependent QBD process. Note that $x(i)$ is the probability vector corresponding to i, the number of customers in the orbit. Then

$$xQ = 0 \text{ and } xe = 1 \qquad (3.1)$$

Even though the generator Q is highly structured, x can not be expressed in a tractable analytical form. So we propose an algorithmic solution based on the Neuts-Rao truncation approach introduced in [18]. This approach can be justified due to the following reason: If the number of customers in the orbit is small, then the likelihood of an idle server and therefore that of a retrial request being successful is not small. When the number of customers in the orbit is sufficiently large, a majority of the retrial requests fail to find a free server and do not result in a change of state. Therefore, if the number of customers in the orbit who can generate retrial requests is restricted to an approximately chosen large number N, then the effect on the system dynamics and the equilibrium probability vector is minimal. For details regarding the selection of N, see [18]. That approximation modifies the infinitesimal generator $\bar{Q}$ which is given below:

$$\bar{Q} = \begin{bmatrix} B_0 & A_0 & & & & & & \\ A_{12} & A_{11} & A_0 & & & & & \\ & A_{22} & A_{12} & A_0 & & & & \\ & \cdots & \cdots & \cdots & & & & \\ & & & A_{2,N-1} & A_{1,N-1} & A_0 & & \\ & & & & A_2 & A_1 & A_0 & \\ & & & & & A_2 & A_1 & A_0 \\ & & \cdots & \cdots & \cdots & \cdots & & \end{bmatrix}$$

where $A_1 = A_{1N}$ and $A_2 = A_{2N}$.

3.1 Stability condition

We first investigate the condition for the system to be stable. We know that the system is stable if and only if $\pi A_0 e < \pi A_2 e$, where π is the stationary probability vector corresponding to the generator matrix $A = A_0 + A_1 + A_2$. For the present model, it is almost impossible to get an explicit expression for the above condition of stability. However, in the particular case that the arrival process is Poisson instead of MAP, we succeed in getting an explicit expression for the stability condition in the form

$$(\lambda + N\mu)\beta[\sigma I - pS^0\beta - S - p\sigma e\beta]^{-1}[(\lambda + \sigma(q-p)e - pS^0)] < N\mu,$$

where λ is the arrival rate.

Because of the special structure of $\bar{Q}$, x can be expressed as

$$x(i+N-1) = x(N-1)R^i, \quad i \geq 0 \tag{3.2}$$

where R is the minimal non-negative (matrix) solution to the equation

$$R^2 A_2 + RA_1 + A_0 = 0. \tag{3.3}$$

For details about the matrix analytic methods, see Neuts [16] and Latouche and Ramaswami [14]. The latter also proposes an efficient algorithm, namely logarithmic reduction algorithm for the computation of the matrix R.

It is to be noted that due to the special structure of A_0, the matrix R has a block partitioned form $R = \begin{bmatrix} 0 & 0 \\ R_0 & R_1 \end{bmatrix}$, which also can be utilized for the computation of R.

The vectors $x(0), x(1), \ldots, x(N-1)$ can be obtained by solving the equations

$$\begin{aligned} & x(0)B_0 + x(1)A_{21} = 0 \\ & x(i-1)A_0 + x(i)A_{1i} + x(i+1)A_{2,i+1} = 0, \quad 1 \leq i \leq N-1 \\ & x(N-2)A_0 + x(N-1)[A_{1,N-1} + RA_2] = 0 \end{aligned} \tag{3.4}$$

subject to the normalizing condition

$$\sum_{i=0}^{N-2} x(i)e + x(N-1)(I-R)^{-1}e = 1 \tag{3.5}$$

Now, from (3.4), we have

$$\begin{aligned} x(0) &= x(1)A_{21}(-B_0)^{-1} \\ &= x(1)A_{21}(-A_0')^{-1} \\ \text{and} \quad x(1) &= -x(2)A_{22}[A_{11} + A_{21}(-A_0')^{-1}A_0]^{-1} \\ &= x(2)A_{22}(-A_1')^{-1} \end{aligned}$$

proceeding like this, in general we get

$$x(i) = x(i+1)A_{2,i+1}(-A_i')^{-1}; \quad 0 \leq i \leq N-1 \tag{3.6}$$

where

$$A_i' = \begin{cases} B_0 & i = 0 \\ A_{1i} + A_{2i}(-A_{i-1}')^{-1}A_0 & 1 \leq i \leq N. \end{cases} \tag{3.7}$$

