Thermal Radiation Effects on MHD Convective Flow over an Inclined Porous Plate Embedded in a Porous Medium with Temperature Dependent Heat Source/Sink

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ABSTRACT

A theoretical analysis of the influence of radiation effect on a two dimensional MHD Convection and mass transfer flow of an electrically conducting fluid past an inclined semi-infinite vertical porous plate embedded in a porous medium, with variable suction and in the presence of heat generation has been analyzed. A uniform magnetic field of constant magnitude is imposed transversely to the plate. Exact solutions are obtained for concentration, temperature and velocity in terms of exponential functions. The effects of physical parameter like Radiation parameter Heat source parameter,Magnetic parameter Grashoffnumber Modified Grashoffnumber Prandtl number on velocity ,temperature and concentration distributions are studiedy graphicaly.

Key words: MHD, Heat generation, Mass transfer, Porous medium, inclined vertical surface.

1. INTRODUCTION

Natural convection flow over a vertical surface immersed in porous media has paramount importance because of its potential applications in soil physics, geohydrology, and filtration of solid from liquids, chemical engineering and biological systems. In recent years due to its diversified applications in geophysics and energy related engineering problems such as natural convection in isothermal reservoirs, heat storage beds, aquifers, quifers, porous insulation, extraction of geothermal energy and grain storage. Magneto hydrodynamics plays an important role in agriculture, engineering and petroleum industries. The study of MHD natural convection flow and heat transfer of an electrically conducting fluid past a heated semi-infinite vertical porous plate finds useful applications in many engineering problems. The effect of radiation on MHD heat transfer problems has become industrially more important, many engineering processes occur at high temperatures and hence the knowledge of radiation heat transfer is essential for designing appropriate equipment.

Seddeek et.al[10] present a radiation effects on unsteady MHD free convection flow of an electrically conducting gray gas near equilibrium in the optically thin limit along an infinite vertical porous plate. Makinde [5] analyzed a convection boundary layer flow with thermal radiation and mass transfer past a moving vertical porous plate. Ouaf [6] studied the effect of radiation on MHD steady asymmetric flow of an electrically conducting fluid past a stretching porous sheet in the presence of radiation. Rashad [9] considered the combined effect of MHD and thermal radiation on convection over a vertical flat plate embedded in a porous medium. It is observed that particle concentration and concentration boundary layer decrease due to increase in either of Lewis number, radiation parameter and buoyancy ratio. Pal et. al [7] presented an analytical study on mixed convection with thermal radiation and chemical reaction on MHD boundary layer flow of a viscous, electrically conducting fluid past a vertical permeable plate using perturbation technique. It has been shown that the effect of thermal radiation and magnetic field decreases the velocity, temperature and concentration profiles in the boundary layer. . Bhuvaneswari et.al [2] presented an analytical solution for the problem of convection heat and mass transfer of a viscous electrically conducting incompressible fluid over a semi-infinite inclined plate in a porous medium with radiation and heat generation.

Bala Anki Reddy [1] studied the effects of radiation on a steady combined free – forced convective and mass transfer flow of a viscous incompressible electrically conducting and radiating fluid over an isothermal semi-infinite vertical porous flat plate embedded in a porous medium. Harikrishan et.al [3] discussed an unsteady free convective flow past an infinite vertical porous plate with heat sink under Hall current effects. Ramana Reddy et.al [8] presented the MHD effects on the unsteady heat and mass transfer convective flow past an infinite vertical porous plate with the same frequency as that of variable suction velocity with the Soret effect. Kartikeyan et.al [4] analyzed the influence of thermal radiation on MHD unsteady flow of an electrically conducting fluid past semi-infinite vertical porous plate embedded in porous medium with variable suction. In this paper we have considered the MHD natural Convection flow of viscous incompressible fluid past an inclined vertical flat plate in a porous medium by considering perturbation technique over a small parameter 'ε'.

