Solution of Fully fuzzy linear system with triangular fuzzy numbers (a, b, c)

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Abstract

Different methods are proposed for finding the solution of fully fuzzy linear system (FFLS) that is, fuzzy linear system with fuzzy coefficients involving fuzzy variables. All of these existing methods assume the non negativity constraint on the solution. Hence all the methods are designed keeping this restriction prior to their development which is an invalid approach. In this paper, we extend the concept of existing methods to solve FFLS with triangular fuzzy numbers of the form (a,b,c) without assuming any restriction on the solution and hence widen the implementation scope of fuzzy linear systems in several engineering applications and sciences like quantum optics, topological spaces and hyperchaotic systems. The proposed method is illustrated with numerical examples.

Key words: Fully fuzzy linear systems (FFLS), Fuzzy matrix, Triangular fuzzy numbers

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1. Introduction

One field of applied mathematics that has many applications in various areas of science is solving a system of linear equations. Systems of simultaneous linear equations play a major role in various areas such as operational research, physics, statistics, engineering and social sciences. When the estimation of the system coefficients is imprecise and only some vague knowledge about the actual values of the parameters is available, it may be convenient to represent some or all of them with fuzzy numbers [22]. Fuzzy number arithmetic is widely applied and useful in computation of linear system whose parameters are all or partially represented by fuzzy numbers.

Dubois and Prade [11,12] investigated two definitions of a system of fuzzy linear equations, consisting of system of tolerance constraints and system of approximate equalities. The simplest method for finding a solution for this system is creating scenarios for the fuzzy system, which is a realization of fuzzy systems. Based on these actual scenarios, Buckley and Qu [7] extended several methods for this category and proved their equivalence. But their approaches are not practicable, because infinite number of scenarios can be driven for a fully fuzzy linear system (FFLS).

Zhao and Govind [23] studied the algebraic equations involving generalized fuzzy numbers (which includes fuzzy numbers, fuzzy intervals, crisp numbers and interval numbers) with continuous membership functions. A general model for solving a fuzzy linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy vector was first proposed by Friedman et al. [13]. Friedman et al. [14] investigated a dual fuzzy linear system by mean of nonnegative matrix theory.

Allahviranloo [4] proposed solution of a fuzzy linear system. by using iterative method (Jacobi and Gauss Seidel methods), later on the same author proposed the solution of such system using Successive over relaxation iterative method [5] and Adomian decomposition method [6] Abbasbandy et al. [3] proposed the Conjugate gradient method, for solving fuzzy symmetric positive definite system of linear equation. Dehghan and Hashemi [9] extended the Adomian decomposition method [6], to find the positive fuzzy vector solution of fully fuzzy linear system. Dehghan et al. [8] proposed classic methods such as Cramer's rule, Gaussian elimination method, LU decomposition method from linear algebra and linear programming for finding the approximated solution of a fully fuzzy linear systems. Abbasbandy and Jafarian [2] applied Steepest descent method for approximation of the unique solution of fuzzy system of linear equation. Abbasbandy et al. [1] used LU decomposition method for solving fuzzy system of linear equation when the coefficient matrix is symmetric positive definite. Muzzioli and Reynaerts [17] pointed out that although several investigations are reported in the literature of the solution of fuzzy systems, very few methods are available for the practical solution of a fuzzy linear system. They introduced an algorithm to find vector solution by transforming the system $A_1x + B_1 = A_2x + B_2$ into the FFLS $Ax = B$ where $A = A_1 - A_2$ and $B = B_1 - B_2$.

Dehghan and Hashemi [10] modified the existing methods employed by Allahviranloo [4] for solving fuzzy linear systems. Mosleh et al. [16] proposed a method to find the solution of fully fuzzy linear system of the form $Ax + B = Cx + D$ with A, C square matrices of fuzzy coefficients and B, D fuzzy number vectors and the unknown vector x is vector consisting of n fuzzy numbers. Nasseri et al. [18] used a certain decomposition methods of the coefficient matrix for solving fully fuzzy linear system of equations. Yin and Wang [21] considered the general case of Splitting iterative methods for solving fuzzy system of linear equation.

Sun and Guo [20] proposed a general model for solving fuzzy linear systems of the standard form and general dual fuzzy linear systems. Nasseri and Zahmatkesh [19] proposed a new method for computing the non-negative solution of fully fuzzy linear system of equations.

