SEMI WEAKLY GENERALLZED CLOSED SET IN INTULTLONLSTIC FUZZY TOPOLOGICAL SPACES

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Abstract : This paper is devoted to the study of intuitionistic fuzzy topological spaces. In this paper, an intuitionistic fuzzy semi weakly generalized closed set and intuitionistic fuzzy semi weakly generalized open set are introduced. Some of their properties are studied.

Keywords : Intuitionistic fuzzy topology, Intuitionistic fuzzy semi weakly generalized closed set, Intuitionistic fuzzy semi weakly generalized open set, Intuitionistic fuzzy $_{sw}T_{1/2}$ space and Intuitionistic fuzzy $_{swg}T_k$ space.

AMS SUBJECT CLASSIFICATION (2000) 54A40, 03F55

1. INTRODUCTION

Fuzzy set(FS), proposed by Zadeh[15] in 1965, as a framework to encounter uncertainity, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. Later on, fuzzy topology was introduced by Chang[3] in 1967. By adding the degree of non-membership to FS, Atanassov[1] proposed intuitionistic fuzzy set (IFS) in 1983 which looks more accurately to uncertainity quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In the last few years various concepts in fuzzy were extended to intuitionistic fuzzy sets. In 1997, Coker[4] introduced the concept of intuitionistic fuzzy topological space. Nagaveni[8] introduced semi weakly generalized closed set in General Topology in 1999. In this paper, we introduce semi weakly generalized closed set in intuitionistic fuzzy topological space. We have studied some of the basic properties regarding it. We also introduce the applications of intuitionistic fuzzy semi weakly generalized closed set namely intuitionistic fuzzy $_{sw}T_{1/2}$ space, intuitionistic fuzzy $_{swg}T_k$ space and obtained some characterizations and several preservation theorems of such spaces.

2. PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the functions

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$\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote the set of all intuitionistic fuzzy sets in X, by IFS(X).

Definition 2.2: **[1]** Let A and B be IFS's of the forms
$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and

$B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$.

Then (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$,

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$,

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of

$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$.

Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_{\sim} = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_{\sim} = \{ \langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: **[4]** An intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFS in X satisfying the following axioms:

(a) $0_{\sim}, 1_{\sim} \in \tau$,

(b) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,

(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.4: **[4]** Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,

$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Result 2.5: **[4]** Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space (X, τ).Then

(a) A is an intuitionistic fuzzy closed set in X $\Leftrightarrow cl(A) = A$,

(b) A is an intuitionistic fuzzy open set in X $\Leftrightarrow int(A) = A$,

(c) $cl(A^c) = (int(A))^c$,

(d) $int(A^c) = (cl(A))^c$,

(e) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$,

(f) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$,

(g) $cl(A \cup B) = cl(A) \cup cl(B)$,

(h) $int(A \cap B) = int(A) \cap int(B)$.

Definition 2.6: [5] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the semi closure of A (scl(A) in short) and semi interior of A (sint(A) in short) are defined as

$sint(A) = \cup \{ G / G$ is an IFSOS in X and $G \subseteq A\}$,

$scl(A) = \cap \{ K / K$ is an IFSCS in X and $A \subseteq K \}$.

Result 2.7: [12] Let A be an IFS in (X, τ), then

(i) $scl(A) = A \cup int(cl(A))$,

(ii) $sint(A) = A \cap cl(int(A))$.

Definition 2.8: [9] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the alpha closure of A ($\alpha cl(A)$ in short) and alpha interior of A ($\alpha int(A)$ in short) are defined as

$\alpha int(A) = \cup \{ G / G$ is an IFαOS in X and $G \subseteq A\}$,

$\alpha cl(A) = \cap \{ K / K$ is an IFαCS in X and $A \subseteq K \}$.

Result 2.9: [9] Let A be an IFS in (X, τ), then

(i) $\alpha cl(A) = A \cup cl(int(cl(A)))$,

(ii) $\alpha int(A) = A \cap int(cl(int(A)))$.

Definition 2.10: An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X,τ) is called an

(a) intuitionistic fuzzy semi closed set [5](IFSCS) if $int(cl(A)) \subseteq A$,

(b) intuitionistic fuzzy α-closed set [5](IFαCS) if $cl(int(cl(A))) \subseteq A$,

(c) intuitionistic fuzzy pre-closed set [5](IFPCS) if $cl(int(A)) \subseteq A$,

(d) intuitionistic fuzzy regular closed set [5](IFRCS) if $cl(int(A))=A$,

(e) intuitionistic fuzzy generalized closed set [13](IFGCS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(f) intuitionistic fuzzy generalized semi closed set [11](IFGSCS) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(g) intuitionistic fuzzy α generalized closed set [9](IFαGCS) if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS and IFαGOS) if the complement of A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS and IFαGCS respectively.

