# Aris Dispersion in a Chiral Fluid Bounded By Porous Layers in the Presence 0f Convective Current

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**Abstract**

This paper discusses the use of Aris analysis to study the dispersion of a chiral fluid flow in a channel in the presence of a convective current bounded by porous layers. Analytical solution for velocity in the presence of a transverse magnetic field is obtained and it is computed for different values of electromagnetic number $W_{em}$. Similarly we have found the analytical solution in the presence of advection of concentration distribution of chiral fluid by using Fourier Transform Method by introducing new space coordinates. Following Aris model, measure of the effective dispersion coefficient $D^*_{eff}/D = 1 + D^*$ for the distribution of chiral fluid which is the sum of the particle diffusivity and the Taylor's dispersion coefficient, where $D^*$ is the Taylor's dispersion coefficient. This $D^*_{eff}/D$ is numerically computed and the results are presented graphically for different values of $W_{em}$, Paclet number $Pe$ and suction Reynolds number $Re$. The results are discussed in the final section of this paper.

*Key words:* Aris method, Dispersion phenomenon, Chiral fluid, Suction Reynolds Number, Fourier series method.

*Mathematics Subject Classification* : **58D30**

## 1. Introduction

In recent years, considerable interest has been evinced in the development of new technologies like Information Technology, Bio-Technology, Nano – Technology, pharmaceutical industry, technologies involving Smart and Chiral materials, using improved and novel processing routes which could replace most of the recent existing technologies, which are going to change every aspect of our living and lead to the generation of new capabilities, new materials and new products. An important aspect associated with these new technologies, is their multi disciplinary nature with applications in Science, Engineering and Technology and their impact on society is expected to be wide spread and all pervasive. By definition, a three dimensional object is chiral if it cannot be brought into congruence with its mirror image by any amount of translation and rotation. In other words chiral, Fluid is the Fluid in which the molecules have the property of handedness and must be either right handed or left handed [23,14]. Therefore, chirality is connected with handedness [9,15].

The molecules like carbohydrates, proteins, nutrients, amino acids, RBC, WBC, enzymes in our body cells, antibodies, hormones, body-fluids etc., in fluids [18,19] like sugar solution, turpentine, glucose, drugs, synovial fluid, blood in arteries [24,1,13,17,5,6] exhibit chirality. Proper functioning of artificial organs like Synovial Joints (SJs) and Coronary Artery Diseases (CAD) in biomedical engineering depend on the dispersion of hyaluronic acid and nutrients in synovial fluid and the dispersion of RBC, WBC and so on in physiological fluids in arteries. In human body synovial joints plays an important role but these are the most vulnerable to one of the following degenerative changes i) Osteoarthritis, ii) Traumatic arthritis and iii) Rheumatic arthritis. These disorders in SJs are mainly due to degenerative changes in cartilages. It is believed [10] that the degenerative changes will not be naturally recouped in articular cartilage and when disorder becomes so severe the replacement by artificial joints is the only alternative to relief from pain. It is a common observation in these days that the artificial joints are manufactured using metals. We know [16,] the difficulties in synthesizing the metallic organs which are lack of compatibility with the human body. These metallic joints will produce either high stresses due to the roughness or low stresses due to the smoothness. Both of them are dangerous to the body because the force produced by the stresses drive the erythrocytes (i.e. red blood cells, RBC) in arteries to a particular place leading to bursting of RBCs which release the haemoglobin. This process of releasing of haemoglobin is a disease called haemolysis leading to some health problems. Recently [5] have studied the effects of dispersion of chiral molecules like hyaluronic acid, glycoprotein's, nutrients and carbohydrates from synovial fluid to natural joints using [25] dispersion model. Also [7] have suggested the use of smart material of nanostructure synthesized using poorly conducting alloys like Nickel-Titanium (i.e. Ni-Ti) as in shape memory alloy to mimic the natural joints as an alternate to the metal joints. This artificial joint made up of smart material of nanostructure may mimic natural joints to achieve effective dispersion of chiral molecules mentioned above from synovial fluid to articular cartilage which are required for the survival of cartilages in addition to achieving higher load supporting capacity and minimizing the friction in biological bearings.