Now, by applying block Gauss elimination, the partitioned subvector $[x(N), x(N+1), \ldots]$ corresponding to non-boundary states, satisfies the relation

$$[x(N), x(N+1), \ldots] \begin{bmatrix} A'_N & A_0 & \cdots & \cdots \\ A_2 & A_1 & A_0 & \cdots \\ & A_2 & A_1 & A_0 \\ & \cdots & \cdots & \end{bmatrix} = 0. \tag{3.8}$$

In order to solve (3.8), we proceed as follows. Let

$$\delta = \sum_{i=N}^{\infty} x(i)e \tag{3.9}$$

$$\text{and} \quad y(i) = \delta^{-1} x(N+1), \quad i \geq 0 \tag{3.10}$$

From (3.8) , we get

$$x(N)A'_N + x(N+1)A_2 = 0$$

and $x(N+i) = x(N+i-1)R$, $i \geq 1$, which implies

$$y(0)A'_N + y(1)A_2 = 0 \quad \text{and} \quad y(i) = y(i-1)R, \quad i \geq 1.$$

Since $\sum_{i=0}^{\infty} y(i)e = 1$ we get $y(0)(I-R)^{-1}e = 1$. Thus

$$x(i) = \delta y(0)R^{i-N}, \quad i \geq N. \tag{3.11}$$

Again by (3.6), we get

$$x(i) = \delta y(0) \prod_{j=1}^{N-i} A_{2N-j+1}(-A'_{N-j})^{-1}, \quad 0 \leq i \leq N-1.$$

Therefore,

$$x(i) = \begin{cases} \delta y(0) \prod_{j=1}^{N-i} A_{2,N-j+1}(-A'_{N-j})^{-1} & 0 \leq i \leq N-1 \\ \delta y(0)R^{i-N}, & i \geq N \end{cases} \tag{3.12}$$

where $y(0)$ is the solution of the system

$$\begin{aligned} y(0)(A'_N + RA_2) &= 0 \\ y(0)(I-R)^{-1}e &= 1. \end{aligned} \tag{3.13}$$

Now, $xe = 1$ implies

$$\delta y(0) \sum_{i=0}^{N-1} \prod_{j=1}^{N-i} A_{2,N-j+1}(-A'_{N-j})^{-1}e + \delta y(0) \sum_{i=N}^{\infty} R^{i-N} = 1.$$

so that

$$\delta = \left[1 + y(0) \sum_{i=0}^{N-1} \prod_{j=1}^{N-i} A_{2,N-j+1}(-A'_{N-j})^{-1}e\right]^{-1} \tag{3.14}$$

since $y(0)(I - R)^{-1} = 1$.

In the next section, we discuss some important descriptors which are very useful in system design.

4 (a) Measures of effectiveness

(1) Probability mass function of the number of customers in the orbit is given by

$$\begin{aligned} a_i &= \Pr[i \text{ customers in the orbit}] = x(i)e \\ &= \begin{cases} \delta y(0) \prod_{j=1}^{N-i} A_{2,N-j+1}(-A'_{N-j})^{-1}e; & 0 \le i \le N-1 \\ \delta y(0) R^{i-N} e & i \ge N. \end{cases} \end{aligned}$$

(2) Expected number of customers in the orbit,

$$\begin{aligned} \mathrm{EN} = \sum_{i=1}^{\infty} i x(i) e = f y(0) \Big[\sum_{i=1}^{N-1} i \prod_{j=1}^{N-i} A_{2,N-j+1}(-A'_{N-j})^{-1}e \\ + R(I-R)^{-2}e + N(I-R)^{-1}e \Big] \end{aligned}$$

