# Stability of parallel couple stress viscous fluid flow in a channel

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**Abstract**

In this paper we study the linear stability of parallel couple stress viscous fluid flow (CSVF) in a channel and estimate the eigenvalues, c, belonging to the solutions of Orr-Sommerfeld equation for couple stress fluid using energy method. It is shown that CSVF causes inhomogenety in the material science processes which is shown that these inhomogeneties can be controlled by understanding the effect of couple stress on the nature of eigenvalues in the presence of a shear due to horizontal basic velocity. The conditions for the nature of stability of parallel CSVF is determined in terms of couple stress parameter $\lambda$. An upper bound for the growth rate $c_i$ is obtained and from this sufficient conditions for stability ($c_i < 0$), instability ($c_i > 0$) and neutral stability($c_i = 0$) are obtained, with or without using Schwartz's inequality.

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## 1 Introduction

The present day unprecedented development of science and technology has contributed for significant progress in the investigations of living beings which has opened new vistas in different branches of human body and organs. Significantly, the developments of science and applied genetic studies and technology have helped to develop human bodies and organs at the molecular level and they will contribute to further advancement of physiological studies. These developments play a significant role in human joints. The junction between two bones is called a joint. Depending on the movement, the joints are classified into the following three groups (see Rudraiah et. al., 2000):
a. Freely movable (i.e. Diarthroidal) joints like hip, knee, ankle, finger joints and so on. They are also called synovial joints (SJs).
b. Slightly movable (i.e.Amphartħroidal) joints like vertebral in bodies
c. Immovable (i.e. Synarthroidal) joints like skull, teeth and so on.

In this paper we are concerned only with SJs because of their importance in human locomotion. The two important parts of SJs are cartilage and rheological nature of synovial fluid (SF see the fig 1). The cartilage forms the covering on the bone ends in SJs and plays a significant role in normal joints functioning. It is a flossy, grayish white substance of thickness 1 to 2 mm covering the articulating ends of the bones meeting in a SJ. The synovial fluid (SF) impregnates the movable joints of the body and is contained in the capsules of the joints in volumes of about 0.2 ml to 2 ml. It serves as a lubricant between cartilage surfaces and carries out metabolic functions by providing nutrients to the articular cartilage (AC). Although SF has some resemblance compositionally to blood plasma but lacks the clotting agents such as fibrinogen. The most important constituent of SF is the hyaluronic acid (HA), which gives the SF such high viscosity of about 1000 times greater than water. The normal SF is non-Newtonian exhibiting decrease in viscosity with an increase in shear rate (see Rudraiah 1998). The most important aspect of SJs is to understand different aspects including its lubrication mechanism in human locomotion because the results obtained may through light on understanding the degenerative changes in SJ's which directly affect the normal physiological functioning of an individual. These changes can evolve through the following four types of arthritis:
(i) Osteo arthritis, common during old age
(ii) Traumatic arthritis, occurs due to injuries
(iii) Rheumatic arthritis, occurs due to diseases
(iv) Kinesthesia arthritis, occurs due to erring gene. Among the recent genetic discoveries reported by the US national institute of arthritis is a single genetic flaw which could cause a very common type of arthritis called kinesthesia arthritis.

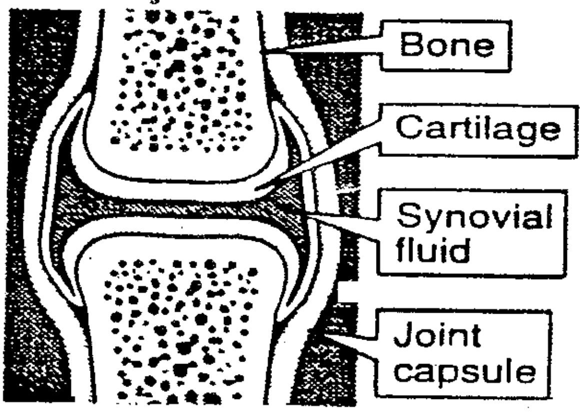


Fig. 1. Synovial joint

According to an orthopedic surgeon although it may not be possible to prevent arthritis there are steps to take to reduce the risk of developing the disease and to slowdown or prevent permanent joint damage. The present measures include reduction in weight as excess weight puts strain on the joints, doing regular exercises help the muscles becoming strong which will protect and support the joints, using joint protecting devices and techniques at work such as proper lifting and posture to protect muscles and joints.