2. MATHEMATICAL ANALYSIS

Consider a unsteady two dimensional laminar flow of incompressible ,electrically conducting fluid past an inclined porous plate embedded in a porous medium it is assume that the plate makes an acute angle 'α' to the vertical. The flow is assumed to be in the x*-direction which is taken along the semi-infinite inclined porous plate and y*-axis normal to it. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. The induced magnetic field is neglected under the assumption that the magnetic Reynolds number is small. It is assumed that there is no applied voltage which implies the absence of an electrical field. The radioactive heat flux in the x*direction is considered negligible in comparison to that in the y*direction. The governing equations for this study are based on the conservation of mass, linear momentum and energy. Taking into consideration the assumptions made above, these equations in Cartesian frame of reference are given by

Equation of continuity :

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Equation of Motion :

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{1}{\rho}\frac{\partial p}{\partial x} + g\beta\left(T^* - T_\infty\right)\cos\alpha$$

$$+ g\beta\left(C^* - C_\infty\right)\cos\alpha - \frac{\sigma B_0^2}{\rho}u^* - \frac{\upsilon}{k^*}u^* \tag{2}$$

Equation of Energy :

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*} - \frac{Q_o}{\rho C_p}(T^* - T_\infty) \tag{3}$$

Equation of Mass:

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Where x*, y* are the dimensional distance along and perpendicular to the plate respectively. u*and v* are the components of the dimensional velocities along x*and y*directions respectively, ρ is the density of the medium, g is acceleration due to gravity, υ is the kinematics viscosity, σ is the fluid electrical conductivity , K is the permeability of the porous medium, β is the coefficient of thermal expansion, T* is the dimensional temperature of the fluid near the plate, T_∞ is the dimensional free stream temperature, κ is the thermal conductivity, C_p is the specific heat at constant pressure, q_r* is the radioactive heat flux and Q_0 is the dimensional heat absorption coefficient, α is the angle of inclination.

The optically thin limit for a non-gray gas near equilibrium the radiative heat flux is represented by the following

$$\frac{\partial q_r^*}{\partial y^*} = 4(T^* - T_\infty)I^* \tag{5}$$

Where $I^* = \int K_{\lambda\omega}\frac{\partial e_{b\lambda}}{\partial T^*}d\lambda$, $K_{\lambda\omega}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is the Planck's function.

Under this assumption, the appropriate boundary conditions for velocity involving slip flow, temperature and concentration fields are given by

$$\left.\begin{aligned} & u^* = u^*_{slip} = \frac{\sqrt{K}}{\alpha_1}\frac{\partial u^*}{\partial y^*}; T = T_\omega + \varepsilon(T_\omega - T_\infty)e^{n^*t^*} \text{ at } y=0 \\ & u^* = U_\infty = U_o(1+\varepsilon e^{n^*t^*}); T \to T_\infty \quad \text{ as } y \to \infty \end{aligned}\right\} \tag{6}$$

Where T_ω is the dimensional temperature at the wall and α_1 is the porous parameter since the suction velocity normal to the plate is a function of time only, it can be taken in the exponential form as

$$v^* = V_0(1+\varepsilon A e^{n^*t^*}) \tag{7}$$

Where A is a real positive constant, ε and εA are small quantities less than unity and $V_0$is a scale of suction velocity which is a non-zero positive constant. Out side the boundary layer, equation (2) gives

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = \frac{\partial U_\infty^*}{\partial t^*}\frac{\sigma B_0^2}{\rho}U_\infty \tag{8}$$

Now we introduce the dimensionless variables quantities as follows.

$$\left.\begin{aligned} & y = \frac{V_0 y^*}{\upsilon},\ u = \frac{u^*}{U_o},\ v = \frac{v^*}{V_o},\ U_\infty = \frac{U_\infty}{U_o},\ t = \frac{t^* V_0^{*2}}{\upsilon},\ \phi = \frac{Q_0 \upsilon}{\rho C_p V_0^2} \\ & \theta = \frac{T^* - T_\infty}{T_\omega - T_\infty},\ C = \frac{C^* - C_\infty}{C_\omega - C_\infty},\ n = \frac{n^*\upsilon}{V_o^2},\ K = \frac{K^* V_0^*}{\upsilon^2},\ F = \frac{4\upsilon I^*}{\rho C_P V_0^2} \\ & G_r = \frac{g\beta\upsilon(T_\omega - T_\infty)}{U_o V_o^2},\ G_c = \frac{g\beta\upsilon(C_\omega - C_\infty)}{U_o V_o^2},\ P_r = \frac{\mu C_p}{\kappa},\ M = \frac{\sigma B_0^2 \upsilon}{\rho V_0^2},\ S_c = \frac{\upsilon}{D} \end{aligned}\right\} \tag{9}$$

