Electrothermoconvection in a dielectric fluid layer in the presence of heat generation

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Abstract

The simultaneous effect of vertical ac electric field, vertical temperature gradient, and uniform internal heating on the onset of convection in a dielectric fluid layer is investigated. It is established that the principle of exchange of stability is valid and the resulting eigenvalue problem is solved numerically using the Galerkin method. It is found that the presence of both internal heating as well as vertical ac electric field is to reinforce each other to hasten the onset of convection as compared to their action in isolation. Besides, increase in the value of dimensionless heat source strength and electric Rayleigh number is to reduce the size of convection cells.

Key words: Thermal convection, electric field, dielectric fluid, heat generation
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1 Introduction

Buoyancy-driven convection in a horizontal fluid layer heated uniformly from below in the presence of external constraints of vertical magnetic field and/or rotation has already been documented in detail (see Chandrasekhar, 1961) because of its applications in many science and engineering problems. The effects of magnetic field become dominant on convective instability only, when the fluid is highly electrically conducting. On the contrary, if the fluid is dielectric with low electrical conductivity then the electric forces play a major role in driving the motion. In dielectric fluids, an applied temperature gradient produces non-uniformities in the electrical conductivity σ and the dielectric permittivity ϵ. If the variation in σ, namely $\Delta\sigma$, is nonzero then free charge builds up which in turn induces an electric field called thermal electric field which in turn produces a force that eventually causes fluid motion due to small scale turbulence. On the other hand, if $\nabla\epsilon$ is nonzero and the electric field is intense, then polarization forces cause fluid motion. In either case, convection can occur in a dielectric fluid layer even if the temperature gradient is stabilizing and such an instability problem is termed electroconvection (EC) which analogous to Rayleigh-Benard convection. In addition, if the applied temperature gradient is also destabilizing then such a convective instability problem is called electrothermoconvection (ETC).

Onset of convection in a dielectric fluid layer in the presence of an electric fluid has been studied both theoretically and experimentally in recent years because of its applications in many practical problems such as liquid heat exchangers, laboratory models of thermal convection in the cores of the earth and planets, understanding of atmospheric electricity and control of convection to mention a few (for details see Saville, 1997). Roberts (1969) was the first to study the effects of ac as well as dc electric fields on the onset of convection in a dielectric fluid layer. Turnbull (1968), Turnbull and Melcher (1969), Takashima and Aldridge (1976) have studied natural convection under an ac or dc electric field and an exhaustive review on this topic has been given by Jones (1978). Of late, Maekawa et al., (1992) have investigated numerically the onset of EC, natural convection and Marangoni convection under an ac and dc electric fields, while Char and Chiang (1994) have analyzed the boundary effects on the onset of coupled Benard-Marangoni instability under an ac electric field. Recently, Douiebe et al., (2001) have studied the combined effects of vertical ac electric field and uniform rotation on coupled buoyancy and thermocapillary instability in an electrically conducting fluid layer. Othman and Sweilam (2002) have applied linear stability theory to study the effect of vertical ac electric field on the stability of natural convection in a dielectric viscoelastic fluid layer in the presence of internal heat generation, whereas the stability of a rotating layer of viscoelastic dielectric fluid (Walters' liquid B') heated from below under an ac electric field has been analyzed by Othman (2004). Rudraiah and Kaloni (2003) have performed linear stability analysis for Marangoni electroconvection in a poorly electrically conducting fluid layer heated from above in the presence of an electric field. Recently, Shivakumara et al., (2007) have investigated the effect of vertical dc electric field and uniform internal heat generation on the onset of convection in a horizontal poorly conducting dielectric fluid layer heated uniformly from below.

In a dielectric fluid it is pertinent that the applied electric field in the presence of a vertical temperature gradient increases the vibrational motion of atoms and eventually this will cause heating throughout the fluid. Therefore, it would be more appropriate to consider the effect of uniform internal heat generation on the onset of ETC in a dielectric fluid layer because it alters the basic temperature distribution from linear to nonlinear which in turn has a profound effect on ETC. Several studies have been undertaken in the past to know the effect of volumetric distribution of heat sources on the onset of thermal convection in a viscous fluid/porous layer owing to its connection with the study of convection in the earth's mantle and also its bearing on the thermal convection in clouds (Sparrow et al., 1964, Roberts, 1967, Rudraiah et al., 1982, Shivakumara and Suma, 2000). Analysis of such convection problems in the presence of electric field helps in better understanding of heat transfer processes since the fluid involved therein is dielectric, in general. However, such a study is lacking in the literature despite its importance in many of the practical problems cited above and also in many technological problems particularly in the design of more efficient heat exchangers.