Carbohydrates are technologically relevant compounds and have significant biological importance [7,8]. Their properties play a pivotal role in the study of reaction conditions of many currently employed industrial processes such as enzymatic conversion of biomass to useful chemicals. Also, they are often used in pharmaceutical, food and biomedical applications (e.g., for stabilization of proteins and membranes), purification of sugar solution in the sugar industry and designing of artificial joints for the human body.

While numerous studies have been carried out on the thermodynamic properties of binary aqueous carbohydrates solutions like sucrose, glucose, but only few have taken into account the transport behavior of these systems in aqueous solutions [11]. Transport properties, particularly diffusion coefficients, provide a direct measure of molecular mobility, an important factor in the preservation of biological materials in sugar matrices. Hopefully, the studies of mathematical model will lead to know how that will allow a better understanding of the physical chemistry conditions underlining the diffusion phenomena occurring in different physiological situations in the human body (i.e., the human oral cavity).

From the results obtained by using [2], which is an improvement over the mathematical model of [26], it will be possible to assess the important parameters involved in dispersion phenomenon. In addition, the measured diffusion coefficients are used to estimate the activity coefficients for aqueous carbohydrates [2].

Experiments of [10] revel that after joint replacement the rheologically modified lubricant behaves like a Newtonian fluid. Dispersion phenomenon [26] in chiral fluid like sugar solution is useful in the purification of sugar solution in sugar industry. The present study shows that the dispersion phenomenon using [2] model is an improvement over [25] model, by relaxing some of the assumptions namely longitudinal diffusion is much less than the transverse diffusion made by [26]

Dispersion mechanism for diffusion measurements are based on the work carried out by [25,26] on the dispersion of concentration of chiral molecules like WBC, RBC, Proteins, carbohydrates and hyaluronic acid in synovial fluid, flowing through a long capillary tube. In the present paper we have assume the length of tube, as arteries in human body, is large compared to the diameter of the tube.

Extensive literature is available on theoretical and experimental aspects of solid chiral materials [3,4,12,28,14,15]. However, much attention has not been given to a detailed study of dispersion in chiral fluids in spite of its importance in many practical problems cited above. The study of it is the main objective of the present paper.

To achieve the objective of this study, the required basic equations for two dimensional flow together with Maxwell's equations and the required constitutive equations for chiral fluids are given in section 2. Using these basic equations with suitable approximations we determine in section 3, the Aris dispersion coefficient in the presence of transverse magnetic field with charge density decreasing continuously with height. The results, discussions and conclusions are given in the final section 4.

## 2. Mathematical formulation

We consider the physical configuration which consists of a flow through a rectangular channel [illegible] $y = \pm h$ and $x$-axis is parallel to the plates, $y$ and $z$ axes are perpendicular to it as shown in the fig 1. We deal with a two-dimensional chiral fluid with $u$ and $v$ the components velocity in the $x$ and $y$ directions respectively and a uniform applied magnetic field $B_0$ in the $z$ direction. We assume the chiral fluid to be incompressible, viscous and Newtonian

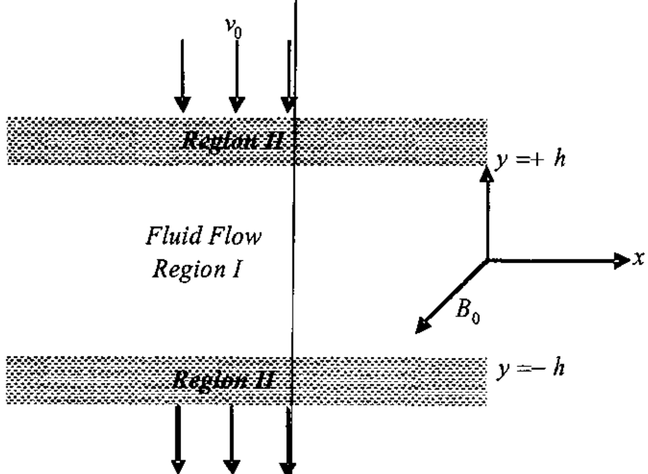


Fig: 1 Physical configuration

and the flow is governed by the modified Navier-Stokes equation (modification means the inclusion of Lorentz force with chirality parameter). Therefore the governing equations, describing a chiral fluid flow in a channel, are:

the conservation of mass

$$\nabla \cdot \vec{q} = 0 \tag{2.1}$$

the conservation of momentum

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}\right) = -\nabla p + \mu_f \nabla^2 \vec{q} + \vec{J} \times \vec{B} \tag{2.2}$$

