A Single Supplier Multiple Corporative Retailers Integrated Inventory Model with Quantity Discount and Permissible Delay in Payments

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ABSTRACT

The paper considers an inventory model with quantity discount and permissible delay in payment by employing the type of fuzzy numbers which are trapezoidal. The EOQ models have been developed using different optimization where the input parameters and the decision variables are fuzzified. The optimal policy for the developed model is determined using the Lagrangean conditions after the defuzzification of the cost function with the graded mean integration method. The proposed method finds optimal lot size with quantity discount and permissible delay in payments. Through an experimental investigation of the problem, we show the effectiveness of our approach in solving large scale problems and illustrate the effect of permissible delay in payments.

Key words: Economic order quantity, Fuzzy inventory, Function principle, Graded mean integration representation, the total inventory cost, quantity discounts, Lagrangean method, Delay in payments.

1. INTRODUCTION

In this paper we propose the EOQ problem where retailers act cooperatively to benefit from the supplier's option namely, the quantity discount and the delay in payments. Retailers can join their orders to save their individual costs. Suppliers generally purpose discounts landings in terms of the purchased quantity. Therefore, joint orders for numerous companies are profitable. At the same time, the supplier can accept to delay the payment for companies in hack of cash.

[3] addressed a distribution channel where a supplier delivers a single product to multiple retailers. [6] incorporated the discount quantity option in the EOQ including two different modes of transportation. More recently [5] proposed a corrected model of [7 , 1] presented an overview of the quantity discount [4] as the first to propose an EOQ model under permissible delay in payment. A fully permissible delay in payments was considered by [2,9,10]

function principle is proposed for arithmetic operation of fuzzy number and Lagrangean method is used for optimization. Graded mean integration is used for defuzzifying the total inventory cost for the EOQ with quantity discount and permissible delay in payments.

In Section 2 deals with basic concepts of fuzzy sets, fuzzy numbers and function principle. Section 3 discuss with fuzzy EOQ inventory models with different situation. Section 4 contains a numerical example illustrates the solution procedure. Finally the conclusions are give in section 5.

2. METHODOLOGY

2.1. Graded Mean Integration Representation Method

[8, 11] introduced Graded mean Integration Representation Method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. Here, we fist define generalized fuzzy number as follows :

Suppose $\tilde{A}$ is a generalized fuzzy number as shown in Fig.1. It is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to [0, 1],
2. $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,
3. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
4. $\mu_{\tilde{A}}(x) = W_A, a_2 \leq x \leq a_3$,
5. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,
6. $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$,

where $0 < W_A \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers.

This type of generalized fuzzy numbers are denoted as $\tilde{A} = (a_1, a_2, a_3, a_4 ; \omega_A)_{LR}$ and $= (a_1, a_2, a_3, a_4 : w_A)_{LR}$. When $\omega_A = 1$, it can be formed as $= (a_1, a_2, a_3, a_4 ; \omega_A)_{LR}$. Second, by Graded Mean Integration Representation Method, L^{-1} and R^{-1} are the inverse functions of L and R respectively and the graded mean h-level value of generalized fuzzy number $= (a_1, a_2, a_3, a_4 ; \omega_A)_{LR}$ is given by (see fig.1).

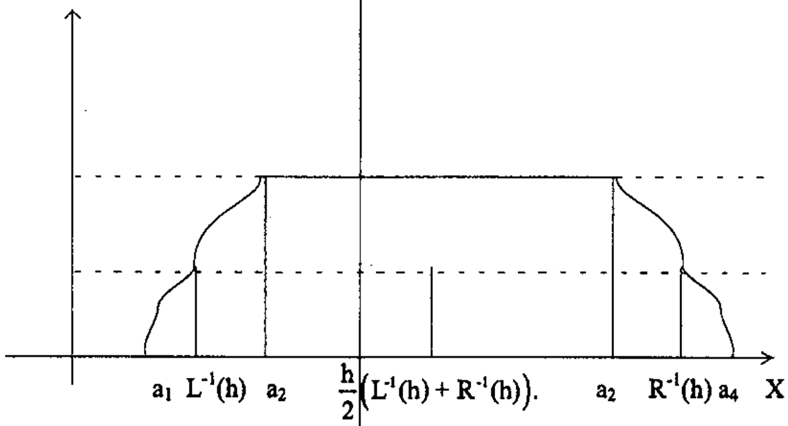

Fig.1: The graded mean h-level value of generalized fuzzy number

$\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR.}$

Then the graded Mean Integration Representation of P($\tilde{A}$) with grade w_A, where