The aim of the present investigation is, therefore, to study theoretically the simultaneous effect of vertical ac electric field, vertical temperature gradient and internal heat generation on the onset of natural convection in a dielectric fluid layer which has not been given due attention in the literature. Such a study throws light in understanding control (suppress or augment) of ETC in a dielectric fluid layer. The linear stability theory is applied and it is shown that the principle of exchange of stability is valid. The resulting eigenvalue problem is solved numerically using the Galerkin method. In addition, comparisons with available published results are presented.

2 Mathematical Formulation

We consider an incompressible dielectric fluid layer of thickness d in the presence of a vertical ac electric field. The lower rigid boundary is maintained at a constant temperature T_0 while the upper rigid boundary is maintained at a constant temperature $T_0 - \Delta T$ with $\Delta T > 0$. Besides, the dielectric fluid layer is heated internally by a uniform distribution of heat sources. A Cartesian coordinate system (x, y, z) is chosen such that the origin is at the bottom of the fluid layer with the z-axis pointing vertically upward in the presence of gravitational field.
The governing equations are (Landau and Lifshitz, 1960):

Conservation of mass

$$\nabla \cdot \vec{q} = 0. \tag{2.1}$$

Conservation of momentum

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}\right] = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{q} + \vec{f_e}. \tag{2.2}$$

Conservation of energy

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T = \kappa \nabla^2 T + Q. \tag{2.3}$$

We consider a Boussinesq fluid for which

$$\rho = \rho_0 \left[1 - \alpha(T - T_0)\right]. \tag{2.4}$$

Here, $\vec{q} = (u, v, w)$ is the velocity vector, T the temperature, p the pressure, ρ the fluid density, μ the fluid viscosity, κ the thermal diffusivity, α the thermal expansion coefficient, ρ_0 the fluid density at $T = T_0$, Q the uniformly distributed volumetric internal heat generation in the dielectric fluid layer and $\vec{f_e}$ is the force of electrical origin which can be expressed as

$$\vec{f_e} = \rho_e \vec{E} - \frac{1}{2} E^2 \nabla \epsilon + \nabla \left[\frac{1}{2} \rho E^2 \left(\frac{\partial \epsilon}{\partial \rho}\right)_T\right]. \tag{2.5}$$

Here, ρ_e is the charge density, ε is the dielectric permittivity and $\vec{E}$ is the electric field. In Eq.(2.5), the first term on the right hand side is the Coulomb force which involves the free charges and usually dominates when dc electric field is present, the second term depends on the gradient of ϵ , and the last electrostriction term can be lumped with the pressure p in Eq.(2.2) and it has no effect on an incompressible fluid. When the ac electric field, the frequency of which is 50 or 60 Hz, is applied the Coulomb force is negligible, because the electric relaxation time of most fluids is of the order of 10-100 s (see Maekawa et al., 1992). Under the circumstances, only the force induced by nonuniformity of the dielectric permittivity is considered and it is assumed to be a linear function of temperature in the form

$$\epsilon = \epsilon_0 [1 - \gamma(T - T_0)] \tag{2.6}$$

where, γ is the thermal expansion coefficient of dielectric constant. Since the fluid is poorly electrically conducting, the induced magnetic field effect is negligible and there is no applied magnetic field, then the Maxwell equations become

$$\nabla \times \vec{E} = 0 \, or \, \vec{E} = -\nabla \varphi. \tag{2.7}$$

and in the absence of free charges

$$\nabla \cdot (\epsilon \vec{E}) = 0 \tag{2.8}$$

where, φ is the electric potential. The quiescent basic state is assumed to have the following form

$$\vec{q} = 0, p = p_b(z), T = T_b(z), \vec{E} = E_b(z)\hat{k}, \rho = \rho_b(z), \epsilon = \epsilon_b(z) \tag{2.9}$$

where the subscript b denotes the basic state. The temperature, density and the dielectric permittivity in the basic state are found to be

$$T_b(z) = T_0 - \Delta T\frac{z}{d} + \frac{Qd^2}{2\kappa}\left(\frac{z}{d} - \frac{z^2}{d^2}\right) \tag{2.10a}$$

$$\rho_b(z) = \rho_0\left(1 + \alpha\Delta T\frac{z}{d} - \alpha\frac{Qd^2}{2\kappa}(\frac{z}{d} - \frac{z^2}{d^2})\right) \tag{2.10b}$$

$$\epsilon_b(z) = \epsilon_0\left(1 + \gamma\Delta T\frac{z}{d} - \gamma\frac{Qd^2}{2\kappa}(\frac{z}{d} - \frac{z^2}{d^2})\right). \tag{2.10c}$$