In this paper, a new computational method for finding the solution of FFLS $\widetilde{A} \otimes \widetilde{x} = \widetilde{b}$, where $\widetilde{A}$ is a non negative fuzzy matrix, $\widetilde{x}$ and $\widetilde{b}$ are fuzzy vectors with appropriate sizes, is proposed. The method eliminates all restrictions on the solution vector $\widetilde{x}$. The method is illustrated by solving numerical examples. The method relies on the brute force strategy to decompose a FFLS into all the possible linear configurations with added constraints. However the researchers are open to use any of the other linear and non linear models and the method presented in the paper is just used to exemplify the concept of solving a FFLS with no restriction on the solution vector.

The rest of this paper is organized as follows: In section 2, shortcomings of the existing methods to solve FFLS are described. In section 3 some basic deûnitions are reviewed. In Section 4 a new method is proposed for solving FFLS. In section 5, to illustrate the proposed method, numerical examples are solved. In section 6 the paper is concluded.

2. Shortcomings of existing methods

In this section the shortcomings in the existing methods [1-8, 9, 10, 18, 19] are pointed out:

1. The existing methods presume the non negativity of the solution vector. This restriction creates difficulty in using the existing methods to solve a FFLS occurring in real life situations for which the solution may not be entirely non negative.

2. In all the existing methods, it is assumed that the system of equations is consistent and then the methods are developed i.e. consistency of the FFLS cannot be checked using the existing methods.

3. Using the existing methods it is not possible to check that the obtained solution is unique or not.

To overcome the above shortcomings, in section 4, a new computational method is proposed for solving a FFLS.

3. Preliminaries

3.1 Basics Definitions

In this section some basic definitions of fuzzy set theory are reviewed [15].

Definition 1

The Characteristic function $\mu_{\tilde{A}}$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ is called a fuzzy set.

Definition 2

A fuzzy set $\tilde{A}$, defined on the universal set of real number R, is said to be a fuzzy number if its membership function has the following characteristics:

(*i*) $\tilde{A}$ is convex i.e., e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \forall x_1, x_2 \in R, \forall \lambda \in [0,1]$

(*ii*) $\tilde{A}$ normal i.e., $\exists x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$

(*iii*) $\mu_{\tilde{A}}$ is piecewise continuous.

Definition 3

A fuzzy number $\widetilde{A}$ is said to be non-negative fuzzy number if and only $\mu_{\widetilde{A}}(x) = 0 \forall x < 0$

3.2 Existing representation of triangular fuzzy numbers

In the literature, triangular fuzzy numbers are represented as follows:

Definition 4

A fuzzy number $\widetilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b < x \leq c \\ 0, & x > c \end{cases}$$

Definition 5

A triangular fuzzy number $\widetilde{A} = (a, b, c)$ is said to be non-negative triangular fuzzy number if and only if $a \geq 0$.

Definition 6

A triangular fuzzy number $\widetilde{A} = (a, b, c)$ is said to be zero triangular fuzzy number if and only if $a = 0, b = 0 \ and \ c = 0$

Definition 7

Two fuzzy number $\widetilde{A} = (a, b, c)$ and $\widetilde{B} = (e, f, g)$ are said to be equal i.e. $\widetilde{A} = \widetilde{B}$ if and only if $a = e, b = f, c = g$.

Definition 8

A matrix $\widetilde{A} = (\widetilde{a}_{ij})$ is called a fuzzy matrix, if each element of this matrix is a fuzzy number. It will be positive (negative) and denoted by $\widetilde{A} > 0$ ($\widetilde{A} < 0$) if each element of this matrix be positive (negative). It will be non positive (non negative) and denoted by $\widetilde{A} \leq 0$ ($\widetilde{A} \geq 0$) if each element be non positive (non negative). We may represent $n \times m$ fuzzy matrix by $\widetilde{A} = (\widetilde{a}_{ij})_{n \times m}$ where $\widetilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij})$

Definition 9

Let $\widetilde{A}=(\widetilde{a}_{ij})$ and $\widetilde{B}=(\widetilde{b}_{ij})$ be two $m\times n$ and $n\times p$ fuzzy matrices. We define $\widetilde{C}=\widetilde{A}\otimes\widetilde{B}=(\widetilde{c}_{ij})$ which is the $m\times p$ matrix where $\widetilde{c}_{ij}=\sum_{k=1,\ldots,n}^{\oplus}\widetilde{a}_{ik}\otimes\widetilde{b}_{kj}$