Where P_r is the Prandtl number, M is the Magnetic field parameter, $G_r$is the thermal Grashoff number, G_c is the mass Grashoff number, S_c is the Schmidt number, θ is the dimensionless temperature, ϕ is the heat source parameter, F is the radiation parameter, C is the dimensionless concentration.

Using equations (7)(8)and (9) the governing equations(2),(3) and (4) reduce to the following non-dimensional forms.

$$\frac{\partial u}{\partial t} - (1+\varepsilon A e^{nt})\frac{du}{dy} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} + G_r\theta\cos\alpha + G_c C\cos\alpha + N(U_\infty - u) \tag{10}$$

Where $N = M + \frac{1}{K}$

$$\frac{\partial\theta}{\partial t} - (1+\varepsilon A e^{nt})\frac{\partial\theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} - (F+\phi)\theta \tag{11}$$

$$\frac{\partial C}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \tag{12}$$

The boundary conditions (6) in the dimensionless form can be written as.

$$\left.\begin{aligned} &u=u_{slip}=\phi_1\frac{\partial u}{\partial y},\theta=1+\varepsilon e^{nt},C=1+\varepsilon e^{nt} \text{ at } y=0\\ &u\to U_\infty=1+\varepsilon e^{nt},\theta\to 0,C\to 0 \quad \text{as } y\to\infty \end{aligned}\right\} \tag{13}$$

Solution of the problem

The partial differential equations (10)-(12) are reduced to ordinary differential equations by assuming the following series expressions for velocity, temperature and concentration fields following .

$$u=f_0(y)+\varepsilon e^{nt}f_1(y)+O(\varepsilon^2) \tag{14}$$

$$\theta=g_0(y)+\varepsilon e^{nt}g_1(y)+O(\varepsilon^2) \tag{15}$$

$$C=h_0(y)+\varepsilon e^{nt}h_1(y)+O(\varepsilon^2) \tag{16}$$

Substituting (14) to (16) into the equation (10)-(12) and equating the harmonic and non-harmonic terms, neglecting the coefficient of O (ε^2), we get the following pars of equations.

Zero order Equations

$$f_0''+f_0'-Nf_0=-N-G_rg_0\cos\alpha-G_ch_0\cos\alpha \tag{17}$$

$$g_0''+P_rg_0'-(F+\phi)P_rg_0=0 \tag{18}$$

$$C_0''+S_cC_0'=0 \tag{19}$$

First order equations

$$f_1''+f_1'-(N+n)f_1=-Af_0'-G_rg_1\cos\alpha-G_Ch_1\cos\alpha-(N+n) \tag{20}$$

$$g_1''+P_rg_1'-(F+\phi+n)P_rg_1=-AP_rg_0' \tag{21}$$

$$C_1''+S_cC_1'-nS_cC_1=-AS_cC_0' \tag{22}$$

Where the primes denote the differentiation with respect to y.

The corresponding boundary condition can be written as.

$$\left.\begin{aligned} &f_0=\phi_1f_0',f_1=\phi_1f_1',g_0=1,g_1=1,C_0=1,C_1=1 \quad \text{at } y=0\\ &f_0\to 1,f_1\to 1,g_0\to 0g_1\to 0,C_0\to 0C_1\to 0 \quad \text{as } y\to\infty \end{aligned}\right\} \tag{23}$$

The solutions of equations (17)-(22) which satisfy the boundary conditions (23)are given by.