It may be noted that the basic temperature, density and the dielectric permittivity distributions are nonlinear with the fluid layer height due to internal heat generation which otherwise vary linearly with respect to z. The basic state pressure distribution can be found from Eq.(2.2) but it is of no consequence here as we are eliminating the same.
From Eq.(2.8) it follows that

$$\epsilon_b E_b = const = \epsilon_0 E_0, say. \tag{2.11}$$

Suppose the basic state is perturbed in the form

$$\begin{gathered}\vec{q} = \vec{q}', p = p_b(z) + p', \rho = \rho_b(z) + \rho', \\ \epsilon = \epsilon_b(z) + \epsilon', T = T_b(z) + T', \vec{E} = \vec{E}_b + \vec{E}'\end{gathered} \tag{2.12}$$

where, $\vec{q}', p', \rho', \epsilon', T'$ and $\vec{E}'$ are perturbed quantities and assumed to be small. Substituting Eq.(2.12) into the governing equations, linearizing, eliminating the pressure term from the momentum equation by operating curl twice and retaining the vertical component, we obtain the following perturbed equations (after dropping the primes):

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\nabla^2 w = \alpha g\nabla_h^2 T + \nabla_h^2\left[-\frac{\epsilon_0 E_0\gamma\Delta T}{\rho_0 d}(\gamma E_0 T - \frac{\partial\varphi}{\partial z})\right] \tag{2.13}$$

$$\left[\frac{\partial}{\partial t} - \kappa\nabla^2\right]T = -\frac{dT_b}{dz}w \tag{2.14}$$

$$\nabla^2\varphi = \gamma E_0 \frac{\partial T}{\partial z} \tag{2.15}$$

where, $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the horizontal Laplacian operator and $\nabla^2 = \nabla_h^2 + \partial^2/\partial z^2$.
On introducing the dimensionless quantities

$$\begin{aligned}(x^*, y^*, z^*) = (x/d, y/d, z/d), w^* = w/(\kappa/d),\\ t^* = t/(d^2/\kappa), T^* = T/\Delta T, \varphi^* = \varphi/(\gamma E_0 \Delta T d)\end{aligned} \tag{2.16}$$

in Eqs.(2.13)-(2.15) and neglecting the asterisks for simplicity, we obtain

$$(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2)\nabla^2 w = R_t \nabla_h^2 T + R_e \nabla_h^2 \left(T - \frac{\partial \varphi}{\partial z}\right) \tag{2.17}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right) T = -f(z) w \tag{2.18}$$

$$\nabla^2 \varphi = \frac{\partial T}{\partial z}. \tag{2.19}$$

In the above equations, $R_t = \alpha g \Delta T d^3/\nu\kappa$ and $R_e = \gamma^2 \epsilon_0 E_0^2 (\Delta T)^2 d^2/\mu\kappa$ are the thermal and electric Rayleigh numbers, respectively, $Pr = \nu/\kappa$ is the Prandtl number and

$$f(z) = \frac{dT_b}{dz} = N_s(1 - 2z) - 1 \tag{2.20}$$

is the non-uniform basic temperature gradient, where $N_s = Qd^2/2\kappa\Delta T$ is the dimensionless heat source strength.
We now assume the normal mode solution for the dependent variables in the form

$$(w, T, \varphi)(x, y, z) = [W, \Theta, \Phi](z) e^{i(lx+my)+\omega t} \tag{2.21}$$

where, ω is the growth rate, l and m are the wave numbers in the x and y- directions, respectively. Substituting Eq.(2.21) into Eqs.(2.17)-(2.19), we obtain

$$\left[\frac{\omega}{Pr} - (D^2 - a^2)\right](D^2 - a^2)W = -R_t a^2 \Theta - R_e a^2 (\Theta - D\Phi) \tag{2.22}$$

