ON α GENERALIZED CLOSED SETS IN IDEAL TOPOLOGICAL SPACES

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Abstract : In this research paper, we are introducing the concept of α-generalized closed sets in Ideal topological space and discussed the characterizations and the properties of α-generalized closed sets in Ideal topological space.

Keywords : *Ig closed sets, $I\hat{g}$ - closed set, αIg- closed sets, Semi- I closed set, Pre- I closed set, α- I closed set, b- I closed set.*

1. INTRODUCTION

The notion of α-open sets was introduced and investigated by Njastad[1]. By using α-open sets, Mashhour et al.[2] defined and studied the concept of α-closed sets, α-closure of a set, α-continuity and α-closedness in topology. Ideals in topological spaces have been considered since 1930. This topic has won its importance by the paper of Vaidyanathaswamy[3]. It was the works of Newcomb[4], Rancin[5], Samuels and Hamlet and Jankovic([6, 7, 8, 9, 10]) which motivated the research in applying topological ideals to generalize the most basic properties in General Topology.

2. PRELIMINARIES

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X, which satisfies the following two conditions:

(i) If $A \in I$ and $B \subseteq A$ implies $B \in I$

(ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$ [11].

An ideal topological space is a topological space (X, τ) with an ideal I on X and it is denoted by (X, τ, I). Given a topological space (X, τ) with an ideal I on X and if $\wp(X)$ is the set of all subsets of X, a set operator (*) : $\wp(X) \to \wp(X)$, called a local function[11] of A with respect to τ and I, is defined as follows: for $A \subseteq X$, $A^*(I,\tau)=\{x \in X / U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x)=\{U \in \tau / x \in U\}$. We simply write A^* instead of $A^*(I,\tau)$. For every Ideal topological space (X, τ, I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I, \tau$

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)={U-i/U ∈ τ &i ∈ I}. But in general (I, τ) is not always a topology. Additionally $cl^*(A)=A\cup A^*$ defines a kuratowski closure operator for $\tau^*(I)$. If $A \subseteq X$, cl(A) and int(A) will, respectively, denote the closure and interior of A in (X, τ) and $int^*(A)$ denote the interior of A in (X, τ^*). A subset A of an ideal space (X, τ, I) is *-closed(resp. *-dense in itself) if $A^* \subseteq A$ (resp. $A \subseteq A^*$).

Definition 2.1[13]:
A subset A of a topological space (X, τ) is called a generalized closed set (briefly g-closed) if $cl(A) \subseteq A$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.2[15]:
A subset A of a topological space (X, τ) is called a α-generalized closed set (briefly αg-closed set) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.3[15]:
A subset A of a topological space (X, τ) is called $\hat{g}$-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.

Definition 2.4[15]:
A subset A of a topological space (X, τ) is said to be
- (i) preclosed set if $cl(int(A)) \subseteq A$.
- (ii) semiclosed set if $int(cl(A)) \subseteq A$.
- (iii) α-closed set if $cl(int(cl(A))) \subseteq A$.
- (iv) b-closed set if $cl(int(A)) \cup int(cl(A)) \subseteq A$.

Definition 2.5[12]:
Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be Ideal generalized closed set (briefly Ig- closed set) if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.6 [14]:
A subset A of an ideal topological space (X, τ, I) is said to be
- (i) pre –I-closed set if $cl^*(int(A)) \subseteq A$.
- (ii) semi-I- closed set if $int(cl^*(A)) \subseteq A$.
- (iii) α-I-closed set if $c^*l(int(cl^*(A))) \subseteq A$.
- (iv) b-I-closed set if $cl^*(int(A)) \cup int(cl^*(A)) \subseteq A$.

Definition 2.7[11]:
Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be $I\hat{g}$-closed set if $A^* \subseteq U$ whenever $A \subseteq U$ and U is semi-open.

Lemma 2.8:[12]
Let (X, τ, I) be an ideal topological space and A, B subsets of X. Then the following properties hold:

(i) $A \subseteq B \Rightarrow A^* \subseteq B^*$,
(ii) $A^* = cl(A^*) \subseteq cl(A)$,
(iii) $(A^*)^* \subseteq A^*$,
(iv) $(A \cup B)^* = A^* \cup B^*$,
(v) $(A \cap B)^* \subseteq A^* \cap B^*$.