It is known that the above degenerative changes in SJs may be due to physiological and mechanical aspects. One of the physiological aspects is the gout that occurs when the body cannot eliminate the natural substance called uric acid (UA). The excess UA forms needle like crystals in the joints that cause gout namely swelling causing severe pain. The mechanical aspects of degenerative changes in SJs are mainly due to degenerative changes in cartilages. It is believed (see Rudraiah et. al., 2006 and references there in) that the degenerative changes in ACs will not be naturally recouped using the above mentioned preventive measures and when disorder becomes severe the replacement by artificial joints is only an alternative to relieve pain. It is a common observation that the artificial joints are manufactured using metals. We know (see Ng et. al., 2005 and references there in) the difficulties in manufacturing the metal joints which are not compatible with natural joints. These metal joints will have either rough or smooth surfaces. Both of them are dangerous to the body because they produce stresses, these stresses produce a force which drives the erythrocytes (i.e. RBC) in joints to a particular place where the high concentration of RBC leads to bursting of RBC's releasing heamoglobin, a disease called heamolysis. This causes health problem. Recently (Ng et. al., 2005) have suggested a mechanism to mimic the natural joints as an alternative to metal joints by studying dispersion of micro polar components in a biological bearing.

This force, produced by either rough surface or smooth surface, may also disturbs the SF causing instabilities which in turn may contribute to malfunctioning of the joints. In the literature considerable work has been done to find the effect of this force by studying dispersion phenomena but, to our knowledge, no work has been done, on the study of the stability of SF model as couple stress fluid to know degenerative changes in the arthritis caused by the instabilities produced by the above mentioned force. The study of it is the main objective of this paper. To achieve the objectives of this paper it is planned as follows. In section 2 on mathematical formulation the required basic equations for couple stress fluid are given by combining conservation of linear momentum and angular momentum. The linear stability equation, called Orr-Sommerfeld equation, is derived in section 3 for couple stress fluid by superposing infinitesimal disturbances on the basic flow. The stability analysis is discussed in section 4 using energy method. Some important conclusions are drawn in the final section 5.

## 2 Mathematical formulation

The physical configuration considered here is shown in figure 1, It consists of a rectangular channel bounded by rigid boundaries separated by a distance 2h.The rheological properties of physiological fluids like synovial fluid in synovial joints, blood in arteries reveal that viscosity varies nonlinearly with concentration exhibiting either shear thinning or shear thickening behaviour. This is one of the non-Newtonian fluids flow properties. Most of the existing literature (see Fung 1981, Rudraiah 1998) on this is silent about micro motions, micro rotation and deformations. These are taken into account in this paper using couple stress fluid as a particular case of micro polar fluid theory developed by Eringen (1966) as described by Rudraiah et al (1998). The basic equations for this fluid are

Conservation of mass for an incompressible fluid

$$\frac{\partial q_i}{\partial x_i} = 0 \tag{2.1}$$

Conservation of linear momentum:

$$\rho \frac{Dq_i}{Dt} = \frac{\partial \tau_{ij}}{\partial x_j} + \pi_i + \rho f_i \tag{2.2}$$

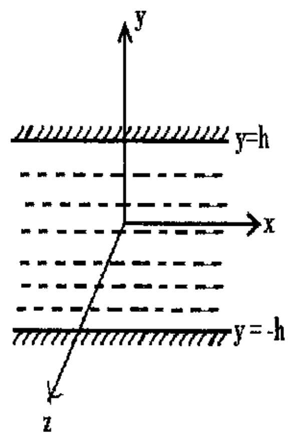


Fig. 2. Physical configuration

Conservation of angular momentum: Angular momentum is usually defined as the moment of the linear moment. If $r_i$ is the position vector of the particles and

$$p_i = \rho q_i \tag{2.3}$$

is the linear momentum, then the angular momentum $L_i$ is

$$L_i = \epsilon_{ijk} r_i p_k \tag{2.4}$$

where

$$\epsilon_{ijk} = \begin{cases} 1 & \text{if i,j,k take values in cyclic order} \\ -1 & \text{if i,j,k take values in acyclic order} \\ 0 & \text{if two or all of i,j,k take the same value} \end{cases}$$

is the Levi - Civita symbol.