$$P(\tilde{A}) = \frac{\int_0^{\omega_A} \frac{h}{2}\left(L^{-1}(h) + R^{-1}(h)\right) dh}{\int_0^{\omega_A} h \, dh} \qquad \ldots (1)$$

where $0 < h \le w_A$ and $0 < w_A \le 1$.

Throughout this paper, we only use popular trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed fuzzy production inventorymodels. Let $\tilde{B}$ be a trapezoidal fuzzy number and be denoted as $= (b_1, b_2, b_3, b_4)$. Then we can get the Graded Mean Integration Representation of by the formula (1) as

$$P(\tilde{B}) = \frac{\int_0^1 \frac{h}{2}\left[(b_1+b_4) + h(b_2 - b_1 - b_4 + b_3)\right] dh}{\int_0^1 h \, dh} = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6} \qquad \ldots (2)$$

2.2. The Fuzzy Arithmetical Operations under Function Principle

Function Principle is introduced by [12] to treat the fuzzy arithmetical operations by trapezoidal fuzzy numbers. We will use this principle as the operation of addition, multiplication, subtract, division of trapezoidal fuzzy numbers, because (1) the Function Principle is easier to calculate than the Extension Principle, (2) the Function Principle will not change the shape of trapezoidal fuzzy number after the multiplication of two trapezoidal fuzzy numbers, but the multiplication of two trapezoidal fuzzy numbers will become drum-like shape fuzzy number by using the Extension Principle, (3) if we have to multiply more than four trapezoidal fuzzy numbers then the Extension Principle cannot solve the operation, but the Function Principle can easily find the result by pointwise computation. Here we describe some fuzzy arithmetical operations under the Function Principle as follows.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ & $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then

(1) The addition of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(2) The multiplication of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oplus \tilde{B} = (c_1, c_2, c_3, c_4)$$

where $T = \{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$

$T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$

$c_1 = \min T_1, c_2 = \min T_1, c_3 = \max T_1, c_4 = \max T_1$

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all zero positive real numbers then

$$\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$$

(3) $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$ then the subtraction of and is

$\tilde{A} \ominus \tilde{B} = \{a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1\}$ where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(4) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} \left(\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1} \right)$ where b_1, b_2, b_3, b_4 are all positive real numbers. If a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are all nonzero positive real numbers then the division $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1} \right)$$

(5) Let $\alpha \in R$, then

(i) $\alpha \geq 0,\ \alpha \otimes \tilde{A} = (\alpha_{a_1}, \alpha_{a_2}, \alpha_{a_3}, \alpha_{a_4})$

(ii) $\alpha \geq 0,\ \alpha \otimes \tilde{A} = (\alpha_{a_4}, \alpha_{a_3}, \alpha_{a_2}, \alpha_{a_1})$

2.3. Extension of the Lagrangean method

[13] discussed how to solve the optimum solution of nonlinear programming problem with equality constraints by using Lagrangean Method, and showed how the Lagrangean method may be extended to solve inequality constraints. The general idea of extending the Lagrangean procedure is that if the unconstrained optimum the problem does not satisfy all constraints, the constrained optimum must occur at a boundary point of the solution space. Suppose that the problem is given by

Minimize $y = f(x)$

Sub to $g_i(x) \geq 0$, $i = 1, 2, \ldots, m$.

The nonnegativity constraints $x \geq 0$ if any are included in the m constraints. Then the procedure of the Extension of the Lagrangean method involves the following steps.

Step 1 : Solve the unconstrained problem

Min $y = f(x)$

If the resulting optimum satisfies all the constraints, stop because all constraints are redundant. Otherwise set $K = 1$ and go to step 2.

Step 2 : Activate any K constraints ((ie) convert them into equality) and optimize f(x) subject to the K active constraints by the Lagrangean method. If the resulting solution is feasible with respect to the remaining constraints and repeat the step. If all sets of active constraints taken K at a time are considered without encountering a feasible solution, go to step 3.