the conservation of species

$$\frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla)C = D(\nabla C) \tag{2.3}$$

the conservation of electric charges

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \vec{J} = 0 \tag{2.4}$$

These equations have to be supplemented with the Maxwell's equations

$$\nabla \cdot \vec{D} = \rho_e \tag{2.5}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.6}$$

$$\nabla \times \vec{H} = \vec{J} \tag{2.7}$$

$$\nabla \cdot \vec{B} = 0 \tag{2.8}$$

together with the following constitutive equations for chiral fluids [20,28]

$$\vec{D} = \varepsilon \vec{E} + i\gamma \vec{B}$$

$$\vec{B}_0 = \mu \vec{H} - i\mu\gamma \vec{E} \qquad 2.9(a,b,c)$$

$$\vec{J} = \rho_e \vec{q} + \frac{\partial \vec{D}}{\partial t} \quad \text{(in the absence of conduction current)}$$

Here $\vec{q} = (u, \upsilon)$ is the velocity, $\vec{B}$ the magnetic induction, $\vec{H}$ the magnetic field, $\vec{J}$ the current density, $\vec{D}$ the dielectric field, $\vec{E}$ the electric field. $P$ the pressure, ρ the density of the fluid, $\rho_e$ the distribution of electric charge density, μ the magnetic permeability, ε the dielectric constant and $\gamma$ the chirality coefficient, $\rho_e \vec{q}$ the convective current, $\frac{\partial \vec{D}}{\partial t}$ the displace current and $\vec{J} \times \vec{B}$ is the Lorentz force. In this paper we consider only the convective current. For the chosen physical configuration, as shown in fig 1, and using the above assumptions, the required basic equations, in Cartesian form for chiral fluid, are

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \rho_e B_0 v \tag{2.10}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \rho_e B_0 u \tag{2.11}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.12}$$

$$\frac{\partial \rho_e}{\partial t} + u\frac{\partial \rho_e}{\partial x} + v\frac{\partial \rho_e}{\partial y} = 0 \tag{2.13}$$

Following [2], we consider the flow to be steady, unidirectional, fully developed and parallel to the plates in the x direction, such that

$$u = u(y), \; \frac{\partial}{\partial t} = 0 \;, \; v = v_0, \; \text{and} \; \frac{\partial p}{\partial x} = \text{constant} \tag{2.14}$$

under these approximations the above eqs (2.10) and (2.11) become

$$\rho v_0 \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} + \rho_e B_0 v_0 \tag{2.15}$$

$$0 = -\frac{\partial p}{\partial y} - \rho_e B_0 u \tag{2.16}$$

These equations are made dimensionless using

$$x^* = \frac{x}{h}, \; \eta = \frac{y}{h}, \; p^* = \frac{p}{\rho v_0^2}, \; u^* = \frac{u}{v_0}, \; \rho_e^* = \frac{\rho_e}{\varepsilon V / h^2} \tag{2.17}$$

where the asterisks (*) denote the dimensionless quantities, h the characteristic height, V the potential difference, $v_0$ the suction(or injection) velocity and other quantities are as defined in eqs. (2.1 to 2.9). Therefore the dimensionless form of the above eqs. (2.15) and (2.16), using eq. (2.17) and for simplicity neglecting the asterisks, take the form

***Region – I***

In this region an incompressible chiral fluid flows between two porous layers and the required equations are:

$$R_e \frac{\partial u}{\partial \eta} = k_2 + \frac{\partial^2 u}{\partial \eta^2} + k_1 \rho_e \tag{2.18}$$

$$0 = \frac{\partial P}{\partial \eta} + \frac{k_1}{R_e} u \rho_e \tag{2.19}$$

***Region – II***

In this region an incompressible chiral fluid is governed by the modified Darcy equation given by

$$\frac{\partial P}{\partial x} = \frac{-\mu}{k} Q_x + \rho_e v_0 B_0 \tag{2.20}$$