$$\left.\begin{aligned} &C_0 = e^{-S_c y}\ ; \quad C_1 = (1+\frac{1}{n}A)e^{-m_2 y} - \frac{1}{n}Ae^{-S_c y}\ ; \quad g_0 = e^{-m_4 y} \\ &g_1 = (1+D_1)e^{-m_6 y} + D_1 e^{-m_4 y} \quad ; \quad f_0 = 1 + D_4 e^{-m_8 y} - D_2 e^{-m_4 y} - D_3 e^{-S_c y} \\ &f_1 = 1 + D_{15}e^{-m_{10} y} + D_{10}e^{-m_8 y} - D_{11}e^{-m_6 y} - D_{12}e^{-m_4 y} - D_{13}e^{-m_2 y} + D_{14}e^{-S_c y} \end{aligned}\right\} \qquad (24)$$

Substituting equations (24) in equations (14)-(16), we obtain the velocity, temperature and concentration distributions in the boundary layer as follows.

$$u(y,t) = (1 + D_4 e^{-m_8 y} - D_2 e^{-m_4 y} - D_3 e^{-S_c y}) +$$
$$\varepsilon e^{nt}(1 + D_{15}e^{-m_{10} y} - D_{10}e^{-m_8 y} - D_{11}e^{-m_6 y} - D_{12}e^{-m_4 y} - D_{13}e^{-m_2 y} + D_{14}e^{-S_c y}) \qquad (25)$$

$$\theta(y,t) = e^{-m_4 y} + \varepsilon e^{nt}((1+D_1)e^{-m_6 y} + D_1 e^{-m_4 y}) \qquad (26)$$

$$C(y,t) = e^{-S_c y} + \varepsilon e^{nt}((1+\frac{1}{n}A)e^{-m_2 y} - \frac{1}{n}Ae^{-S_c y}) \qquad (27)$$

The localSkin-friction :

$$C_{f_x} = \frac{\tau_\omega}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(-m_8 D_4 + m_4 D_2 + S_c D_3\right) +$$
$$\varepsilon e^{nt}\left(-m_{10}D_{15} + D_{10}m_8 + D_{11}m_6 + D_{12}m_4 + D_{13}m_2 - S_c D_{14}\right)$$

The local Nusselt number :

$$N_u == \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -m_4 + \varepsilon e^{nt}\left[-m_6(1-D_1) - m_4 D_1\right]$$

The local Sherwood number :

$$S_h == \left(\frac{\partial C}{\partial y}\right)_{y=0} = -S_c + \varepsilon e^{nt}\left(-m_2(1+\frac{A}{n}) + \frac{S_c A}{n}\right)$$

3. DEDUCTIONS

1) When the angle of inclination 'α' tends to zero the velocity, temperature and concentration reduce to

$$u(y,t) = (1 + b_3 e^{-a_1 y} - b_1 e^{-m_4 y} - b_2 e^{-S_c y}) +$$
$$\varepsilon e^{nt}(1 + b_{10}e^{-a_2 y} - b_7 e^{-a_1 y} - a_{10}e^{-m_6 y} - b_8 e^{-m_4 y} - b_{12}e^{-m_2 y} + b_9 e^{-S_c y}) \qquad (18)$$

$$\theta(y,t) = e^{-m_4 y} + \varepsilon e^{nt}((1+D_1)e^{-m_6 y} + D_1 e^{-m_4 y})$$

$$C(y,t) = e^{-S_c y} + \varepsilon e^{nt}((1+\frac{1}{n}A)e^{-m_2 y} - \frac{1}{n}Ae^{-S_c y})$$

2) When $Gc \to 0, C \to 0$ the above solution reduce to

$$u(y,t) = (1+b_{11}e^{-a_1 y} - b_1 e^{-m_4 y}) + \varepsilon e^{nt}(1+b_{16}e^{-a_2 y} - b_{14}e^{-a_1 y} - a_{101}e^{-m_6 y} - b_{15}e^{-m_4 y})$$

$$\theta(y,t) = e^{-m_4 y} + \varepsilon e^{nt}((1+D_1)e^{-m_6 y} + D_1 e^{-m_4 y}) \quad (19)$$

These results are in good agrement by karthikeyan et al [7]