Definition 2.11: [9] An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X,τ) is said to be an

(i) intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq cl(int(A))$,

(ii) intuitionistic fuzzy α-open set (IFαOS in short) if A ⊆ int(cl(int(A))),

(iii) intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)).

3. INTUITIONISTIC FUZZY SEMI WEAKLY GENERALIZED CLOSED SET

In this section we introduce intuitionistic fuzzy semi weakly generalized closed set and have studied some of its properties.

Definition 3.1 An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy semi weakly generalized closed set (IFSWGCS) if cl(int(A)) ⊆ U whenever A ⊆ U, U is IFSOS in X .
The family of all IFSWGCSs of an IFTS (X, τ) is denoted by IFSWGCS(X).

Example 3.2: Let X = {a, b} and let τ = {0~, T, 1~} be an IFT on X,
where T = ⟨ x, (0.5, 0.4), (0.2, 0.3) ⟩.
Then the IFS A = ⟨ x, (0.1, 0.2), (0.5, 0.4) ⟩ is an IFSWGCS in X.

Theorem 3.3: Every IFCS is an IFSWGCS but not conversely.

Proof: Let A be an IFCS in (X, τ). Let U be an intuitionistic fuzzy semi open set in (X, τ) such that A ⊆ U. Since A is an intuitionistic fuzzy closed, cl(A) = A and hence cl(A) ⊆ U. But cl(int(A)) ⊆ cl(A) ⊆ U. Therefore cl(int(A)) ⊆ U. Hence A is an IFSWGCS in X.

Example 3.4: Let X = {a, b} and let τ = {0~, T, 1~} be an IFT on X, where
T = ⟨ x, (0.3, 0.4), (0.5, 0.4) ⟩. Then the IFS A = ⟨ x, (0.1, 0.2), (0.6, 0.7) ⟩ is an IFSWGCS in X but not an IFCS in X.

Theorem 3.5: Every IFαCS is an IFSWGCS but not conversely.

Proof: Let A be an IFαCS in X and let A ⊆ U and U is an IFSOS in (X, τ). By hypothesis, cl(int(cl(A)))⊆ A. Therefore cl(int((A)) ⊆ cl(int(cl(A))) ⊆ A⊆ U. Therefore cl(int((A)) ⊆ U. Hence A is an IFSWGCS in X.

Example 3.6: Let X = {a, b} and let τ = {0~, T, 1~} be an IFT on X,
where T = ⟨ x, (0.4, 0.2), (0.4, 0.8) ⟩. Then the IFS A = ⟨ x, (0.3, 0.1), (0.6, 0.8) ⟩ is an IFSWGCS but not an IFαCS in X.

Theorem 3.7: Every IFGCS is an IFSWGCS but not conversely.

Proof: Let A be an IFGCS in X and let A ⊆ U and U is an IFSOS in (X, τ). Since cl(A) ⊆ U, cl(int(A)) ⊆ cl(A). That is cl(int(A)) ⊆ cl(A) ⊆ U. Therefore A is an IFSWGCS in X.

Example 3.8: Let X = {a, b} and let τ = {0~, T, 1~} be an IFT on X, where T = ⟨ x, (0.2, 0.3), (0.8, 0.7) ⟩. Then the IFS A = ⟨ x, (0.1, 0.3), (0.9, 0.7) ⟩ is an IFSWGCS but not an IFGCS in X.

Theorem 3.9: Every IFRCS is an IFSWGCS but not conversely.

Proof: Let A be an IFRCS in X. Let A ⊆ U and U is an IFSOS in (X, τ). Since A is IFRCS, cl(int(A)) = A ⊆ U. This implies cl(int(A)) ⊆ U. Hence A is an IFSWGCS in X.

Example 3.10: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} be an IFT on X, where T = ⟨ x, (0.5, 0.7), (0.3, 0.1) ⟩. The IFS A = ⟨ x, (0.1, 0.1), (0.7, 0.8) ⟩ is an IFSWGCS but not an IFRCS in X.

