PERFORMANCE MEASURES FOR BAYESIAN CHAIN SAMPLING PLAN WITH GEOMETRIC DISTRIBUTION

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ABSTRACT

This paper is concerned with the set of tables for the construction and selection of performance measures for BChSp-1 with Beta prior indexed through the entry parameters. The AQL, LQL and IQL values are obtained and these measures are compared with conventional parameters.

Key words: Bayesian chain sampling, Beta Geometric distribution, Acceptance quality level, limiting quality level (LQL), Indifference Quality Level (IQL).

2. INTRODUCTION

Acceptance Sampling is a form of inspection applied to lots or batches of items before or after a process to judge conformance to predetermined standards. Classical statistics is directed towards the use of sample information. In addition to the sample information two other types of information are typically relevant. The first is knowledge of the possible consequences of the decision and the second source of non-sample information is prior information in inductive inference and the approach to statistics, which formally seeks to utilize prior information, is called Bayesian analysis. In Bayesian acceptance sampling the conclusion about the lot is given by sample information and prior information.

Most of the customers understand that 100% inspection is impractical and are generally willing to accept that a certain level of defectives will be produced. Hald (1981) introduced the statistical theory of sampling inspection by Attributes. To overcome the disadvantages of single sampling plan Govindaraj (1990) studied the average probability of acceptance for the given quality level. Latha and Jeyabharathi (2012) has provided a formula to compute the average probability of acceptance of chain sampling plans (ChSP-1) by attributes under Beta- Geometric

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distribution. Sampling inspection in which the criteria for acceptance and non -acceptance of the lot depends on the results of the inspection of immediately preceding lot is adopted in chain sampling plan (ChSP–1) which is proposed by Dodge (1955).This plan make use of cumulative results of several samples help to overcome the short coming of the single sampling plan. Suresh and Latha (2001) discussed the average probability of acceptance of Bayesian Chain sampling plan using Gamma distribution as the prior distribution. Latha and Jeyabharathi (2013) studied the average probability of Acceptance of Bayesian chain sampling plans (ChSP-1) by attributes under Beta-Binomial distribution.

In this paper, the selection procedures are provided for Bayesian Chain Sampling plan BChSP-1 with Beta Geometric (when the product quality follows Beta prior distribution) under the following circumstances. For specified n and s, the parameter of the prior distribution with 1.AQL and LQL

2. Indifference Quality Level or Point of Control

3. OPERATING PROCEDURE OF CHAIN SAMPLING

- The plan is implemented in the following way.
- For each lot, select sequence of sample of 'n' units and test each unit for conformance to the specified requirements.
- Accept the lot if d (the observed number of defectives) in zero in the sequence of sample of 'n' units, and reject if $d > 1$.
- Accept the lot if $d = 1$ and if no defectives are formed in the immediately preceding 'i' sequence of sample size 'n'.

The procedures and tables for the construction and selection of ChSP-1 plans developed by Soundararajan (1978) give plans approximately satisfying the conditions specified such as given Acceptance Quality level (AQL), and producer's risk (2), Limiting Quality Level (LQL), and consumer's risk β. Suresh and Latha (2001) discussed the construction And Evaluation of Performance Measures of Bayesian Chain Sampling Plan with Gamma prior distribution.

4. BETA - GEOMETRIC DISTRIBUTION

The Beta-Geometric (s,t) distribution models that the no of success that will occur in a binomial process before the first failure is observed and where, the Geometric probability 'p' is itself a random variable taking Beta (s,t) distribution.

Geometric variable 'x' giving the number of success that will occur before the first failure is given by

$$g(x,p) = p^x(1-p)$$

Where 'n' is the number of trails in which the first failure occurs ($n = 1,2 \ldots\ldots$) and 'p' is the probability of success on each trail assuming 'p' has Beta-Geometric distribution is given by

$$g_w(x,p) = \int_0^1 g(x,p)w(p)dp$$

4. BAYESIAN CHAIN SAMPLING PLAN

According to Dodge (1955) the operating characteristic function of ChSP-1 is

$$Pa\,(p) = P_0 + P_1\,(P_0)^i$$

Using the past history of inspection, it is observed that 'p' follows a Beta distribution with density function.