3.α-IDEAL GENERALIZED CLOSED SETS

Definition 3.1:
Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be α -Ideal generalized closed set (briefly αIg- closed set) if $A^* \subseteq U$ whenever $A \subseteq U$ and U is α-open.

Example 3.2:
Let X={a,b,c} with topology τ ={Φ,X,{a},{b},{a,b}} and I={Φ,{b}}. The set A={a,c}, where A^* ={a,c} is an αIg- closed set.

Definition 3.3:
Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be α -Ideal generalized open set (briefly αIg- open set) if X– A is αIg- closed set.

Theorem 3.4:
Every *-closed set is αIg-closed set but not conversely.
Proof:
Let A be a *-closed, then $A^* \subseteq A$. Let $A \subseteq U$, and U is α-open. This implies $A^* \subseteq U$. Hence A is αIg-closed.

Example 3.5:
Let X={a,b,c} with topology τ ={Φ,X,{a},{b,c}} and I={Φ,{c}}. It is clear that A= {b} is αIg-closed set since A^*={b,c} ⊆ U where U is α-open. But A is not a *-closed set.

Remark 3.6:
αIg- closed set and αI-closed set are independent to each other, as seen from the following examples.

Example 3.7:
Let X={a,b,c} with topology τ ={Φ,{a},{b,c},X} and I={Φ,{c}}. Clearly, the set A={b} which is an αIg-closed set is not an αI- closed set since $cl^*(int(cl^*(A)))$={b,c}$\not\subseteq$ A.

Example 3.8:
Let X={a,b,c} with topology τ ={Φ,{a},{a,c},X} and I={Φ,{b}}. It is clear that A={c} is an αI- closed set. But A is not an αIg-closed set since A^*={b,c}⊄ U.

Remark 3.9:
αIg- closed set and semi I-closed set are independent to each other, as seen from the following examples.

Example 3.10:
Let X={a,b,c} with topology τ ={Φ,{a},{b,c},X} and I={Φ,{c}}. Clearly, the set A={b} is an αIg-closed set but not semi I- closed set since int(cl*(A))={b,c}⊄ A.

Example 3.11:
Let X={a,b,c} with topology τ ={Φ,{a},{b},{a,b},X} and I={Φ,{a}}. It is clear that A={b} which is semi I-closed set. But A is not an αIg-closed set since A^*={b,c}⊄ U .

Remark 3.12:
Every pre I-closed set need not be an αIg- closed set.

Example 3.13:
Let X={a,b,c} with topology τ ={Φ,{a},{a,c},X} and I={Φ,{b}}. Clearly, the set A={c} is pre I- closed set but not an αIg- closed set since A^*={b,c}⊄ U.

Remark 3.14:
αIg- closed set and b I-closed set are independent to each other, as seen from the following examples.

Example 3.15:
Let X={a,b,c} with topology τ ={Φ,{a},{b,c},X}and I={ Φ,{c}}. Clearly, the set A={a,b} is an αIg-closed set, but not a b I-closed set, since cl*(int(A))∪ int (cl*(A)) =X⊄A.

Example 3.16:
Let X={a,b,c} with topology τ ={Φ,{a},{a,c},X} and I={ Φ, {b}}. It is clear that A={c} is bI-closed set. But A is not an αIg-closed set since A^*={b,c}⊄ U.

Theorem 3.17:
Every αIg-closed set is an Ig- closed set but not conversely.

Proof:
Let A⊆ U and U is open. Clearly every open set is α-open. Since A is αIg-closed set, A^*⊆ U, which implies that A is an Ig-closed set.

Example 3.18:

Let X={a,b,c} with topology τ ={Φ,{a},{a,c},X} and I={Φ,{b}}. Clearly, the set A={a,b} is Ig- closed set but not an αIg- closed set since A^*=X $\not\subset$ U.

Theorem 3.19:

Every I$\hat{g}$-closed set is an αIg-closed set.

Proof:

Let A$\subseteq$ U and U is α-open. Clearly, every α-open set is semi-open. Since A is I$\hat{g}$-closed set, $A^*\subseteq$ U, which implies that A is an αIg- closed set.

Theorem 3.20:

If (X, τ,I) is an ideal space, then every αIg-closed, which is α-open is *-closed set.

Proof:

Let A be an αIg-closed and α-open set. Then A $\subseteq$ A implies $A^* \subseteq$A since A is α-open. Therefore , A is *-closed set.

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