After satisfying the equilibrium condition, we have

$0 = \frac{-\mu}{k} Q_y$, which implies $Q_y = 0$

where $\Omega_1 = W_{em} R_e$, $\Omega_2 = -R_e P$, $P = \frac{\partial p}{\partial x}$, $W_{em} = \frac{\varepsilon V B_0}{h \rho v_0}$ the electromagnetic parameter and $R_e = \frac{h v_0}{\nu}$ is the suction Reynolds number. We assume a stable density of charge distribution, $\rho_e$, in chiral fluid which decreases continuously in the vertical direction of the form

$$\rho_e = \rho_{eo} e^{-\beta \eta} \tag{2.21}$$

where $\beta$ is the charge density stratification factor. Therefore eqn (2.18), using eq. (2.21), takes the form

$$R_e \frac{\partial u}{\partial \eta} = \Omega_2 + \frac{\partial^2 u}{\partial \eta^2} + \Omega_1 e^{-\beta\eta} \tag{2.22}$$

At the interface between porous layers and chiral fluid there exists a slip postulated by Beaver and Joseph 1967, called the BJ-slip boundary conditions, given by

$$\frac{du}{d\eta} = \mathrm{m}\frac{\alpha(u_b - Q_x)}{\sqrt{k}} \qquad at\ \ \eta = \pm 1 \tag{2.23}$$

Here $u$ is the velocity of thin fluid parallel to the porous layers, $Q_x$ the Darcy velocity in the porous layers, $k$ the permeability of a porous cartilage or the endothelium walls and $\alpha$ is the BJ-slip coefficient at the interface between chiral fluid and saturated porous layers. eq. (2.22) is solved analytically using the boundary conditions eq. (2.23) and obtained,

$$\begin{aligned} u = &-\frac{\Omega_1\eta}{\mathrm{Re}} - \frac{\Omega_2 e^{-\beta\eta}}{\beta(\beta+\mathrm{Re})} + \frac{\Omega_3}{\Omega_4}(\Omega_1\Omega_7 + \Omega_2\Omega_8) - \frac{\Omega_2\Omega_5}{\alpha\sigma} - \frac{\Omega_2\Omega_6}{\beta} \\ &+ \frac{1}{\sigma^2}(\Omega_1 - \Omega_2 Cosh\beta) + \frac{1}{\Omega_4}(\Omega_1\Omega_7 + \Omega_2\Omega_8)e^{R_e\eta} \end{aligned} \tag{2.24}$$

where the constants $\Omega_i$ $(i = 1\, to\, 8)$ are given in the appendix.

### 3. Concentration Distribution of Chiral Fluid

If C is the concentration of chiral fluid such as proteins, nutrients, amino acids, carbohydrates and sugar solution [1,17] in a chiral fluid and diffuses in a fully developed flow, then C satisfies the advection-diffusion eqn (2.3).

In the Aris dispersion mechanism, one has to consider as in [27] model, a quasi-steady flow involving the chiral fluid to understand the hydrodynamic dispersion. Following [2], we assume that the longitudinal diffusion, transverse diffusion, diffusivity $D_m$, viscosity $\mu_a$, and the pressure gradient $P$ are all constants. Then the advection of concentration of chiral fluid satisfies the equation

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v_0\frac{\partial C}{\partial y} = D_m\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right] \tag{3.1}$$

where $D_m$ is the molecular diffusivity. This equation is made dimensionless using the non-dimensional quantities

$C^* = C/C_0,\ \tau = Dt/h^2,\quad t^* = t/t_0,\ t_0 = L/\bar{u},\quad x^* = x/L,\quad \eta = y/h$ And the moving coordinate

$$\xi = \left(x - \bar{u}t\right)/L \tag{3.2}$$

where $C_0$ is the initial slug, L the characteristic length and $\bar{u}$ is the average velocity given below by the eq. (3.3). Making eq. (3.1) dimensionless using eq. (3.2), we get

$$\frac{D}{h^2}\frac{\partial C}{\partial t} + \frac{W(\eta)}{L}\frac{\partial C}{\partial x} + \frac{v_0}{h}\frac{\partial C}{\partial y} = D_m\left[\frac{1}{L^2}\frac{\partial^2 C}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 C}{\partial y^2}\right] \tag{3.2a}$$

where

$$W(\eta) = u - \bar{u} = -\frac{\Omega_1\eta}{\mathrm{Re}} - \frac{\Omega_2 e^{-\beta\eta}}{\beta(\beta+\mathrm{Re})} + \frac{1}{\Omega_4}(\Omega_1\Omega_7 + \Omega_2\Omega_8)e^{R_e\eta} - \frac{\Omega_2\Omega_5}{\beta^2} - \frac{1}{\Omega_4}\frac{Sinh\,\mathrm{Re}}{\mathrm{Re}}$$