The conservation of angular momentum neglecting the body couple and contact couples $c_i$ (see Rudraiah 1998) can be obtained by taking the cross product of $r_i$ with equation (2.2) and using equation (2.3) in the form

$$\epsilon_{ijk}r_i\frac{\partial p_k}{\partial t} + \epsilon_{ijk}r_i\frac{\partial q_j}{\partial x_k}p_k = \epsilon_{ijk}r_i\frac{\partial \tau_{jk}}{\partial x_k} + \epsilon_{ijk}r_i\rho f_k \tag{2.5}$$

In equations (2.1) to (2.5), $q_i(i = 1,2,3)$ are the velocity components, $\tau_{jk}$ is the stress tensor and $\rho f_k$ is the body force.
We note that the limitations encountered in the continuum theory is the lack of taking into account the micro rotation of HA molecules present in SF. In that case the intrinsic motions of the micro elements must be taken into account where microelement motions and deformations play a significant role which has to be taken into account in deriving the required constitutive equations. In such situations, the Eringen's (1966) "micro polar fluid theory is useful and he showed that the couple stress theory results as a special case of micro polar fluid theory if the micro rotation vector is constrained to equal the fluid bulk vorticity through out the flow field, and the deformation of the fluid microelements is very small. Then, the constitute equations for couple stress fluid, following Stokes (1968) as in Rudraiah (1998), are

$$\tau_{ij} = (-p + \mu' e_{kk})\delta_{ij} + 2\mu e_{ij} \tag{2.6}$$

$$\tau'_{ij} = -2\lambda\Omega_{ij,kk} - \frac{\rho}{2}\epsilon_{ijs}G_s \tag{2.7}$$

$$M_{ij} = 4\lambda\Omega_{j,i} + 4\lambda'\Omega i, j \tag{2.8}$$

where

$$e_{ij} = \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i}\right) \tag{2.9}$$

is the strain tensor

$$\Omega_i = \epsilon_{ijk}q_{k,j} = \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j} - \frac{\partial q_j}{\partial x_i}\right) \tag{2.10}$$

is the vorticity tensor.
$G_s$ is the angular velocity vector and $M_{ij}$ is the body moment. Here the dimensions of $\mu'$ and $\mu$ are those of viscosity and $\lambda$ and $\lambda'$ are those of momentum. The ratio $\frac{\lambda}{\mu}$ has the dimensions of length squared.
For an incompressible fluid, when the body moments are absent, the basic equations of motion for couple stress fluid, using equations (2.5) to (2.7), following Rudraiah (1998), are

$$\rho\left(\frac{\partial q_i}{\partial t} + q_i\frac{\partial q_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 q_i}{\partial x_i^2} - \lambda\frac{\partial^4 q_i}{\partial x_i^4} \tag{2.11}$$

These equations have to be solved satisfying the no-slip and couple stress boundary conditions given in the subsequent section.

## 3 Linear Stability Equations For Couple Stress Fluid

We consider a two-dimensional parallel flow of an incompressible couple stress fluid between two fixed rigid parallel plates of width 2h as shown in figure 2. The basic flow is assumed to be fully developed and unidirectional parallel to the plates driven by a constant pressure gradient $\frac{\partial p}{\partial x}$ given by

$$0 = -\frac{\partial p_b}{\partial x} + \mu\frac{\partial^2 u_b}{\partial y^2} - \lambda\frac{\partial^4 u_b}{\partial y^4}, \tag{3.1}$$

$$0 = -\frac{\partial p_b}{\partial y}$$

where the suffix b represents the basic quantities.
The boundary conditions are the no-slip conditions

$$u = 0\, at\ y = \pm h \tag{3.2}$$

and the couple stress conditions

$$\frac{\partial^2 u}{\partial y^2} = 0\, at\ y = \pm h \tag{3.3}$$

We make eqns (3.1) to (3.3) dimensionless, using

$$u^* = \frac{u}{\bar{u}}, p^* = \frac{p}{\rho\bar{u}^2}, y^* = \frac{y}{h}, x^* = \frac{x}{h} \tag{3.4}$$

where $\bar{u}$ is the average velocity and the asterisks (*) denote the dimensionless quantities. Substituting eqn (3.4) into eqn (3.1) and simplifying using (3.2) and (3.3) and for simplicity neglecting the asterisks, we get