(3) Probability mass function of the server state is given by

$$\begin{aligned} P_0 = \Pr[\text{sever is idle}] &= \sum_{i=0}^{\infty} x_i(0)e \\ &= \sum_{i=0}^{N-1} x_i(0)e + x_{N-1}(1)(I - R_1)^{-1} R_0 \end{aligned}$$

and

$$\begin{aligned} P_1 = \Pr[\text{sever is busy}] = \sum_{i=0}^{\infty} x_i(0)e = \sum_{i=0}^{N-1} x_i(1)e \\ + x_{N-1}(1)(I - R_1)^{-1} R_1 e. \end{aligned}$$

Note that by $x_i(0)$ and $x_i(1)$, we mean the probability vectors corresponding the idle server and busy server respectively. R_0 and R_1 are the components of R.

(4) The overall rate of retrials, $\mu_1^* = \sum_{i=1}^{\infty} i\mu x(i)e = \mu E(N)$.

(5) The successful rate of retrials, $\mu_2^* = \sum_{i=1}^{\infty} i\mu x_i(0)e$

$$= \mu[\sum_{i=1}^{N-1} ix_i(0)e + x_{N-1}(1)[R_1(I-R_1)^{-2} + N(I-R_1)^{-1}]R_0]$$

(6) Fraction of successful retrials, $r = \frac{\mu_2^*}{\mu_1^*}$.

(7) Expected number of customers in the system,

$$ES = EN + p_1$$

(8) The mean time spent by an arbitrary customer in the system

$$W_S = \frac{1}{\lambda}ES.$$

4 (b) Numerical Results

Using the results obtained in the previous section, we present some numerical results for illustrating the performance of the system. For this purpose, we have taken the following values for different parameters under consideration.

$$D_0 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -5 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.9 & 0 & 0 \\ 2 & 0 & 3 \end{bmatrix},$$

$$S = \begin{bmatrix} -20 & 2.5 & 0 \\ 0 & -20 & 5 \\ 2.5 & 2.5 & -20 \end{bmatrix}, \quad \beta = [0.3 \quad 0.3 \quad 0.4],$$

$\sigma = 0.5$, $q = 0.5$, $p = 0.9$.

For this set of parameters, the impact of the variation in the retrial rate μ, on the different measures of effectiveness are shown in table 4. From table 4, it is clear that as μ increases, the mean orbit size decreases, where as the overall retrial rate as well as successful retrial rate increases as expected.

For the preparation of table 5, we keep the values of all parameters as such in the case of table 4, except those for p and q, for which the values are assigned as 0.5 and 0.9, respectively. Comparing tables 4 and 5 we see that for a particular value of μ, the mean orbit size as given in table 5 is larger than the corresponding one in table 4. This is due to the fact that in table 5, the

Table 4

μ	2	4	6	8	10	12	14
EN	0.00422	0.00356	0.00329	0.00315	0.00305	0.00299	0.00294
P_0	0.96180	0.96180	0.96180	0.96180	0.96180	0.96180	0.96180
P_1	0.03820	0.03820	0.03820	0.03820	0.03820	0.03820	0.03820
μ_1^*	0.00844	0.01423	0.01975	0.02517	0.03053	0.03586	0.04117
μ_2^*	0.00296	0.00342	0.00365	0.00379	0.00389	0.00396	0.00402
r	0.35033	0.24018	0.18455	0.15047	0.12727	0.11039	0.09752

Table 5

μ	2	4	6	8	10	12	14
EN	0.01389	0.00888	0.00701	0.00602	0.00540	0.00498	0.00467
P_0	0.96732	0.96732	0.96732	0.96732	0.96732	0.96732	0.96732
P_1	0.03868	0.03868	0.03868	0.03868	0.03868	0.03868	0.03868
μ_1^*	0.02777	0.03550	0.04204	0.04813	0.05401	0.05975	0.06540
μ_2^*	0.2080	0.02267	0.02355	0.02408	0.02444	0.02470	0.02490
r	0.74904	0.63868	0.56016	0.50023	0.45253	0.41346	0.38078

probability of customer remaining in the system after the breakdown is much higher than that in table 4, which results in an increase in orbit size. The same relationship can be observed in the values of μ_1^* and μ_2^* as well.