The Aris dispersion model deals with the advection across the plane moving with the mean speed $\bar{u}$ and is given by

$$\bar{u} = \frac{1}{2}\int_{-1}^{1} u\,d\eta = \Omega_2\Omega_5\left(\frac{1}{\beta^2} - \frac{1}{\alpha\sigma}\right) + \frac{1}{\Omega_4}(\Omega_1\Omega_7 + \Omega_2\Omega_8)\left[\Omega_3 + \frac{Sinh\,\mathrm{Re}}{\mathrm{Re}}\right] - \frac{\Omega_6\Omega_2}{\beta} + \frac{1}{\sigma^2}(\Omega_1 - \Omega_2 Cosh\beta) \tag{3.3}$$

Following Aris, we consider the $P^{th}$ moment of the distribution of the chiral fluid bounded by porous layers at a time $t$ and the sufficient condition to be imposed on concentration of $C$, as $\xi \to \pm\infty$ is thus that these moments should exit and finite, a condition fulfilled if the concentration of the chiral fluid is originally contained in a finite length of the tube.

Let $$C_p(\eta,\tau) = \int_{-\infty}^{+\infty} \xi^p C(\xi,\eta,\tau)\,d\xi \tag{3.3a}$$

Defining $m_p(\tau)$ the average of $C_p$ over a cross-section S, the area of the channel, we get

$$m_p(\tau) = \bar{C}_p = \frac{1}{S}\int_s C_p\,d\eta \tag{3.3b}$$

The velocity $u(\eta)$ is in the $x$-direction given by

$$u(\eta) = \bar{u}(1 + \chi(\eta)) \tag{3.3c}$$

where $\bar{u}$ is the mean velocity and $\chi(\eta)$ defines the velocity relative to the mean. The sufficient condition to be imposed on concentrations of $C$ as $\xi \to \pm\infty$ [2] is thus that these moments should exit and finite, a condition fulfilled if the concentration of the chiral molecules are originally contained in a finite length of the tube. Multiplying eq. (3.2a) by $\xi^p$ and integrating with respect to $\xi$ from $-\infty$ to $+\infty$, we have

$$\frac{\partial C_p}{\partial \tau} = \frac{\partial^2 C_p}{\partial \eta^2} - P_e \frac{\partial C_p}{\partial \eta} + U_o \chi(\eta) P C_{p-1} + P(P-1) C_{p-2} \tag{3.4}$$

we solve this equation using the initial condition

$$C_p(\eta, 0) = G_{p0}(\eta) \qquad at \quad t = 0 \tag{3.5}$$

and the boundary conditions

$$\frac{\partial C_p(\eta, t)}{\partial \eta} = 0 \qquad at \quad \eta = 1, 0 \tag{3.6}$$

Eq. (3.4), subject to conditions eqs. (3.5) and (3.6) for *P=0, 1, 2...*.now form a sequence of inhomogeneous equations which can be solved for the moments $C_p$. Let $P = 0$ in equation (3.4), and obtain

$$\frac{\partial C_0}{\partial \xi} = \frac{\partial^2 C_0}{\partial \eta^2} - P_e \frac{\partial C_0}{\partial \eta} \tag{3.7}$$

satisfying the conditions eqs. (3.5) and (3.6) [21], we introduce another coordinate $\varsigma$, mathematically this implies that we change our space coordinate $\eta$ to $\varsigma$, i.e., $\varsigma = \eta - P_e \tau$ and $\tau_1 = \tau$. The solution of eq. (3.7), using finite Fourier cosine transform, is

$$C_0(\eta, \tau) = \frac{G_0(\eta, \tau)}{2} + \sum_{n=1}^{\infty} G_0(\eta, \tau)\, Exp[-n^2 \pi^2 \tau]\, Cos(n\pi\eta - Pen\pi\tau) \tag{3.8}$$

Averaging eq. (3.4) over the cross-section of unit breadth and using eqs. (3.3b) and (3.6), we get

$$\frac{\partial m_p}{\partial \tau} = P(P-1) m_{p-2} + \frac{U_0 P}{2} \int_{-1}^{1} \chi(\eta) C_{p-1}\, d\eta \tag{3.8a}$$

where $\chi(\eta) = \Omega_{11}\eta - \Omega_{12} Exp[-\beta\eta] + \Omega_{13} Exp[\mathrm{Re}\,\eta] - \Omega_{14}$ (3.8b)

and $\Omega_i$ $(i = 11 to 14)$ are given in the appendix.