Theorem 3.11: Every IFαGCS is an IFSWGCS but not conversely.
Proof: Let A be an IFαGCS in X and let A $\subseteq$ U and U is an IFSOS in (X, τ). By Definition, A$\cup$cl(int(cl(A))) $\subseteq$ U. This implies cl(int(cl(A))) $\subseteq$ U and cl(int(A)) $\subseteq$ cl(int(cl(A))) $\subseteq$ U. Therefore cl(int(A)) $\subseteq$ U. Hence A is an IFSWGCS in X.

Example 3.12: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X, where T = ⟨ x, (0.5, 0.7), (0.5, 0.3) ⟩. Then the IFS A = ⟨ x, (0.3, 0.5), (0.7, 0.5) ⟩ is an IFSWGCS but not an IFαGCS in X.

Proposition 3.13: IFSCS and IFSWGCS are independent to each other which can be seen from the following examples.

Example 3.14: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} be an IFT on X, where T = ⟨ x, (0.4, 0.2), (0.4, 0.5) ⟩. Then the IFS A = T is an IFSCS but not an IFSWGCS in X.

Example 3.15: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} be an IFT on X, where T = ⟨ x, (0.7, 0.7), (0.3, 0.3) ⟩. Then the IFS A = ⟨ x, (0.7, 0.6), (0.3, 0.3) ⟩ is an IFSWGCS but not an IFSCS in X.

Proposition 3.16: IFGSCS and IFSWGCS are independent to each other.

Example 3.17: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} be an IFT on X, where T = ⟨ x, (0.2, 0.5), (0.8, 0.5) ⟩. Then the IFS A = T is an IFGSCS but not an IFSWGCS in X.

Example 3.18: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} bean IFT on X, where T = ⟨ x, (0.6, 0.8), (0.4, 0.2) ⟩. Then the IFS A = ⟨ x, (0.5, 0.5), (0.5, 0.5) ⟩ is an IFSWGCS but not an IFGSCS in X.

Remark 3.19: The union of any two IFSWGCS's need not be an IFSWGCS in general as seen from the following example.

4. INTUITIONISTIC FUZZY SEMI WEAKLY GENERALIZED OPEN SETS

In this section we introduce intuitionistic fuzzy semi weakly generalized open set and have studied some of its properties.

Definition 4.1: An IFS A is said to be an intuitionistic fuzzy semi weakly generalized open set (IFSWGOS in short) in (X, τ) if its complement A^c is an IFSWGCS in X.

The family of all IFSWGOS of an IFTS (X, τ) is denoted by IFSWGO(X).

Example 4.2: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} be an IFT on X, where T = ⟨ x, (0.6, 0.4), (0.2, 0.4) ⟩. Then the IFS A = ⟨ x, (0.7, 0.6), (0.1, 0.1) ⟩ is an IFSWGOS in X.

Theorem 4.3: For any IFTS (X, τ), we have the following:

(i) Every IFOS is an IFSWGOS.

(ii) Every IFPOS is an IFSWGOS.

(iii) Every IFαOS is an IFSWGOS.

(iv) Every IFGOS is an IFSWGOS. But the converses are not true in general.

Proof: The proof is straight forward.

The converse of the above statement need not be true in general as seen from the following examples.

Example 4.4: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} be an IFT on X, where T = ⟨ x, (0.2, 0.4), (0.7, 0.6) ⟩. Then the IFS A = ⟨ x, (0.7, 0.7), (0.2, 0.2) ⟩ is an IFSWGOS in X but not an IFOS in X.

Example 4.5: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X, where T = ⟨ x, (0.4, 0.3), (0.6, 0.7) ⟩. Then the IFS A = ⟨ x, (0.6, 0.8), (0.2, 0.1) ⟩ is an IFSWGOS but not an IFαOS in X.

Example 4.6: Let X = {a, b} and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X, where T = ⟨ x, (0.2, 0.4), (0.8, 0.6) ⟩. Then the IFS A = ⟨ x, (0.8, 0.7), (0.1, 0.3) ⟩ is an IFSWGOS but not an IFGOS in X.

5. CONCLUSION

In this paper we have introduced intuitionistic fuzzy semi weakly generalized closed set, intuitionistic fuzzy semi weakly generalized open set, and studied some of its basic properties. Also we have to prove some characterizations of intuitionistic fuzzy semi weakly generalized closed sets in near future.

6. REFERENCES

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