Acknowledgement A. Krishnamoorthy acknowledges financial support from Indian Science Lab (General Motors), Bangalore.

References

[1] A. Aissani. A retrial queue with redundancy and unreliable server. *Queueing Systems*, 17:431–449, 1994.

[2] A. Aissani and J. R. Artalejo. On the single server retrial queue subject to breakdowns. *Queueing Systems*, 30(3–4):309–321, 1998.

[3] J. R. Artalejo. A classified bibliography of research on retrial queues: Progress in 1990–1999. *TOP*, 7(2):187–211, December 1999.

[4] J. R. Artalejo and A. Gomez-Corral. Steady state solution of a single-server

queue with linear repeated requests. *Journal of Applied Probability*, 36:907–918, 1999.

[5] J. R. Artalejo, V. C. Joshua, and A. Krishnamoorthy. An $M/G/1$ retrial queue with orbital search by the server. In *Advances in Stochastic Modelling*, pages 41–54. Notable Publications Inc., New Jersey, 2002.

[6] P. Bocharov, C. D'Apice, B. D'Auria, and S. Salerno. A queueing system of finite capacity with the server requiring a priority search for customers. *Seroa Prikladnaia Matematika i informatika*, 1:50–61, 2000.

[7] S. R. Chakravarthy, A. Krishnamoorthy, and V. C. Joshua. Analysis of a multi server retrial queue with search of customer from the orbit. *Performance Evaluation*, 63(8):776–798, 2006.

[8] B. D. Choi and V. G. Kulkarni. Retrial queues with server subject to breakdown and repairs. *QUESTA*, 7(2):191–208, 1990.

[9] A. N. Dudin, A. Krishnamoorthy, V. C. Joshua, and G. V. Tsarankov. Analysis of the BMAP$|g|$1 retrial system with search of customers from the orbit. *European Journal of Operational Research*, 157:169–179, 2004.

[10] G. Falin. A survey of retrial queues. *Queueing System*, 7:127–168, 1990.

[11] G. Falin and J. G. C. Templeton. *Retrial Queues*. Chapman & Hall, 1997.

[12] D. P. Gaver, P. A. Jacobs, and G. Latouche. Finite birth-and-death models in randomly changing environments. *Adv. in Appl. Probab.*, 16:715–731, 1984.

[13] A. Krishnamoorthy and P. V. Ushakumari. On an $M/G/1$ retrial queue with disaster to the customer in service. In *International Workshop on Retrial Queues*, Madrid, September 1998.

[14] G. Latouche and V. Ramaswami. *Introduction to matrix analytic methods in stochastic modeling*. ASA-SIAM: Philadelphia, 1999.

[15] D. M. Lucantoni. New results on the single server queue with a batch Markovian arrival process. *Commun. Statist.-Stochastic Models*, 7:1–46, 1991.

[16] M. F. Neuts. *Matrix-geometric solutions in stochastic models-An algorithmic approach*. John-Hopkins Univ. Press, 1981. Also published by Dover, 2002.

[17] M. F. Neuts and M. F. Ramalhoto. A service model in which the server is required to search for customers. *Journal of Applied Probability*, 21:157–166, 1984.

[18] M. F. Neuts and B. M. Rao. Numerical investigation of a multiserver retrial model. *Queueing systems*, 7:169–190, 1990.

[19] H. White and L. S. Christie. Queueing with pre-emptive priorities or with breakdown. *Operations Research*, 6:79–95, 1958.

[20] T. Yang and J. G. C. Templeton. A survey of retrial queues. *QUESTA*, 2(3):201–233, 1987.