This shows that $\frac{\partial m_0}{\partial \tau}=0$, so that $m_0$ is constant which can be taken as unity. This merely expresses the face that the total quantity of the chiral molecule in the channel remains constant. For $P=1$, the eq. (3.8a) takes the form

$$\frac{\partial m_1}{\partial \tau}=\frac{U_0}{2}\int_{-1}^{-1}\chi(\eta)C_0\,d\eta \tag{3.9}$$

Then eq. (3.9), using eqs. (3.8) and (3.8b), takes the form

$$\frac{\partial m_1}{\partial \tau}=\frac{U_0}{2}\Omega_{15}G_0+\frac{U_0}{2}\sum_{n=1}^{\infty}\begin{pmatrix}(\Omega_{11}-\Omega_{14})\frac{Sinn\pi(1-Pe\tau)}{n\pi}-\frac{\Omega_{12}\Omega_{16}}{n^2\pi^2+\beta^2}\\ -(\Omega_{11}+\Omega_{14})\frac{Sinn\pi(1+Pe\tau)}{n\pi}+\frac{\Omega_{13}\Omega_{17}}{n^2\pi^2+\mathrm{Re}^2}\end{pmatrix}G_o Exp[-n^2\pi^2\tau] \tag{3.10}$$

As $\tau\to\infty$, $\frac{\partial m_1}{\partial \tau}\to\frac{U_0\Omega_{15}G_0}{2}$ which is due to chirality effect and integrating eq. (3.9) and choosing the origin in the original plane of the centre of the gravity we have

$$m_1(\tau)=\frac{U_0}{2}\left(\Omega_{15}G_0+\sum_{n=1}^{\infty}\Omega_{18}G_0(1-Exp[-n^2\pi^2\tau])\right) \tag{3.11}$$

as $\tau\to\infty$, $m_{1,\infty}(\tau)=\frac{U_0}{2}\left(\Omega_{15}G_0+\sum_{n=1}^{\infty}\Omega_{18}G_0\right)$ (3.12)

where $\Omega_i$ $(i=15to18)$ are given in appendix, which implies the centre of gravity ultimately moves with the mean speed of the stream. Let $P=1$ in eq (3.4) and we get

$$\frac{\partial C_1}{\partial \tau}=\frac{\partial^2 C_1}{\partial \eta^2}-P_e\frac{\partial C_1}{\partial \eta}+U_o\chi(\eta)C_0 \tag{3.13}$$

This, using $\varsigma=\eta-Pe\tau$ and $\tau_1=\tau$ becomes

$$\frac{\partial C_1}{\partial \tau_1}=\frac{\partial^2 C_1}{\partial \varsigma^2}+U_o\chi(\varsigma)C_0 \tag{3.14}$$

which is analogous to the one given by [2] and [22]. The complete solution of eq (3.14) consists of the following three parts: (i) the term in the particular integral arising out of the constant term $C_0$, (ii) the remaining part of the particular integral which may be written as

$$U_0\sum_{n=1}^{\infty}\phi_n(\varsigma)Exp[-n^2\pi^2\tau_1] \qquad (3.15)$$

where $\phi_n(\varsigma)$ is given by

$$\frac{d^2\phi_n}{d\varsigma^2}+c_n\chi(\varsigma)Cosn\pi\varsigma=0 \qquad (3.16)$$

and (iii) the complimentary function which may be written as

$$\frac{D_1}{2}+\sum_{n=1}^{\infty}D_nCosn\pi\varsigma\, Exp[-n^2\pi^2\tau_1] \qquad (3.17)$$

Where $D_1$ and $D_n$ are arbitrarily constants, thus finally the centre of gravity is distributed across the tube according to the equation

$$C_1\cong\Delta_1(const)+\frac{U_0G_0\Omega_{14}}{4}\eta^2 \qquad (3.18)$$

Now by choosing $p=2$ in (3.8a), we get

$$\frac{\partial m_2}{\partial\tau}=2m_0+U_0\int_{-1}^{-1}\chi(\eta)C_1\,d\eta \qquad (3.19)$$