$$\frac{\partial^4 u_b}{\partial y^4} - a^2\frac{\partial^2 u_b}{\partial y^2} = -R_e a^2\frac{\partial p_b}{\partial x} \tag{3.5}$$

satisfying the boundary conditions

$$u_b = \frac{\partial^2 u_b}{\partial y^2} = 0,\ at\ y = \pm 1 \tag{3.6}$$

where $a^2 = \frac{h^2}{l^2}, l = \sqrt{\frac{\lambda}{\mu}}$ and $R_e = \frac{\bar{u}h}{\nu}$ is the Reynolds number. Integrating eqn (16) twice, we get

$$\frac{\partial^2 u_b}{\partial y^2} - a^2 u_b = -R_e a^2\frac{\partial p_b}{\partial x}\frac{y^2}{2} + Ay + B \tag{3.7}$$

Solving this non-homogenous partial differential equation using the boundary conditions (3.6), we get

$$u_b = \frac{R_e}{a^2}\frac{\partial p_b}{\partial x}\left[1 - \frac{coshay}{cosha}\right] - \frac{R_e}{2}\frac{\partial p_b}{\partial x}(1-y^2) \tag{3.8}$$

We note that the first term on the r.h.s of eqn (3.8) represents the effect of couple stress and the second term represents the effect of the usual Newtonian nature of parallel flow. The average velocity, $\bar{u_b}$, using eqn (3.8), is

$$\bar{u_b} = \frac{1}{2}\int_{-1}^{1} u_b dy = -\frac{R_e}{3}\frac{\partial p_b}{\partial x}\left[1 - \frac{3}{a^2}\left(1 - \frac{tanha}{a}\right)\right] \tag{3.9}$$

In two - dimensional incompressible homogeneous fluid flow with $q_i = (u, v, 0)$, the couple stress momentum eqn (2.11) takes the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u - \frac{\lambda}{\rho}\nabla^4 u \tag{3.10}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v - \frac{\lambda}{\rho}\nabla^4 v \tag{3.11}$$

together with the conservation of mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.12}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$.

To study the stability of couple stress fluid in a channel shown in fig. 2, we superimpose an infinitesimal disturbance, denoted by the primes, over the basic state of the form

$$u = u_b + u', v = v', p = p_b + p' \tag{3.13}$$

Substituting eqn (3.13) into eqns (3.10) to (3.12), linearizing by neglecting the product of prime quantities compared to the basic state, we obtain

$$\frac{\partial u'}{\partial t} + u_b\frac{\partial u'}{\partial x} + v'\frac{\partial u_b}{\partial y} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} + \nu\left(\frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u'}{\partial y^2}\right) - \frac{\lambda}{\rho}\left(\frac{\partial^4 u'}{\partial x^4} + 2\frac{\partial^4 u'}{\partial x^2 \partial y^2} + \frac{\partial^4 u'}{\partial y^4}\right) \tag{3.14}$$

$$\frac{\partial v'}{\partial t} + u_b\frac{\partial v'}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial y} + \nu\left(\frac{\partial^2 v'}{\partial x^2} + \frac{\partial^2 v'}{\partial y^2}\right) - \frac{\lambda}{\rho}\left(\frac{\partial^4 v'}{\partial x^4} + 2\frac{\partial^4 v'}{\partial x^2 \partial y^2} + \frac{\partial^4 v'}{\partial y^4}\right) \tag{3.15}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0 \tag{3.16}$$

These perturbed equations are made dimensionless using

$$u^* = \frac{u}{\bar{u}}, v^* = \frac{v}{\bar{v}}, y^* = \frac{y}{h}, x^* = \frac{x}{h}, p^* = \frac{p}{\rho\bar{u}^2}, t^* = \frac{t}{(\frac{h}{\bar{u}})} \tag{3.17}$$