3.3 Arithmetic operations on triangular fuzzy numbers

In this subsection addition and multiplication operations between two triangular fuzzy numbers are reviewed. Let there be two fuzzy numbers $\widetilde{A}=(a,b,c)$ and $\widetilde{B}=(e,f,g)$ then

(i) $(a,b,c)\oplus(e,f,g)=(a+e,b+f,c+g)$

(ii) $(a,b,c)\ominus(e,f,g)=(a-g,b-f,c-e)$

(iii) If $\widetilde{A}=(a,b,c)$ is non negative, then

$(a,b,c)\otimes(e,f,g)=(\min(ae,ce),bf,\max(ag,cg))$

where

$$\min(ae,ce)=\left(\frac{a+c}{2}\right)e-\left|\frac{c-a}{2}\right||e| \text{ and } \max(ag,cg)=\left(\frac{a+c}{2}\right)g+\left|\frac{a-c}{2}\right||g|$$

4. Proposed method

In this section a new computational method is proposed to find solutions of FFLS $\widetilde{A}\otimes\widetilde{x}=\widetilde{b}$ where $\widetilde{A}\geq 0$

i.e.

$$(\widetilde{a}_{11}\otimes\widetilde{x}_1)\oplus(\widetilde{a}_{12}\otimes\widetilde{x}_2)\oplus\ldots\oplus(\widetilde{a}_{1n}\otimes\widetilde{x}_n)=\widetilde{b}_1$$

$$(\widetilde{a}_{21}\otimes\widetilde{x}_1)\oplus(\widetilde{a}_{22}\otimes\widetilde{x}_2)\oplus\ldots\oplus(\widetilde{a}_{2n}\otimes\widetilde{x}_n)=\widetilde{b}_2$$

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$$(\widetilde{a}_{n1}\otimes\widetilde{x}_1)\oplus(\widetilde{a}_{n2}\otimes\widetilde{x}_2)\oplus\ldots\oplus(\widetilde{a}_{nn}\otimes\widetilde{x}_n)=\widetilde{b}_n$$

The steps of the proposed method are as follows:

Step1 Substituting $\widetilde{A}=(\widetilde{a}_{ij})_{n\times n}$, $\widetilde{x}=(\widetilde{x}_j)_{n\times 1}$ and $\widetilde{b}=(\widetilde{b}_j)_{n\times 1}$ the FFLS may be written as:

$$\sum_{j=1,\dots,n}^{\oplus}\widetilde{a}_{ij}\otimes\widetilde{x}_j=\widetilde{b}_i\ \forall i=1,2,\dots,n. \qquad (4.1)$$

Step 2 If all the parameters $\widetilde{a}_{ij},\widetilde{x}_j$ and $\widetilde{b}_j$ are represented by triangular fuzzy numbers (a_{ij},b_{ij},c_{ij}), (x_j,y_j,z_j) and (b_i,g_i,h_i) respectively, then the FFLS obtained in step 1, may be written as:

$$\sum_{j=1,2,\dots,n}^{\oplus}(a_{ij},b_{ij},c_{ij})\otimes(x_j,y_j,z_j)=(b_i,g_i,h_i)\ \forall i=1,2,\dots,n \qquad (4.2)$$

Step 3 Assuming $(a_{ij},b_{ij},c_{ij})\otimes(x_j,y_j,z_j)=(f_{ij},p_{ij},q_{ij})$, the FFLS, obtained in step 2, may be written as:

$$\sum_{j=1,2,\dots,n}^{\oplus}(f_{ij},p_{ij},q_{ij})=(b_i,g_i,h_i)\ \forall i=1,2,\dots,n \qquad (4.3)$$

where

$$f_{ij}=\min(a_{ij}x_j,c_{ij}x_j)=(\frac{a_{ij}+c_{ij}}{2})x_j-\left|\frac{a_{ij}-c_{ij}}{2}\right||x_j| \qquad (4.4)$$

$$p_{ij}=b_{ij}y_j \qquad (4.5)$$

$$q_{ij}=\max(a_{ij}z_j,c_{ij}z_j)=(\frac{a_{ij}+c_{ij}}{2})z_j+\left|\frac{a_{ij}-c_{ij}}{2}\right||z_j| \qquad (4.6)$$