Step 3 : If $K = m$, stop ;

no feasible solution exists.

Otherwise set $K = K + 1$ and go to step 2.

3. THE EOQ INVENTORY MODEL WITH QUANTITY DISCOUNT AND PERMISSIBLE DELAY IN PAYMENTS

In this section we develops a sequential optimization method using Lagrangean method. We use this method to find the optimal economic order quantity. EOQ of the fuzzy inventory model.

3.1. Notations

The following notations are used throughout to develop the EOQ inventory model.

n	-	the number of retailers
N	-	A set of retailers 1, ….. n

i	-	the retailers index $i \in N$
q_i	-	the ordered quantity of retailer i
d_i	-	the demand for retailer i
a	-	the order cost
h_i	-	the holding cost per unit and for a unit time for retailer i
Q_{max}	-	A threshold quantity to manage the discount
c	-	the initial unit purchasing cost proposed by the supplier
c_p	-	the unit purchasing cost depending on the ordered quantity and the delay period
c_p^{min}	-	the fixed unit purchasing cost when the order quantity exceeds Q_{max}
e	-	the discount quantity rate
c_i	-	the cost supported by retailer I in a non cooperative strategy
α	-	the payment rate fixed by the supplier

3.2. Mathematical model

Discount : We adopt the purchasing price's formulating of [1] stated interms of the ordered quantity as follows.

$$C_p = \begin{cases} C, & q_i = 0 \\ C - E < q_i & 0 < q_i \leq Q_{max} \\ C_p^{min} & q_i > Q_{max} \end{cases}$$

equation ( ) reports three expressions where :

- The initial unit purchasing cost c is the announced price of the supplier in the market.
- e is a discount rate using in the landing $0 < q_i$ d" Q_{max}
- Beyond Q_{max} and no matter how large the order quantity is the supplier charges a fixed minimum price equal to c_p^{min}, Q_{max} can be computed, as proposed in through the following equation.

3.3. Discount and Delay

Besides the discount option that generally encourages retailers to order greater quantities, delay can also be a powerful tool to increase the retailer – supplier trade. Retailers are of course motivated by the discount , however they may not dispose of enough cash and need to payoff by loan from the bank. In this case, the supplier can purpose to be a source of immediate credit by delaying payments with charged interest .

From the supplier's standpoint, delay in payments incurs capital loss during the delay period. This loss can be balanced by the increase of the sale volume. We purpose, an extension of the purchasing cost that takes into account both of the discount and the delay options. The additional cost of delay is c_p x α x p_i, where pi denotes the delay period. We develop the purchasing cost in terms of two main option, namely: the quantity discount and the delay in payments. We start by stating c_p only with discount, then add the delay option. The general purchasing cost with discount and delay is reported as follows.

$$C_p = \begin{cases} C, & q_i = 0 \\ (C - E < q_i)(1 + \alpha \times p_i) & 0 < q_i \leq Q_{max} \\ C_p^{min}(1 + \alpha \times p_i) & q_i > Q_{max} \end{cases}$$

3.4. The Average Individual Cost Function for Retailer (I) C_i

If $0 < q_i$ d" Q_{max}

$$C_i = a \times \frac{d_i}{q_i} + h_i \times \frac{q_i}{2} + (C - E < q_i)(1 + \alpha \times p_i) \times d_i \quad \forall i \in N$$

We keep the holding and ordering costs and propose a purchasing cost that inserts a fractional cost incurred by postponing the payment, where $\frac{di}{q_i}$ is the number of placed orders, $\frac{q_i}{2}$ is the average size of inventory and (c – e x q_i)(1 + α x p_i) x d_i is the annual purchasing cost. The objective is to find the optimal order quantity which minimize the average individual cost.