Substituting eqn (3.17) into eqns (3.14) to (3.15) and for simplicity neglecting the asterisks (*), we get

$$\frac{\partial u}{\partial t}+u_b\frac{\partial u}{\partial x}+Du_b v = -\frac{\partial p}{\partial x}+\frac{1}{R_e}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{1}{a^2 R_e}\left(\frac{\partial^4 u}{\partial x^4}+2\frac{\partial^4 u}{\partial x^2 \partial y^2}+\frac{\partial^4 u}{\partial y^4}\right) \tag{3.18}$$

$$\frac{\partial v}{\partial t}+u_b\frac{\partial v}{\partial x} = -\frac{\partial p}{\partial y}+\frac{1}{R_e}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{1}{a^2 R_e}\left(\frac{\partial^4 v}{\partial x^4}+2\frac{\partial^4 v}{\partial x^2 \partial y^2}+\frac{\partial^4 v}{\partial y^4}\right) \tag{3.19}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \tag{3.20}$$

where a and Re are as defined in eqn (3.5).
We assume that all perturbed quantities vary as

$$f(x,y,z) = f(y)e^{i\alpha(x-ct)} \tag{3.21}$$

where $c = c_r + ic_i$ is the perturbed velocity and $\alpha$ , real and positive wave number. Eqns (3.18) to (3.20), using eqn (3.21), and after simplification, take the form

$$[D^2-\alpha^2-i\alpha R_e(u_b-c)-\frac{1}{a^2}(D^2-\alpha^2)^2]u = i\alpha R_e p + R_e Du_b v \tag{3.22}$$

$$[D^2-\alpha^2-i\alpha R_e(u_b-c)-\frac{1}{a^2}(D^2-\alpha^2)^2]v = R_e Dp \tag{3.23}$$

$$i\alpha u + Dv = 0 \tag{3.24}$$

Eliminating the pressure p between eqns (3.22) and (3.23), by operating D on eqn (3.22), multiplying eqn (3.23) by $i\alpha$ then subtracting and expressing u in terms of v using eqn (3.24), we get the stability equation

$$\alpha R_e(D^2-\alpha^2)v-\alpha R_e\frac{D^2 u_b}{(u_b-c)}v+i\frac{(D^2-\alpha^2)^2}{(u_b-c)}v-\frac{i}{a^2}\frac{(D^2-\alpha^2)^3}{(u_b-c)}v = 0 \tag{3.25}$$

Equation (3.25) is the required stability equation which can be called the Orr-Sommerfeld equation for a couple stress fluid satisfying the boundary conditions

$$v = Dv = D^3 v = 0 \, at \; y = \pm 1 \tag{3.26}$$

The condition over $Dv$ and $D^3v$ are obtained from eqn (3.24) using the conditions (3.2) and (3.3) on $u$.

## 4 Stability analysis

From the nature of the solution given by eqn (3.21) it follows that if $c_i > 0$ the motion is unstable and if $c_i < 0$ the motion is stable and if $c_i = 0$ the motion is neutrally stable. Hence, to discuss the stability or instability of the basic flow, we have to find, following Drazin and Reid (1981) and Rudraiah (1962, 1963) the nature of c using stability eqn (3.25), subject to the boundary conditions (3.26). For this, we multiply equation (3.25) by $\bar{v}$ , the complex conjugate of $v$, and integrating the resulting equation with respect to y from -1 to 1 and using the boundary conditions (3.26) and after some simplification, we get

$$a^2 I_2^2 + 2a^2\alpha^2 I_1^2 + a^2\alpha^4 I_0^2 + I_3^2 + 3\alpha^2 I_2^2 + 3\alpha^4 I_1^2 + \alpha^6 I_0^2 = \tag{4.1}$$

$-i\alpha R_e a^2(Q_r + iQ_i) + i\alpha R_e a^2 c(I_1^2 + \alpha^2 I_0^2)$ where

$$Q = \int_{-1}^{1}\left[u_b|Dv|^2 + (\alpha^2 u_b + D^2 u_b)|v|^2\right]dy + \int_{-1}^{1}\bar{v}DvDu_b dy = Q_r + iQ_i \tag{4.2}$$

$I_n^2 = \int_{-1}^{1}|D^n v|^2 dy$

$$Q_r = RelQ = \int_{-1}^{1}\left[u_b|Dv|^2 + \left(\alpha^2 u_b + \frac{1}{2}D^2 u_b\right)|v|^2\right]dy \tag{4.3}$$

$$Q_i = ImQ = \frac{i}{2}\int_{-1}^{1}(vD\bar{v} - \bar{v}Dv)Du_b dy \tag{4.4}$$

Observe now that $|ImQ| \leq \int_{-1}^{1}|v||Dv||Du_b|dy$
To study the stability of the basic flow, we estimate the nature of the eigenvalues, $c_i$ using the cases, one without using Schwartz's inequality and the other using Schwartz's inequality.