$$w\,(p)\,dp = \frac{1}{\beta(s,t)} \int_0^1 p^{s-1}\,(1-p)^{t-1}dp$$

The Average probability of acceptance is given by

$$\bar{P} = \int_0^1 p\,(n,i/p)\,w\,(p)\;dp$$

$$\bar{P} = \frac{t}{s+t} + \frac{s(t+i)(t+i-1)(t+i-2)\ldots\ldots}{(s+t+i+1)(s+t+i)(s+t+i-1)\ldots\ldots(s+t)} \ldots\ldots\ldots\ldots \qquad (A)$$

For, $i = 1$, the above equation (A) can be reduced to $\bar{P} = (1-\mu)\;\;1 + \frac{s\mu\,(s-\mu s+\mu)}{(s+2\mu)(s+\mu)}$ (1)

For, $i = 2$, the above equation (A) can be reduced to

$$\bar{P} = (1-\mu)\;\left\{1 + \frac{s\mu\,(s-\mu s+2\mu)(s-\mu s+\mu)}{(s+3\mu)(s+3\mu)(s+\mu)}\right\} \qquad (2)$$

For, $i = 3$, the above equation (A) can be reduced to

$$\bar{P} = (1-\mu)\;\left\{1 + \frac{s\mu\,(s-\mu s+3\mu)(s-\mu s+2\mu)\,(s-\mu s+\mu)}{(s+4\mu)(s+3\mu)(s+2\mu)(s+\mu)}\right\} \qquad (3)$$

For, $i = 4$, the above equation (A) can be reduced to

$$\bar{P} = (1-\mu)\left\{1 + \frac{s\mu\,(s-\mu s+4\mu)(s-\mu s+3\mu)\,(s-\mu s+2\mu)(s-\mu s+\mu)}{(s+5\mu)(s+4\mu)(s+3\mu)(s+2\mu)(s+\mu)}\right\} \qquad (4)$$

For, $i = 5$, the above equation (A) can be reduced to

$$\overline{P} = (1-\mu) \times \left\{1 + \frac{s\mu\,(s-\mu s+5\mu)(s-\mu s+4\mu)\,(s-\mu s+3\mu)(s-\mu s+2\mu)(s-\mu s+\mu)}{(s+6\mu)(s+5\mu)(s+4\mu)(s+3\mu)(s+2\mu)(s+\mu)}\right\} \quad (5)$$

The Probability of Acceptance obtained in equations (1),(2),(3),(4), and (5) are equated to 0.5 and Indifference Quality Level μ_0 values are obtained for various values i and s are presented in the Table.2.1

Example:1

For $i = 1, s = 1$ the corresponding IQL value $\mu_0 = 0.5725$, for $i = 2, s = 1$ the corresponding IQL value $\mu_0 = 0.5450$,for $i = 3, s = 1$ the IQl value $\mu_0 = 0.5300$ and for i=4,s=1 the corresponding IQL value $\mu_0 = 0.5223$. Therefore the value of i increased, the Indifference Quality Level is decreased.

In conventional sampling plan $P_0 = 0.5725$ for $i = 3$, $P_0 = 0.5450$ for i= 3 and $\mu_0 = 0.5300$ for $i = 4$ .It is observed that when compared to the conventional sampling plan, the i values are very small in Bayesian Chain sampling plan.Hence this sampling plan will be more advantageous to the consumers.

The equations 1),(2),(3),(4), and (5) are equated to the probability values 0.95 and 0.10, the coresponding AQL and LQL values are obtained using Newtons Mehtod of approximation.For these values the operating ratio is also calculated and presented the Table 2.1.Using the operating ratio one can easily obtain the corresponding i value.

Example:2

For i=1,s=1 it is given that AQL $\mu_1 = 0.1300$, LQL $\mu_2 = 0.9144$ and IQL $\mu_0 = .5725$

For conventional sampling plan AQL = 0.1700, LQL = 0.9844 and IQL $\mu_0 = .5825$

For i=2,s=1 it is given that AQL $\mu_1 = 0.1100$, LQL $\mu_2 = 0.9000$ and IQL $\mu_0 = 0.5450$

For conventional sampling plan AQL = 0.1500, LQL = 0.9944 and IQL $\mu_0 = .5925$

For i=3,s=1 it is given that AQL $\mu_1 = 0.0993$, LQL $\mu_2 = 0.9000$ and IQL $\mu_0 = 0.5300$

For conventional sampling plan AQL = 0.1100, LQL = 0.9962 and IQL $\mu_0 = .5895$

It is observed that when 'i' increased, AQL,LQL and IQL values are decreased and the IQL value lies between AQL and LQL.When compared to the conventional sampling plan,

AOL, LQL and IQL values very high. Hence Bayesain approach will be more advantageous to the consumers.