Substituting this value of $C_1$ and eq (3.8b) into eq (3.19) and neglecting terms which tend to zero as $\tau\rightarrow\infty$, we get

$$\frac{\partial m_2}{\partial\tau}=2+\frac{\bar{u}^2h^2}{4D^2}\left(\frac{\Omega_{18}}{4}+\frac{\Omega_{19}}{2}-\frac{\Omega_{14}^2G_0}{12}\right) \qquad (3.20)$$

Now, if $\psi$ is the rate of change of variance of the distribution of the chiral fluid about the moving origin, we get

$$\psi=m_p(\tau)=\bar{C}_p=\frac{1}{s}\int_s d\eta\int_{-\infty}^{+\infty}(x-\bar{u}t)^2C(\xi,\eta,\tau)d\xi \qquad (3.21)$$

From this we have

$$\lim_{t\to\infty}\frac{D}{2}\frac{d\psi}{dt}\propto D+\frac{\bar{u}^2h^2}{D}\left(\frac{\Omega_{18}}{4}+\frac{\Omega_{19}}{2}-\frac{\Omega_{14}^2G_0}{12}\right) \qquad (3.22)$$

That is $\frac{d\psi}{dt}\propto D+D^*=D_{eff}^*$ (3.22a) where $D^{*}_{eff}$ is the effective diffusion coefficient is the sum of the molecular diffusion coefficient, D, and $D^{*}=\frac{\bar{u}^{2}h^{2}}{D}\left(\frac{\Omega_{18}}{4}+\frac{\Omega_{19}}{2}-\Omega\frac{k_{14}^{2}G_{0}}{12}\right)$ is the [27] diffusion coefficient. This amounts to

$$\frac{D^{*}_{eff}}{D}\propto 1+\left(\frac{\bar{u}h}{D}\right)^{2}\left(\frac{\Omega_{18}}{4}+\frac{\Omega_{19}}{2}-\frac{\Omega_{14}^{2}G_{0}}{12}\right) \tag{3.23}$$

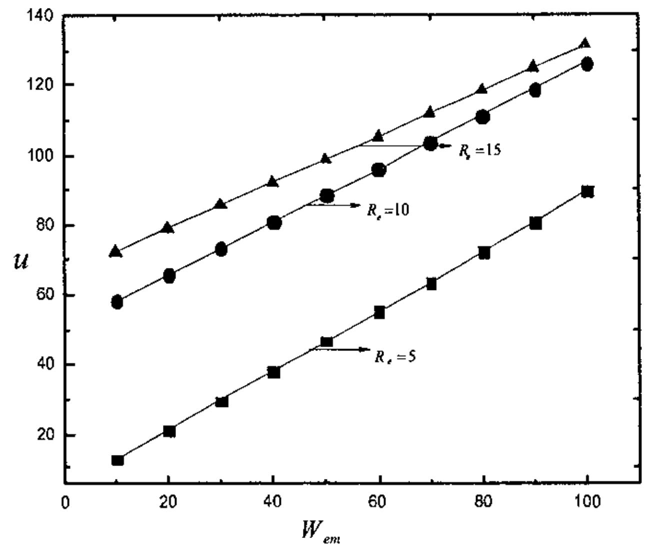


**Figure 2**

Variation of u v/s $W_{em}$ for different values of $R_e$=5, 10 and 15 $C_2$

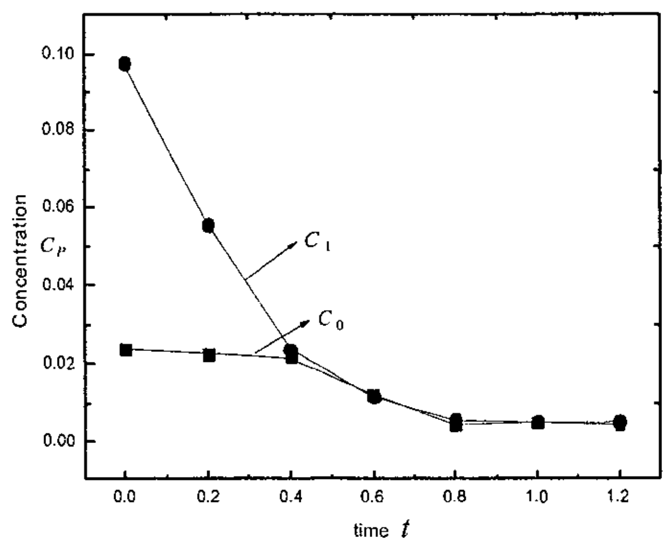


**Figure 3**

Variation of $C_p$ v/s time $t$ for constant value of Paclet number.