Step 4 Using the arithmetic operations, defined in section 3, the FFLS, obtained in step 3, may be written

$$(\sum_{j=1}^{n}f_{ij},\sum_{j=1}^{n}p_{ij},\sum_{j=1}^{n}q_{ij})=(b_i,g_i,h_i)\quad \forall i=1,2,\dots,n \qquad (4.7)$$

Step 5 The FFLS, obtained in step 4, may be converted into the following crisp system of equations

$$\sum_{j=1}^{n}p_{ij}=g_i\Rightarrow\sum_{j=1}^{n}b_{ij}y_j=g_i\quad \forall i=1,2,\dots,n \qquad (4.8)$$

$$\sum_{j=1}^{n}f_{ij}=b_i\Rightarrow\sum_{j=1}^{n}\left(\frac{a_{ij}+c_{ij}}{2}\right)x_j-\left|\frac{a_{ij}-c_{ij}}{2}\right||x_j|=b_i\quad \forall i=1,2,\dots n \qquad (4.9)$$

$$\sum_{j=1}^{n}q_{ij}=h_i\Rightarrow\sum_{j=1}^{n}\left(\frac{a_{ij}+c_{ij}}{2}\right)z_j+\left|\frac{a_{ij}-c_{ij}}{2}\right||z_j|=h_i\quad \forall i=1,2,\dots n \qquad (4.10)$$

Step 6 Solve the $n \times n$ crisp linear system of equations (4.8) using any classical method like LU decomposition, matrix inversion, Cramer's rule, row reduced echelon form etc. i.e. any method from the existing literature can be applied to solve this crisp system. Check that the linear system obtained from equation (1) is consistent or not.

Case (i) If the system of equations is inconsistent then the given FFLS is inconsistent.

Case (ii) If the system of equation is consistent then find solution of (4.8) i.e. evaluate $y_j \forall j = 1,2,...,n$ and Go to step 7.

Step 7 Solve the system of equations (4.9) by assuming appropriate constraints. Since $x_j \forall j = 1,2,...,n$ may be either negative or non negative, so there are 2^n possible choices to break the n equations. Hence break the system of equations (4.9) into 2^n number of sets of $n \times n$ crisp linear system of equations and n constraints each. Find the solution of this system, using any classical method. Compute all the solutions $x_j \forall j = 1,2,...,n$ and Go to step 8. However if there doesn't exist any solution of (4.9), then the $n \times n$ FFLS has no solution. We have used the brute force strategy to decompose the system into set of all possible linear systems. However we may use any other like linear programming method to solve this system. The method is used just to clarify the concept.

Step 8 Repeat the procedure of step 7 for the system of equations (4.10) and Go to step 9.

Step 9 Find all the unique tuples $\tilde{x}_j = (x_j, y_j, z_j) \forall j = 1,2,...,n$ and Go to step 10.

Step 10 Find all the feasible solutions of the FFLS $\tilde{x}_j = (x_j, y_j, z_j) \forall j = 1,2,...,n$ i.e. all the tuples that satisfy the constraint $z_j \geq y_j \geq x_j$

Remark 1 The $n \times n$ FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ will have a feasible solution $\tilde{x}_j = (x_j, y_j, z_j) \forall j = 1,2,...,n$ iff $z_j \geq y_j \geq x_j$ else the solution will be infeasible.

5 Numerical examples

Example 1

Let us consider the following FFLS and solve it by the proposed method

$(2,2,3) \otimes \tilde{x}_1 \oplus (2,3,4) \otimes \tilde{x}_2 = (1,8,15)$

$(1,1,2) \otimes \tilde{x}_1 \oplus (1,2,3) \otimes \tilde{x}_2 = (0,5,11)$

Solution: The given FFLS can be written as

$$\sum_{j=1,\ldots,n}^{\oplus} \tilde{a}_{ij} \otimes \tilde{x}_j = \tilde{b}_i \;\forall i = 1,2,\ldots,n. \tag{5.1}$$