4. DISCUSSIONS

The velocity of the fluid thus obtained are discussed through graphically for various parameters like the Hartmann number M, Hall parameter m, permeability parameter k, Prandtl number P_r, Thermal Grashoff number G_r, Mass Grashoff number G_c,

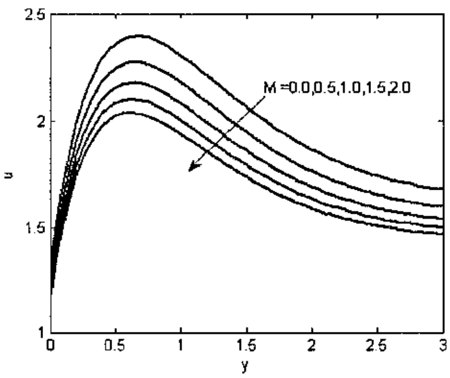

Fig.1: The variation of velocity for various values Sc=0.1,n=0.1,Pr=0.72, F=1,Φ=0.5,K=1,G_r=-10, Gc=0.5,Φ_i=0.3,ε=0.2,t=1, α=π/4, A=1

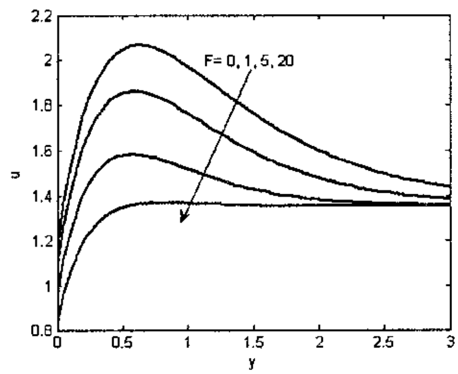

Fig.2: The variation of velocity for various values Sc=0.1,n=0.1, Pr=0.72, Φ=0.5,M=4,K=1,G_r=-10,Gc=0.5, Φ_i=0.3, ε=0.2,t=1, α=π/4,A=1

Schmdit number S_c, Angle of the inclination α, Radiation parameter F, Heat source parameter Φ, Real positive constant A. Fig.(3),(4)&(5) shows that the fluid velocity increasing for increasing the m,G_c,ε & t respectively.

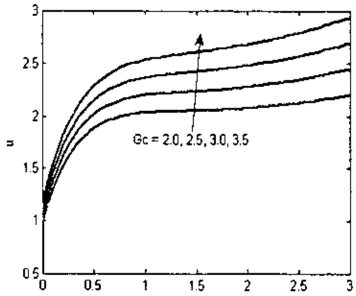

Fig.3: The variation of velocity for various values Sc=0.20,n=0.1, Pr=0.72,F=2,Φ=0.5,M=4,K=4, G_r=5,$Φ_I$=0.3, ε=0.2,t=1, α=π/4,A=1

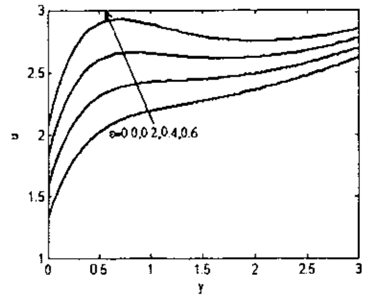

Fig.4: The variation of velocity for various values Sc=0.20,n=0.1,Pr=0.72, F=2,Φ=0.5,M=4,K=4,G_r=5, Gc=3, $Φ_I$=0.6,t=1,α=π/4,A=1

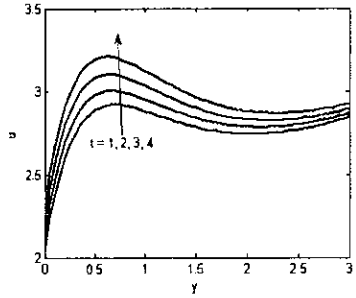

Fig.5: The variation of velocity for various values Sc=0.20,n=0.1,Pr=0.72, F=2,Φ=0.5,M=4,K=4,G_r=5, Gc=3,$Φ_I$=0.6,ε=0.6,α=π/4,A=1

Fig.(1),(2)and (6) shows that the fluid velocity is decreases for increasing the M,F&P_r respectively. Fig.(7) shows that the temperature decreases for increasing F and Fig.(8) shows that the concentration decreases for increasing Schmidt number.