The necessary conditions for minimum

$$\frac{\partial d_i}{\partial q_i} = 0$$

Therefore the optimal order quantity is

$$q_i^* = \sqrt{\frac{2a \times d_i}{h_i - 2e \times (1 + \alpha \times p_i) d_i}} \quad \forall i \in N$$

If $q_i > Q_{max}$

$$C_i = a \times \frac{d_i}{q_i} + h_i \times \frac{q_i}{2} + C_p^{min}(1 + \alpha \times p_i) \times d_i \quad \forall i \in N$$

The optimal quantity in this case is equal to

$$q_i^* = \sqrt{\frac{2a \times d_i}{h_i}} \quad \forall i \in N$$

Throughout this paper we use the following variables inorder to simplify the treatment of the fuzzy inventory models. $\tilde{a}$, $\tilde{L}_i$, $\tilde{C}$, $\tilde{e}$, $\tilde{d}_i$, $\tilde{\alpha}_i$, $\tilde{p}_i$ are fuzzy parameters. The fuzzy average individual cost function for retailer i is

$$\tilde{T}C_i(q_i) = \Big\{ a_1 \times \frac{d_{i_1}}{q_i} + h_{i_1} \times \frac{q_i}{2} + (c_1 - e_4 \times q_i)(1 + \alpha_1 \times p_{i_1}) \times d_{i_1},$$

$$a_2 \times \frac{d_{i_2}}{q_i} + h_{i_2} \times \frac{q_i}{2} + (c_2 - e_3 \times q_i)(1 + \alpha_2 \times p_{i_2}) \times d_{i_2},$$

$$a_3 \times \frac{d_{i_3}}{q_i} + h_{i_3} \times \frac{q_i}{2} + (c_3 - e_2 \times q_i)(1 + \alpha_3 \times p_{i_3}) \times d_{i_3},$$

$$a_4 \times \frac{d_{i_4}}{q_i} + h_{i_4} \times \frac{q_i}{2} + (c_4 - e_1 \times q_i)(1 + \alpha_4 \times p_{i_4}) \times d_{i_4} \Big\}$$

$$\tilde{T}C_i(q_i) = a \otimes (d_i | q_i) \oplus h_i \otimes \frac{q_i}{2} \oplus (C \ominus e \otimes q_i) \otimes (1 \oplus \alpha \otimes p_i) \otimes d_i$$

where $\oslash, \otimes, \ominus, \oplus$ are the fuzzy arithmetical operation under function principle [14].

Suppose, $\tilde{a} = (a_1, a_2, a_3, a_4)$

$\tilde{h}_i = (h_{i_1}, h_{i_2}, h_{i_3}, h_{i_4})$

$\tilde{C} = (c_1, c_2, c_3, c_4)$

$\tilde{e} = (e_1, e_2, e_3, e_4)$

$\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$

$\tilde{P}_i = (P_{i_1}, P_{i_2}, P_{i_3}, P_{i_4})$

$\tilde{d}_i = (d_{i_1}, d_{i_2}, d_{i_3}, d_{i_4})$

are nonnegative trapezoidal fuzzy numbers. Then we solve the optimal order quantity as the following steps. Second we defuzzify the fuzzy total inventory cost. Using the Graded Mean Integration Representation Method. The result is

$$P(TC_i(q_i)) = \frac{1}{6}\Big\{ a_1 \times \frac{d_{i_1}}{q_i} + h_{i_1} \times \frac{q_i}{2} + (c_1 - e_4 \times q_i)(1 + \alpha_1 \times p_{i_1}) \times d_{i_1}$$

$$+ 2\left[a_2 \times \frac{d_{i_2}}{q_i} + h_{i_2} \times \frac{q_i}{2} + (c_2 - e_3 \times q_i)(1 + \alpha_2 \times p_{i_2}) \times d_{i_2} \right]$$

$$+ 2\left[a_3 \times \frac{d_{i_3}}{q_i} + h_{i_3} \times \frac{q_i}{2} + (c_3 - e_2 \times q_i)(1 + \alpha_3 \times p_{i_3}) \times d_{i_3} \right]$$

$$+a_4 \times \frac{d_{i_4}}{q_i} + h_{i_4} \times \frac{q_i}{2} + \left(c_4 - e_1 \times q_i\right)(1 + \alpha_4 \times p_{i_4}) \times d_{i_4} \Big\}$$