Using step 2, let $\tilde{x}_1 = (x_1, y_1, z_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2)$, the given FFLS can be rewritten as:

$$\sum_{j=1,2,\ldots,n}^{\oplus} (a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) = (b_i, g_i, h_i) \;\forall i = 1,2,\ldots,n \tag{5.2}$$

where $a_{11} = 2, b_{11} = 2, c_{11} = 3, a_{12} = 2, b_{12} = 3, c_{12} = 4, a_{21} = 1, b_{21} = 1, c_{21} = 2, a_{22} = 1, b_{22} = 2, c_{23} = 3$

Using step 3,

$$\sum_{j=1,2,\ldots,n}^{\oplus} (f_{ij}, p_{ij}, q_{ij}) = (b_i, g_i, h_i) \;\forall i = 1,2,\ldots,n \tag{5.3}$$

where

$$f_{ij} = \min(a_{ij}x_j, c_{ij}x_j) = \left(\frac{a_{ij} + c_{ij}}{2}\right)x_j - \left|\frac{a_{ij} - c_{ij}}{2}\right| |x_j| \tag{5.4}$$

$$p_{ij} = b_{ij} y_j \tag{5.5}$$

$$q_{ij} = \max(a_{ij}z_j, c_{ij}z_j) = \left(\frac{a_{ij} + c_{ij}}{2}\right)z_j + \left|\frac{a_{ij} - c_{ij}}{2}\right| |z_j| \tag{5.6}$$

Using step 4 and 5, we break the given FFLS into following set of 3 system of equations

$$\begin{cases} 2y_1 + 3y_2 = 8 \\ y_1 + 2y_2 = 5 \end{cases} \tag{5.7}$$

$$\begin{cases} 2.5x_1 + 3x_2 - 0.5|x_1| - |x_2| = 1 \\ 1.5x_1 + 2x_2 - 0.5|x_1| - |x_2| = 0 \end{cases} \tag{5.8}$$

$$\begin{cases} 2.5z_1 + 3z_2 + 0.5|z_1| + |z_2| = 15 \\ 1.5z_1 + 2z_2 + 0.5|z_1| + |z_2| = 11 \end{cases} \tag{5.9}$$

Using step 6, we solve the linear system of equation (5.7)

$y_1 = 1, y_2 = 2$

Using step 7, we solve the linear system of equation (5.8):

The given set of equations may be converted into the following 4 sets of 2×2 linear system of equations with 2 constraints each

case 1	case 2	case 3	case 4
$2x_1 + 2x_2 = 1$	$2x_1 + 4x_2 = 1$	$3x_1 + 2x_2 = 1$	$3x_1 + 4x_2 = 1$
$x_1 + x_2 = 0$	$x_1 + 3x_2 = 0$	$2x_1 + x_2 = 0$	$2x_1 + 3x_2 = 0$
$x_1 \geq 0, x_2 \geq 0$	$x_1 \geq 0, x_2 \leq 0$	$x_1 \leq 0, x_2 \geq 0$	$x_1 \leq 0, x_2 \leq 0$
no solution	no solution	$x_1 = -1, x_2 = 2$	no solution

Using step 8, we solve the linear system of equation (5.9) :

The given set of equations may be converted into the following 4 sets of 2×2 linear system of equations with 2 constraints each

case 1	case 2	case 3	case 4
$2z_1 + 2z_2 = 15$	$2z_1 + 4z_2 = 15$	$3z_1 + 2z_2 = 15$	$3z_1 + 4z_2 = 15$
$z_1 + z_2 = 11$	$z_1 + 3z_2 = 11$	$2z_1 + z_2 = 11$	$2z_1 + 3z_2 = 11$
$z_1 \geq 0, z_2 \geq 0$	$z_1 \geq 0, z_2 \leq 0$	$z_1 \leq 0, z_2 \geq 0$	$z_1 \leq 0, z_2 \leq 0$
no solution	no solution	$z_1 = 7, z_2 = -3$	$z_1 = 1, z_2 = 3$

Using step 9, we find all unique tuples $\tilde{x}_j = (x_j, y_j, z_j) \forall j = 1,2,...,n$

1. $\tilde{x}_1 = (-1,1,1)$ and $\tilde{x}_2 = (2,2,3)$

2. $\tilde{x}_1 = (-1,1,7)$ and $\tilde{x}_1 = (2,2,-3)$

Using step10, we find the unique and feasible solution of the FFLS: $\tilde{x}_1 = (-1,1,1)$ and $\tilde{x}_2 = (2,2,3)$