Case-1: Stability Analysis without using Schwartz's inequality: From the real and imaginary parts of eqn (4.1), using eqns (4.3) and (4.4), we get

$$c_i = \frac{Q_i - (\alpha R_e)^{-1}\left[\left(3\alpha^2 + \frac{h^2\mu}{\lambda}\right)I_2^2 + \alpha^2\left(3\alpha^2 + \frac{2h^2\mu}{\lambda}\right)I_1^2 + \alpha^4\left(\alpha^2 + \frac{h^2\mu}{\lambda}\right)I_0^2 + I_3^2\right]}{(I_1^2 + \alpha^2 I_0^2)} \tag{4.5}$$

and

$$c_r = \frac{Q_r}{(I_1^2 + \alpha^2 I_0^2)} \tag{4.6}$$

From (4.5), if $Q_i > 0$ that is $\int_{-1}^{1} v_r Dv_i dy > \int_{-1}^{1} v_i Dv_r dy$ and

$$\alpha R_e Q_i < \left[\left(3\alpha^2 + \frac{h^2\mu}{\lambda}\right)I_2^2 + \alpha^2\left(3\alpha^2 + \frac{2h^2\mu}{\lambda}\right)I_1^2 + \alpha^4\left(\alpha^2 + \frac{h^2\mu}{\lambda}\right)I_0^2 + I_3^2\right] \tag{4.7}$$

$c_i < 0$ then the system is stable, if

$$\alpha R_e Q_i > \left[\left(3\alpha^2 + \frac{h^2\mu}{\lambda}\right) I_2^2 + \alpha^2\left(3\alpha^2 + \frac{2h^2\mu}{\lambda}\right) I_1^2 + \alpha^4\left(\alpha^2 + \frac{h^2\mu}{\lambda}\right) I_0^2 + I_3^2\right] \tag{4.8}$$

$c_i > 0$ then the system is unstable, if

$$\alpha R_e Q_i = \left[\left(3\alpha^2 + \frac{h^2\mu}{\lambda}\right) I_2^2 + \alpha^2\left(3\alpha^2 + \frac{2h^2\mu}{\lambda}\right) I_1^2 + \alpha^4\left(\alpha^2 + \frac{h^2\mu}{\lambda}\right) I_0^2 + I_3^2\right] \tag{4.9}$$

$c_i = 0$ then the system is neutrally stable.

Case - 2: Stability Analysis with Schwarz's inequality
$|ImQ| \le qI_0I_1$ where $q = max_{-1\le y\le 1}|Du_b|$.
This gives the upper bound for $c_i$

$$c_i < \frac{\alpha R_e q I_0 I_1 - \left[\left(3\alpha^2 + \frac{h^2\mu}{\lambda}\right) I_2^2 + \alpha^2\left(3\alpha^2 + \frac{2h^2\mu}{\lambda}\right) I_1^2 + \alpha^4\left(\alpha^2 + \frac{h^2\mu}{\lambda}\right) I_0^2 + I_3^2\right]}{\alpha R_e (I_1^2 + \alpha^2 I_0^2)} \tag{4.10}$$

From this it follows that, if

$$\alpha R_e q < \frac{\left[\left(3\alpha^2 + \frac{h^2\mu}{\lambda}\right) I_2^2 + \alpha^2\left(3\alpha^2 + \frac{2h^2\mu}{\lambda}\right) I_1^2 + \alpha^4\left(\alpha^2 + \frac{h^2\mu}{\lambda}\right) I_0^2 + I_3^2\right]}{I_0 I_1} \tag{4.11}$$

Then $c_i < 0$ and hence the condition (4.11) is a sufficient condition for stability which is identical with the condition (4.7) obtained without using Schwartz inequality.

## 5 Conclusions

Stability of parallel couple stress fluid flow in a rectangular channel is studied using linear stability analysis. The eigenvalues, $c_i$, of the system are obtained from the stability eqn (3.25) using energy method as given in eqn (4.5). Sufficient conditions for stability, instability and for neutral stability are obtained as given in eqns (4.7) to (4.9) respectively without using the approximation of Schwartz inequality. However, Using Schwartz inequality it is shown that the same eqns (4.7) to (4.9) also give sufficient condition for stability, instability and stability. This condition for stability are helpful to manufacture artificial cartilages without any side effect like haemolysis.

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