Third, we can get the optimal order quantity q_i^* when $P\left(\tilde{T}C_i(q_i)\right)$ is minimization. In order to find when $P\left(\tilde{T}C_i(q_i)\right)$ the derivative of $P\left(\tilde{T}C_i(q_i)\right)$ with q_i is

$$\frac{\partial P\left(\tilde{T}C_i(q_i)\right)}{\partial q_i} = 0$$

Hence we find the optimal order quantity

$$q_i^* = \sqrt{\frac{2\left[\left(a_1 \times d_{i_1}\right) + 2\left(a_2 \times d_{i_2}\right) + 2\left(a_3 \times d_{i_3}\right) + \left(a_4 \times d_{i_4}\right)\right]}{\begin{array}{l}\left(h_{i_1} + 2h_{i_2} + 2h_{i_3} + h_{i_4}\right) - 2\left[e_4 \times (1 + \alpha_1 \times p_{i_1}) \times d_{i_1}\right] \\ + 2\left[e_3 \times (1 + \alpha_2 \times p_{i_2}) \times d_{i_2}\right] \\ + 2\left[e_2 \times (1 + \alpha_3 \times p_{i_3}) \times d_{i_3}\right] + \left[e_1 \times (1 + \alpha_4 \times p_{i_4}) \times d_{i_4}\right]\end{array}}}$$

3.5. Fuzzy Inventory EOQ Model with Fuzzy Order Quantity

In this section, we introduce the fuzzy inventory EOQ models by changing the crisp order quantity q_i^* be a trapezoidal fuzzy number $q_i^* = \left(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}\right)$ with $0 < q_{i_1} \le q_{i_2} \le q_{i_3} \le q_{i_4}$.

Then we can get the fuzzy average individual cost function for retailer (i) as

$$\tilde{T}C_i(q_i) = \Big\{ a_1 \times \frac{d_{i_1}}{q_i} + h_{i_1} \times \frac{q_i}{2} + \left(c_1 - e_4 \times q_i\right)(1 + \alpha_1 \times p_{i_1}) \times d_{i_1},$$

$$a_2 \times \frac{d_{i_2}}{q_i} + h_{i_2} \times \frac{q_i}{2} + \left(c_2 - e_3 \times q_i\right)(1 + \alpha_2 \times p_{i_2}) \times d_{i_2},$$

$$a_3 \times \frac{d_{i_3}}{q_i} + h_{i_3} \times \frac{q_i}{2} + \left(c_3 - e_2 \times q_i\right)(1 + \alpha_3 \times p_{i_3}) \times d_{i_3},$$

$$a_4 \times \frac{d_{i_4}}{q_i} + h_{i_4} \times \frac{q_i}{2} + \left(c_4 - e_1 \times q_i\right)(1 + \alpha_4 \times p_{i_4}) \times d_{i_4} \Big\}$$

Secondly we defuzzify the fuzzy average individual cost function for retailer (i) using the Graded Mean Integration Representation Method.

The result is

$$\tilde{P}\big(TC_i(q_i)\big) = \frac{1}{6}\Bigg\{a_1 \times \frac{d_{i_1}}{q_i} + h_{i_1} \times \frac{q_i}{2} + \big(c_1 - e_4 \times q_i\big)(1 + \alpha_1 \times p_{i_1}) \times d_{i_1}$$

$$+ 2\left[a_2 \times \frac{d_{i_2}}{q_i} + h_{i_2} \times \frac{q_i}{2} + \big(c_2 - e_3 \times q_i\big)(1 + \alpha_2 \times p_{i_2}) \times d_{i_2}\right]$$

$$+ 2\left[a_3 \times \frac{d_{i_3}}{q_i} + h_{i_3} \times \frac{q_i}{2} + \big(c_3 - e_2 \times q_i\big)(1 + \alpha_3 \times p_{i_3}) \times d_{i_3}\right]$$

$$+ a_4 \times \frac{d_{i_4}}{q_i} + h_{i_4} \times \frac{q_i}{2} + \big(c_4 - e_1 \times q_i\big)(1 + \alpha_4 \times p_{i_4}) \times d_{i_4}\Bigg\}$$

with $0 < q_{i_1} \le q_{i_2} \le q_{i_3} \le q_{i_4}$.

It will not change the meaning of formula if we replace inequality conditions with $0 < q_{i_1} \le q_{i_2} \le q_{i_3} \le q_{i_4}$ into the following inequality. $q_{i_2} - q_{i_1} \ge 0$, $q_{i_3} - q_{i_2} \ge 0$, $q_{i_4} - q_{i_3} \ge 0$, $q_{i_1} > 0$. In the following steps, extension of the Lagrangean method is used to find the solutions of q_{i_1}, q_{i_2}, q_{i_3} and q_{i_4} to minimize $\tilde{P}\big(TC_i(q_i)\big)$.