$$\left[\omega - (D^2 - a^2)\right]\Theta = -f(z) W \tag{2.23}$$

$$\left(D^2 - a^2\right)\Phi = D\Theta \tag{2.24}$$

where, $D = d/dz$ and $a = \sqrt{l^2 + m^2}$ is the overall horizontal wave number. Since the principle of exchange of stability is valid (see Appendix), we drop the time derivative terms in the above equations and arrive at the following stability equations:

$$(D^2 - a^2)^2 W = R_t a^2 \Theta + R_e a^2 (\Theta - D\Phi) \tag{2.25}$$

$$(D^2 - a^2)\Theta = f(z)W \tag{2.26}$$
$$\left(D^2 - a^2\right)\Phi = D\Theta. \tag{2.27}$$

While seeking the solution of the above equations, the following boundary conditions are used:

$$W = DW = \Theta = \Phi = 0 \quad at\ z = 0, 1. \tag{2.28}$$

3 Method of Solution

Equations (2.25) - (2.27) with the boundary conditions (2.28) constitute the eigenvalue problem with R_t or R_e as an eigenvalue. The resulting eigenvalue problem is solved numerically using the Galerkin method. Accordingly, the unknown variables are written in a series of base functions as

$$W = \sum_{i=1}^{N} A_i W_i \tag{3.1a}$$
$$\Theta = \sum_{i=1}^{N} B_i \Theta_i \tag{3.1b}$$
$$\Phi = \sum_{i=1}^{N} C_i \Phi_i \tag{3.1c}$$

where A_i , B_i and C_i are constants and the base functions W_i, Θ_i and Φ_i are represented by modified Chebyshev polynomials as trial functions satisfying the boundary conditions. Substituting Eqs.(3.1a-c) into Eqs.(2.25)-(2.27) and the Galerkin procedure of demanding the residues be orthogonal to the basis functions is applied, we get the following system of homogeneous algebraic equations:

$$C_{ji}A_i + D_{ji}B_i + E_{ji}C_i = 0 \tag{3.2}$$
$$F_{ji}A_i + G_{ji}B_i = 0 \tag{3.3}$$
$$H_{ji}B_i + I_{ji}C_i = 0. \tag{3.4}$$

The coefficients C_{ji} to I_{ji} in the above equations involve inner products of the basis functions and are given by

$$\begin{aligned}
C_{ji} &= \left\langle D^2W_j D^2W_i + a^4 W_j W_i + 2a^2 DW_j DW_i \right\rangle \\
D_{ji} &= -(R_t + R_e)a^2 \left\langle W_j \Theta_i \right\rangle \\
E_{ji} &= R_e a^2 \left\langle W_j D\Phi_i \right\rangle \\
F_{ji} &= \left\langle f(z)\Theta_j W_i \right\rangle \\
G_{ji} &= \left\langle D\Theta_j D\Theta_i + a^2 \Theta_j \Theta_i \right\rangle \\
H_{ji} &= \left\langle \Phi_j D\Theta_i \right\rangle \\
I_{ji} &= \left\langle D\Phi_j D\Phi_i + a^2 \Phi_j \Phi_i \right\rangle
\end{aligned} \tag{3.5}$$

where the inner product is defined as $< fg >= \int_0^1 fgdz$.
We note that the system of homogeneous algebraic Eqs. (3.2)- (3.4) will have the nontrivial solution if and only if

$$\begin{vmatrix} C_{ji} & D_{ji} & E_{ji} \\ F_{ji} & G_{ji} & 0 \\ 0 & H_{ji} & I_{ji} \end{vmatrix} = 0. \tag{3.6}$$

We select the trial functions as

$$W_i = (z^4 - 2z^3 + z^2)T^*_{i-1}, \; \Theta_i = z(1-z)T^*_{i-1} = \Phi_i \tag{3.7}$$

where, T^*_i are the modified Chebyshev polynomials, such that they satisfy the corresponding boundary conditions. The inner products involved in the elements of the determinant are evaluated analytically rather than numerically to avoid errors in the numerical integration. For a fixed value of R_e and N_s , Eq.(3.6) gives a relation between R_t and the wave number a which enables us to plot a locus in the $(R_t, a)-$ plane. The minimum point of R_t as a function of a gives the critical thermal Rayleigh number R_{tc} and the corresponding critical wave number a_c.