In the similarway, the Average Probability Acceptance(A) is equated to various probability values,we can get various product quality 'μ' and those values are presented in Table 1.1 to 1.4.

Example:3

For, i=1,s=2, $\bar{P} = 0.90$,the corresponding product quality μ=0.2113.

But in conventional sampling $\bar{P} = 0.95$, the corresponding product quality μ=0.2113.

For, i=2,s=2, $\bar{P} = 0.95$, the corresponding product quality μ=0.1825 where as in conventional sampling plan$\bar{P} = 0.97$, the corresponding product quality is μ=0.1825

It is observed that, when 'i'values are increased, the average product quality is decreased and when compared to the conventional sampling plan, chance of getting defective items are very less in Bayesian chain sampling plan. Hence this plan will be more advantageous to the consumers.

5.Conclusion

Bayesian Acceptance Sampling is the technique which deals with the procedures in which decision to accept or reject the lots or process is based on the examination of past history or knowledge of samples. But in conventional or non-Bayesian plans, only a limited amount of quality history can be used. Hence, Bayesian approach can be useful to the producer who may have complete control of the process. When compared to the conventional sampling, Bayesian approach will be more advantageous to the consumer. This paper is mainly related to the construction and selection of performance measures for BChSp-1 with Beta prior indexed through AQL and LQL.

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Table 1.1: μ values for Bayesian Chain Sampling Plan (BChSP-1) for given average Probability of Acceptance for i=1

1	$\bar{P}$ / S	0.99	0.95	0.90	0.50	0.10	0.05
i=1	1	0.0533	0.1300	0.2000	0.5725	0.9144	0.9571
	2	0.0600	0.1430	0.2113	0.5840	0.9151	0.9573
	3	0.0635	0.1500	0.2200	0.6000	0.9145	0.9569
	4	0.0653	0.1524	0.2230	0.6000	0.9138	0.9564
	5	0.0665	0.1546	0.2257	0.6000	0.9132	0.9560

Table 1.2: μ values for Bayesian Chain Sampling Plan (BChSP-1) for given average Probability of Acceptance for i=2

	$\bar{P}$ / S	0.99	0.95	0.90	0.50	0.10	0.05
i=2	1	0.0444	0.1100	0.1697	0.5450	0.9000	0.9539
	2	0.0500	0.1212	0.1825	0.5498	0.9000	0.9533
	3	0.0526	0.1260	0.1881	0.5500	0.9000	0.9527
	4	0.0541	0.1287	0.1913	0.5512	0.9000	0.9521

	5	0.0550	0.1300	0.1933	0.5513	0.9000	0.9518

Table 1.3: μ values for Bayesian Chain Sampling Plan (BChSP-1) for given average Probability of Acceptance for i=3

	$\bar{p}$ / S	0.99	0.95	0.90	0.50	0.10	0.05
i=3	1	0.0390	0.0993	0.1551	0.5300	0.9000	0.9524
	2	0.0439	0.1000	0.1654	0.5321	0.9000	0.9517
	3	0.0461	0.1122	0.1699	0.5317	0.9000	0.9512
	4	0.0474	0.1145	0.1724	0.5310	0.9000	0.9500
	5	0.0482	0.1159	0.1740	0.5300	0.9000	0.9500

Table 1.4: μ values for Bayesian Chain Sampling Plan (BChSP-1) for given average Probability of Acceptance for i=4

	$\bar{p}$ / 5	0.99	0.95	0.90	0.50	0.10	0.05
i=4	1	0.0354	0.0918	0.1455	0.5223	0.9000	0.9517
	2	0.0397	0.0995	0.1539	0.5219	0.9000	0.9510
	3	0.0417	0.1000	0.1576	0.5200	0.9000	0.9500
	4	0.0428	0.1000	0.1596	0.5198	0.9000	0.9500
	5	0.0435	0.1000	0.1600	0.5190	0.5190	0.9500

Table 2.1 CERTAIN PARAMETERIC VALUES OF BAESIAN CHAIN SAMPLING PLAN

i=1	S	μ_1	μ_2	μ_0	μ_2 / μ_1
	1	0.1300	0.9144	.5725	0.5223
	2	0.1430	0.9151	0.5840	6.399301