<u>**Step 1 :**</u> Solve the unconstraint problem.

To find the min $P\left[\tilde{T}C_i(q_i)\right]$, we have to find the derivative of $P\left[\tilde{T}C_i(q_i)\right]$ with respect to q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}.

$$\frac{\partial P}{\partial q_{i_1}} = \frac{1}{6}\left\{\frac{h_{i_1}}{2} - \frac{a_4 d_{i_4}}{q_{i_1}^2} - e_1 \times \left(1 + \alpha_4 \times P_{i_4}\right) \times d_{i_4}\right\}$$

$$\frac{\partial P}{\partial q_{i_2}} = \frac{1}{6}\left\{\frac{h_{i_2}}{2} - \frac{a_3 d_{i_3}}{q_{i_2}^2} - e_2 \times \left(1 + \alpha_3 \times P_{i_3}\right) \times d_{i_3}\right\}$$

$$\frac{\partial P}{\partial q_{i_3}} = \frac{1}{6}\left\{\frac{h_{i_3}}{2} - \frac{a_2 d_{i_2}}{q_{i_3}^2} - e_3 \times \left(1 + \alpha_2 \times P_{i_2}\right) \times d_{i_2}\right\}$$

$$\frac{\partial P}{\partial q_{i_4}} = \frac{1}{6}\left\{\frac{h_{i_4}}{2} - \frac{a_1 d_{i_1}}{q_{i_4}^2} - e_4 \times \left(1 + \alpha_1 \times P_{i_1}\right) \times d_{i_1}\right\}$$

Let all the above partial derivatives equal to zero and solve q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}.

$$q_{i_1} = \sqrt{\frac{2a_4 d_{i_4}}{h_{i_1} - 2e_1 \times \left(1+\alpha_4 \times P_{i_4}\right) \times d_{i_4}}}$$

$$q_{i_2} = \sqrt{\frac{2(2a_3 d_{i_3})}{h_{i_2} - 2e_2 \times \left(1+\alpha_3 \times P_{i_3}\right) \times d_{i_3}}}$$

$$q_{i_3} = \sqrt{\frac{2(2a_2 d_{i_2})}{h_{i_3} - 2e_3 \times \left(1+\alpha_2 \times P_{i_2}\right) \times d_{i_2}}}$$

$$q_{i_4} = \sqrt{\frac{2a_1 d_{i_1}}{h_{i_4} - 2e_4 \times \left(1+\alpha_1 \times P_{i_1}\right) \times d_{i_1}}}$$

Because the above show that $q_{i_1} > q_{i_2} > q_{i_3} > q_{i_4}$. It does not satisfy the constraint

$0 < q_{i_1} \le q_{i_2} \le q_{i_3} \le q_{i_4}$. Therefore set $K = 1$ and go to step 2.

Step 2 : Convert the inequality constraint $q_{i_2} - q_{i_1} \ge 0$ into equality constraint $q_{i_2} - q_{i_1} = 0$ and optimize $P\left(\tilde{T}C_i(q_i)\right)$ subject to $q_{i_2} - q_{i_1} = 0$ by the Lagrangean method.

$$L(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda) = P\left(\tilde{T}C_i(q_i)\right) - \lambda\left(q_{i_2} - q_{i_1}\right).$$

Taking the partial derivatives of $L(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda)$ with respect to $q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}$ and λ to find the minimization of $L(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda)$.