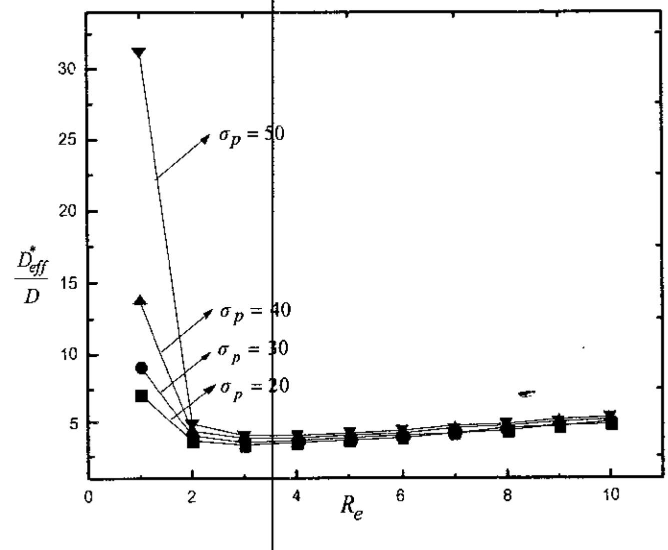


**Figure 4**

Variation of Concentration $D^*_{eff}/D$ v/s Reynolds number for different values of $\sigma_p$ =20, 30,40 and 50

Thus the effective diffusion coefficient may be regarded as the sum of the molecular coefficient $D$ and effective diffusion coefficient $D^*$. Hence, the restriction imposed on [28] has been removed in this Aris model. The $\frac{D^*_{eff}}{D}$ given by eq (3.23) is computed for different values of the suction Reynolds number, $R_e$, and Peclet number, $Pe$ and the results are depicted graphically in Figures 2 to 4 and the results are discussed in the section 4.

### 4. Results and Discussion

In this paper, we have developed a mathematical model for chiral fluid following the analysis of [2] dispersion mechanism to investigate the effect of magnetic field, chirality factor and porous parameter on the dispersion coefficient modeling the cartilage or arterial veins as porous material. The variation of velocity against electro magnetic parameter $W_{em}$ for different values Reynolds number Re is shown in figure 2. From this figure it is evident that the velocity increases with an increase in $W_{em}$ and for different values of= 5, 10 and 15. From this figure it is clear that as increasing Reynolds number increases the velocity. The variation of concentration of chiral fluid with respect to time is as shown in the figure 3, form the figure it is evident that the concentration of chiral fluid decreases like parabola, it concludes that the dispersion is along the velocity of the flow in channel. The variation of dispersion coefficient ( $D^{*}_{eff}/D$ ) v/s Reynolds number ( $R_e$ ) for different values of porous parameter $\sigma_p$ is shown in the figure 4. From this figure it is evident that the dispersion coefficient decreases initially and increases continuously, which concludes that the porous parameter is more effective in dispersing the chiral fluid.

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**Appendix:**

$$\Omega_1 = \mathrm{Re}\, P$$

$$\Omega_2 = We\,\mathrm{Re}$$

$$\Omega_3 = \frac{\mathrm{Re}}{\alpha\sigma} Sinh\,\mathrm{Re} + Cosh\,\mathrm{Re}$$

$$\Omega_4 = \mathrm{Re}\, Cosh\,\mathrm{Re} - \alpha\sigma Sinh\,\mathrm{Re}$$

$$\Omega_5 = \frac{Sinh\beta}{\beta + \mathrm{Re}}$$

$$\Omega_6 = \frac{Cosh\beta}{\beta + \mathrm{Re}}$$

$$\Omega_7 = \frac{1}{\mathrm{Re}} - \alpha\sigma\frac{\Omega_1}{\mathrm{Re}}$$

$$\Omega_8 = \frac{\Omega_5}{\beta} + Sinh\beta - \Omega_6$$