4 Results and Discussion

The combined effect of vertical ac electric field, vertical temperature gradient, and uniform internal heating on the onset of convection in a horizontal dielectric fluid layer is investigated. The eigenvalue problem is solved numerically using the Galerkin technique and the effect of various physical parameters on the stability of the system is analyzed. To validate the solution procedure, computations are carried out first under the limiting conditions of $N_s = 0$ and also when $R_e = 0$. It is observed that six terms ($i.e., N = 6$) in the series expansion of Eqs.(3.1a-c) are sufficient to get convergent results. The critical thermal Rayleigh number R_{tc} and the corresponding critical wave number a_c obtained for $R_e = 0$ and for various values of N_s are compared with the results of Sparrow et al., (1964) in Table 1, while the critical electric Rayleigh number R_{ec} and corresponding a_c obtained for $N_s = 0$ and for different values of R_t are compared with those of Roberts (1969) in Table 2. From the tables we note that the agreement is excellent and thus verifies the accuracy of the method employed.

The neutral stability curves in the $(R_t, a)-$plane for different values of dimensionless heat source strength N_s and electric Rayleigh number R_e are shown in Fig.1. It can be seen from the figure that the neutral curves exhibit single minimum with respect to the wave number. Also, increase in the strength of internal heating for a fixed value of R_t and increase in the strength of vertical ac electric field for a fixed value of N_s is to decrease the Rayleigh number and thus they have a destabilizing effect on the system. Figure 2 shows the neutral stability curves in the $(R_e, a)-$ plane for different values of thermal Rayleigh number R_t for two values of $N_s = 0$ and 5 . The case $R_t = 0$ corresponds to electroconvective instability in the absence of gravity and note that convection sets in when R_e exceeds a certain critical value. From the figure, it is also evident that heating from above ($R_t < 0$, gravitationally stable thermally stratified fluid layer) is to delay the onset of convection contrary to the case of heating from below ($R_t > 0$, gravitationally unstable thermally stratified fluid layer) as expected on the physical grounds.

The lowest point of R_t as a function of a gives the critical thermal Rayleigh number R_{tc} and the corresponding critical wave number a_c . By repeating this procedure for different values of R_e, R_{tc} and a_c are obtained as a function of N_s. The variation of R_{tc} and a_c as a function of N_s for different values of R_e is depicted in Figs. 3 and 4, respectively. From Fig. 3, we note that the critical Rayleigh number is high in the case of either $N_s = 0$ or $R_e = 0$ as compared to the simultaneous presence of N_s and R_e. That is, the simultaneous presence of internal heating and vertical ac electric field is to reinforce together and to hasten the onset of convection when compared to their individual influence on the stability of the system. This is due to an increase in energy supply to the system by both the mechanisms. From Fig. 4, it is evident that increase in the values of R_e and N_s is to increase the critical wave number which corresponds to tall thin convection cells. Further, the deviation in a_c values with R_e is found to be significant with increasing N_s.

5 Conclusions

In this paper, we have analyzed theoretically the simultaneous effect of vertical ac electric field, vertical temperature gradient, and internal heating on the onset of convection in a dielectric fluid layer. It is observed that the vertical ac electric field and internal heating have a destabilizing effect on the system, and moreover their simultaneous action makes the system more unstable when compared to their influence in isolation. Besides, increase in R_e and N_s is to increase the critical wave number and thus their effect is to reduce the size of convection cells. The implication of the present investigation is that by controlling the magnitude of an ac electric field and internal heating it is possible to control (suppress or augment) convective instability in a dielectric fluid layer.

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6 Appendix

To prove that the principle of exchange of stability is valid, the moment approach which does not suffer from the ambiguities of satisfying some of the higher boundary conditions is used (Mikaelian, 1993).

For this, we operate $(D^2 - a^2)$ on Eq.(2.21) and use Eq.(2.23) to get

$$\left[\frac{\omega}{Pr} - (D^2 - a^2)\right](D^2 - a^2)^2 W = -(R_t + R_e)a^2(D^2 - a^2)\Theta + R_e a^2 D^2\Theta. \quad (.1)$$

Equation (2.22) is considered as it is and is given by

$$\left[\omega - (D^2 - a^2)\right]\Theta = -f(z)W. \quad (.2)$$

To derive the moment equations, we multiply Eq.(.1) by W^m and Eq.(.2) by Θ^m and then integrate from $z = 0$ to 1. The exponent m is taken to be a non-negative number, but not necessarily an integer. Many terms are integrated by