Let all the above partial derivatives $\frac{\partial L}{\partial q_{i_1}}, \frac{\partial L}{\partial q_{i_2}}, \frac{\partial L}{\partial q_{i_3}}, \frac{\partial L}{\partial q_{i_4}}, \frac{\partial L}{\partial \lambda}$ equal to zero and solve to $q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}$. Then we get

$$q_{i_1} = q_{i_2} = \sqrt{\frac{2\left(a_4 d_{i_4} + 2a_3 d_{i_3}\right)}{\left(h_{i_1} + 2h_{i_2}\right) - 2e_1 \times \left(1+\alpha_1 \times P_{i_4}\right) \times d_{i_4} - 2e_2 \times \left(1+\alpha_3 \times P_{i_3}\right) \times d_{i_3}}}$$

$$q_{i_3} = \sqrt{\frac{2(2a_2 d_{i_2})}{h_{i_3} - 2e_3 \times (1+\alpha_2 \times P_{i_2}) \times d_{i_2}}}$$

$$q_{i_4} = \sqrt{\frac{2a_1 d_{i_1}}{h_{i_4} - 2e_4 \times (1+\alpha_1 \times P_{i_1}) \times d_{i_1}}}$$

Because the above results show that $q_{i_3} > q_{i_4}$, it does not satisfy the constraint $0 < q_{i_1} \le q_{i_2} \le q_{i_3} \le q_{i_4}$. Therefore it is not a local optimum, set K = 2 and go to step 3.

Step 3 :

Convert the inequality constraint $q_{i_2} - q_{i_1} \ge 0$, $q_{i_3} - q_{i_2} \ge 0$ into equality constraints $q_{i_2} - q_{i_1} = 0$ and $q_{i_3} - q_{i_1} = 0$. We optimize $P(\tilde{T}C_i(q_i))$ subject to $q_{i_2} - q_{i_1} = 0$ and $q_{i_3} - q_{i_2} = 0$ by the Lagrangean Method. Then the Lagrangean Method is

$$L(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda_1, \lambda_2) = P\left[\tilde{T}C_i(q_i)\right] - \lambda_1(q_{i_2} - q_{i_1}) - \lambda_2(q_{i_3} - q_{i_2}).$$

In order to find the minimization of $L(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda_1, \lambda_2)$. We take the partial derivatives of $L(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda_1, \lambda_2)$ with respect to $q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda_1, \lambda_2$ and let all the partial derivatives $\frac{\partial L}{\partial q_{i1}}, \frac{\partial L}{\partial q_{i2}}, \frac{\partial L}{\partial q_{i3}}, \frac{\partial L}{\partial q_{i4}}, \frac{\partial L}{\partial \lambda_1}, \frac{\partial L}{\partial \lambda_2}$ equal to zero and to solve $q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}$.

$$q_{i_1} = q_{i_2} = q_{i_3} = \sqrt{\frac{2(a_4 d_{i_4} + 2a_3 d_{i_3} + 2a_2 d_{i_2})}{(h_{i_1} + 2h_{i_2} + 2h_{i_3}) - 2e_1 \times (1+\alpha_1 \times P_{i_4}) \times d_{i_4} - 2e_2 \times (1+\alpha_3 \times P_{i_3}) \times d_{i_3} - 2e_3 \times (1+\alpha_2 \times P_{i_2}) \times d_{i_2}}};$$

$$q_{i_4} = \sqrt{\frac{2a_1 d_{i_1}}{h_{i_4} - 2e_4 \times (1+\alpha_1 \times P_{i_1}) \times d_{i_1}}}$$

The above result $q_{i_1} > q_{i_4}$ does not satisfy the constraint $0 < q_{i_1} \le q_{i_2} \le q_{i_3} \le q_{i_4}$. Therefore set K = 3 and go to step 4.

Step 4 :

Convert the inequality constraint $q_{i_2} - q_{i_1} \geq 0$, $q_{i_3} - q_{i_2} \geq 0$ and $q_{i_4} - q_{i_3} \geq 0$ into equality constraints $q_{i_2} - q_{i_1} = 0$, $q_{i_3} - q_{i_2} = 0$, $q_{i_4} - q_{i_3} = 0$. We optimize $P(\tilde{T}C_i(q_i))$ subject to $q_{i_2} - q_{i_1} = 0$, $q_{i_3} - q_{i_2} = 0$, $q_{i_4} - q_{i_3} = 0$ by the Lagrangean Method. The Lagrangean Function is given by

$$L(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda_1, \lambda_2, \lambda_3) = P(\tilde{T}C_i(q_i)) - \lambda_1(q_{i_2} - q_{i_1}) - \lambda_2(q_{i_3} - q_{i_2}) - \lambda_3(q_{i_4} - q_{i_3})$$