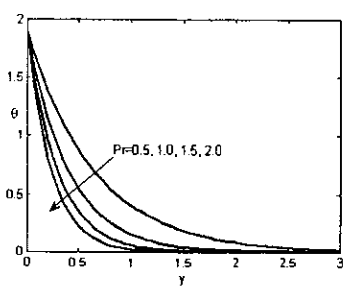

Fig.6: The variation of velocity for various values n=0.1,F=2,Φ=0.5, ε=0.6, t=4,A=1

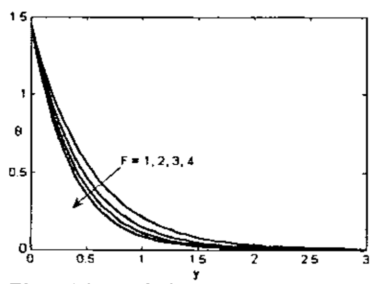

Fig.7: The variation of temperature profiles n=0.1,Pr=1,Φ=0.1,ε=0.4, t=1, A=1

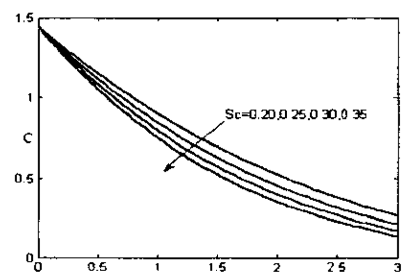

Fig.8: The variation of mass profiles n=0.1,ε=0.4,t=1,A=1

REFERENCES

1. Bala Anki Reddy, P. and Bhaskar Reddy, N. "Radiation effects on MHD combined convection and Mass transfer flow past a vertical porous plate embedded in a porous medium with heat generation". Int. J. of Appl. Math and Mech. 2010, Vol.6(18), p.33-49.

2. Bhuvaneswari, M., S. Sivasanaran and Y.J. Kim 'Exact Analysis of Radiation convective flow Heat and Mass transfer over an Inclined plate in a porous medium", World Applied Science Journal 2010, Vol.10(7), p.774-778.

3. Hari Krishna, L., and Jagadeeswara Pillai, K. "Hall Current Effects on unsteady free convection flow past an infinite vertical plate with constant suction and heat sink" International Review of pure and applied mathematics, 2010, Vol.6(1), pp.11-21.

4. Karithikeyan.S, Sivasankaran.S, & Rajan.S, "Thermal radiation effects on mhd convective flow over a vertical porous plate embedded in a porous medium by perturbation technique", Proc. Int. Conf. on Fluid Dynamics and its Applications B.N.M.Inst.of Tech.Bangalore, India, 2011, July 20-22, pp484-491.

5. makinde.O.D, "Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate", Int.comm.Heat Mass Transfer, 2005, No.32, pp.1411-1419.

6. Ouaf.M.E.M, "exact solution of thermal radiation on MHD flow over a stretching porous sheet", Appl.Math.Comput, 2005, No.170, pp.1117-1125.

7. Pal.D,Talukdar.B, "Perturbation analysis of unsteady magneto hydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction", Comm.Nomlinear Sci.Numer.Simul., 2010, No.15, pp.1813-1830.

8. Ramana Reddy.G.V, Ramanamurthy.Ch.V and Bhaskar reddy.N, "Unsteady MHD free convective Mass transfer flow past an infinite vertical porous plate with veriable suction and soret effect",JAMM, 2011, Vol.7, pp.70-84.

9. Rashad .A.M, "Influence of radiation on MHD free convection from a vertical flat plate embedded in porous media with thermophoretic deposition of particles", Comm.Nonlinear.Sci.Numer Simul, 2008, No.13, pp.2213-2222.

10. Seddeek.M.A,Aboeldahab.E.M., "Radiation effects on unsteady MHD free convection with Hall current near an infinite vertical porous plate", IJMMS 2001, Vol.26, No.4, pp.249-255.