Example 2

Let us consider the following FFLS and solve it by the proposed method

$$(2,2,3) \otimes \tilde{x}_1 \oplus (2,4,4) \otimes \tilde{x}_2 = (6,12,18)$$

$$(4,4,5) \otimes \tilde{x}_1 \oplus (4,5,5) \otimes \tilde{x}_2 = (12,18,25)$$

Solution

The given FFLS can be written as

$$\sum_{j=1,\ldots,n}^{\oplus} \tilde{a}_{ij} \otimes \tilde{x}_j = \tilde{b}_i \ \forall i = 1,2,\ldots,n. \qquad (5.10)$$

Using step 2, let $\tilde{x}_1 = (x_1, y_1, z_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2)$, the given FFLS can be rewritten as:

$$\sum_{j=1,2,\ldots,n}^{\oplus} (a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) = (b_i, g_i, h_i) \ \forall i = 1,2,\ldots,n \qquad (5.11)$$

where $a_{11} = 2, b_{11} = 2, c_{11} = 3, a_{12} = 2, b_{12} = 4, c_{12} = 4, a_{21} = 4, b_{21} = 4, c_{21} = 5, a_{22} = 4, b_{22} = 5, c_{23} = 5$

Using step 3,

$$\sum_{j=1,2,\ldots,n}^{\oplus} (f_{ij}, p_{ij}, q_{ij}) = (b_i, g_i, h_i)\ \forall i = 1,2,\ldots,n \tag{5.12}$$

where

$$f_{ij} = \min(a_{ij}x_j, c_{ij}x_j) = \left(\frac{a_{ij} + c_{ij}}{2}\right)x_j - \left|\frac{a_{ij} - c_{ij}}{2}\right||x_j| \tag{5.13}$$

$$p_{ij} = b_{ij}y_j \tag{5.14}$$

$$q_{ij} = \max(a_{ij}z_j, c_{ij}z_j) = \left(\frac{a_{ij} + c_{ij}}{2}\right)z_j + \left|\frac{a_{ij} - c_{ij}}{2}\right||z_j| \tag{5.15}$$

Using step 4 and 5, we break the given FFLS into following set of 3 system of equations

$$\begin{cases} 2y_1 + 4y_2 = 12 \\ 4y_1 + 5y_2 = 18 \end{cases} \tag{5.16}$$

$$\begin{cases} 2.5x_1 + 3x_2 - 0.5|x_1| - |x_2| = 6 \\ 4.5x_1 + 4.5x_2 - 0.5|x_1| - 0.5|x_2| = 12 \end{cases} \tag{5.17}$$

$$\begin{cases} 2.5z_1 + 3z_2 + 0.5|z_1| + |z_2| = 18 \\ 4.5z_1 + 4.5z_2 + 0.5|z_1| + 0.5|z_2| = 25 \end{cases} \tag{5.18}$$

Using step 6, the system of equation (5.16) has the following solution:

$$y_1 = 2, y_2 = 2$$

Using step 7, The system of equation (5.17) has infinite solutions:

$$x_1 = u, x_2 = 3 - u\ \forall u \in [0,3]$$

Using step 8, the system of equation (5.18) has following solutions

$$z_1 = 2, z_2 = 3 \text{ and } z_1 = 11, z_2 = -7.5$$

Using step 9, we find all unique tuples $\tilde{x}_j = (x_j, y_j, z_j)\forall j = 1,2,\ldots,n$

1. $\tilde{x}_1 = (u,2,2)$ and $\tilde{x}_2 = (3-u,2,3)$ (Feasible solution for $u \in [1,2]$)
2. $\tilde{x}_1 = (u,2,11)$ and $\tilde{x}_1 = (3-u,2,-7.5)$

Using step10, we find the feasible solutions of the FFLS: $\tilde{x}_1 = (u,2,2)$ and $\tilde{x}_2 = (3-u,2,3)$ for $u \in [1,2]$.Clearly the FFLS has infinitely many solutions.

6. Conclusion

In this paper, a new computational method for solving a fully fuzzy linear system with triangular fuzzy numbers of the form (a,b,c) is presented. The method eliminates the non negative restriction on the solution vector. The proposed method is easy to understand and apply in real life scenarios. The method is illustrated with the help of numerical examples.

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