In order to find the minimization of $L(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda_1, \lambda_2, \lambda_3)$. We take the partial derivatives of $L(q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda_1, \lambda_2, \lambda_3)$ with respect to $q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}, \lambda_1, \lambda_2$ and λ_3. Let all the partial derivatives $\frac{\partial L}{\partial q_{i_1}}, \frac{\partial L}{\partial q_{i_2}}, \frac{\partial L}{\partial q_{i_3}}, \frac{\partial L}{\partial q_{i_4}}, \frac{\partial L}{\partial \lambda_1}, \frac{\partial L}{\partial \lambda_2}, \frac{\partial L}{\partial \lambda_3}$ equal to zero.

$$q_i^* = \sqrt{\frac{2\left(a_1 d_{i_1} + 2a_2 d_{i_2} + 2a_3 d_{i_3} + a_4 d_{i_4}\right)}{\left(h_{i_1} + 2h_{i_2} + 2h_{i_3} + h_{i_4}\right) - 2\left[(e_4 \times (1+\alpha_1 \times P_{i_1}) \times d_{i_1}) + 2(e_3 \times (1+\alpha_2 \times P_{i_2}) \times d_{i_2}) + 2(e_2 \times (1+\alpha_3 \times P_{i_3}) \times d_{i_3}) + (e_1 \times (1+\alpha_4 \times P_{i_4}) \times d_{i_4}\right.}}$$

4. NUMERICAL EXAMPLES

Consider an inventory system with the following characteristics.

Retailer's data and costs for n = 1, o = 60, e = 0.005, α = 0.01, $d_1 = 500$, $h_1 = 15$, $p_1 = 3$,

$m_1 = 7$, $a_i^* = 78.047$ & $C_i = 26554.384$

Suppose Fuzzy initial unit purchasing cost is "more or less than 50"

$\tilde{C} = (C_1, C_2, C_3, C_4) = (30, 40, 60, 70)$

Fuzzy annual demand for retailer (i = 1) is "more or less than 500"

$\tilde{d}_1 = (d_{11}, d_{12}, d_{13}, d_{14}) = (480, 490, 570, 520)$

Fuzzy holding cost per unit and for a unit time for retailer (i = 1) is "more or less than 15"

$$\tilde{h}_1 = (h_{11}, h_{12}, h_{13}, h_{14}) = (11, 13, 17, 19)$$

Fuzzy delay period for retailer (i = 1) is "more or less than 3"

$$\tilde{P}_1 = (P_{11}, P_{12}, P_{13}, P_{14}) = (2, 2.5, 3.5, 4)$$

Fuzzy number of orders per period for retailer (i = 1) is "more or less than 7"

$$\tilde{m}_1 = (m_{11}, m_{12}, m_{13}, m_{14}) = (6, 6.5, 7.5, 8)$$

Fuzzy order cost is "more or less than 60"

$$\tilde{a} = (a_1, a_2, a_3, a_4) = (50, 55, 65, 70)$$

Fuzzy discount quantity rate is "more or less than 0.005"

$$\tilde{e} = (e_1, e_2, e_3, e_4) = (0.003, 0.004, 0.006, 0.007)$$

Fuzzy payment rate fixed by the supplier is "more or less than 0.01"

$$\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (0.008, 0.009, 0.011, 0.012)$$

Fuzzy order quantity

$$\tilde{q}_i^* = (78.4, 78.4, 78.4, 78.4)$$

Optimal average individual cost function for retailer i(i = 1)

$$(15100.08, 18226.22, 32701.24, 39228.11)$$

5. CONCLUSION

This paper presents two fuzzy EOQ model with quantity discount and the delay in payments and minimizing the total inventory cost function for the EOQ. In this model. initial unit purchasing cost (c) annual demand for retailer (d_i), holding cost (h_i), delay period for retailer (p_i), number of orders per period for retailer (m_i), order cost (a_i), discount quantity rate (e), payment rate fixed by the supplier (α) and the average individual cost function for retailer i (C_i) are represented by fuzzy numbers. For each fuzzy model, a method of defuzzification namely the Graded Mean Integration

Representation is employed to find the estimate of average individual cost function for retailer i (C_i) in the fuzzy sense and then the corresponding optimal order lotsize is derived from Lagrangean Method. Numerical Examples are carried out to investigate the behavior of our proposed model.

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