A Novel approach for Image Segmentation using Level Sets

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Abstract

The class of geometric deformable models, so called level sets has brought tremendous impact to image segmentation because of its capability to preserve topology and fast shape recovery. Robust and efficient segmentation algorithm on digital images are challenging research topic of increasing interest in the decade. In this paper a fuzzy stopping force for the level sets is proposed to detect the Region of Interest

Keywords: Level sets, Stopping force, Fuzzy

1. Introduction

The application of level sets in image segmentation became extremely popular because of its ability to capture the topology of shapes. Image segmentation is one of the most important steps leading to the analysis of processed image data. Its main goal is to divide an image into parts that have a strong correlation with objects.

A recent approach towards segmentation is the contour detection introduced by Kass et al.[14]. In this model, active contour (snakes) start their search for a contour taking the advantage of user-provided knowledge about approximate position and shape of the required contour. The snake's energy depends on its shape and location within the image. Another class of deformable models is level sets. These deformable models for segmentation were started by Osher and Sethian [15]. The classical active contour models solve the objective function to obtain the required boundary, if an approximate or initial location of the contour is available. The level sets methods are governed by the curvature dependent speeds of moving curves or fronts.

1.1 Level Sets

The class of geometric deformable models, called level sets, has brought tremendous impact in image segmentation. The diversity of application of level set has reached into several fields such as grid computing, device fabrication, morphing, stereo vision, shape from shading, color image segmentation, 3-D reconstruction etc. The fundamental equation of level sets called "curve evolution"[7] is derived as, let $\tau(t)$ be a closed motion for the hyper surface along the direction of the normal.

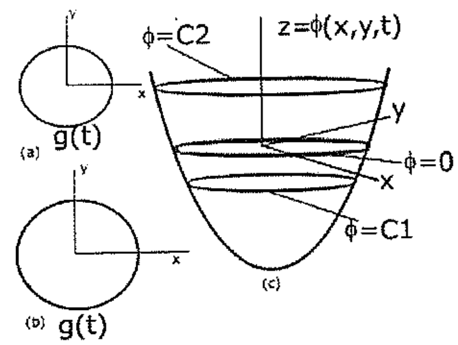

Figure 1 Level set formulation of equations of motion (a) the initial curve g(t=0) propagating in normal direction (b) shape of g(t) with t>0 (c) The initial curve embedded into x=ϕ(x,y,t), x=0 indicates the zero level set of g(t=0) and x=C1 and C2 are positive constants representing possible level sets for g(t)\t>0.

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It is represented in an Euclidian formulation with motion of propagating font in the normal direction and speed V(k), which is a scalar and curvature dependent function. This front can be embedded as zero level set of a higher dimension function ϕ. This closed interface can either be a curve in 2D space or a surface in 3D space.

Let $\phi(x,t=0)$ where $x \in R^N$ and it is defined as $\phi(x,t=0)=\pm d$, where d is the distance from position x to $\tau(0)$ and the plus (minus) is chosen if the point x is outside (inside) the initial zero level set. Thus we have an initial function: $\phi(x,t=0)=R^N \rightarrow R$ with the property $\tau(t=0)=(x \mid \phi(x,t=0)=0)$. This can be understood from Figure 1. Let $x(t), t \in [0,\infty]$ be the path of a point on the propagating front i.e. $x(t=0)$ is the point on the initial front $\tau(t=0)$ and $x_t = V(x(t))$ is the vector x_t normal to the front at *x(t)*. Since the evolving function ϕ is always zero on the propagating front. The intrinsic close curve propagation with speeds dependent on local geometric characteristic (local mean curvature k) embedded in a level set gives curve evolution equation as [7],

$$\frac{\partial \phi}{\partial t} = V(k) \mid \nabla \phi \quad \text{or} \quad \frac{\partial \phi}{\partial t} = V(k) \mid \nabla \phi = \phi \qquad (1)$$

where ϕ is the level set function, *V*(k) is the speed with which the front(or zero level curve) propagates and k is the curvature function

1.2 Active Contour Model

Based on the theory of curve evolution, Caselles et. al.[1] and Malladi [7] proposed an active contour model. In this model the curve or surface is propagating by means of velocity, which is a function of the curvature. This velocity function contains two terms, one is related to the regularity of the curve and the other shrinks or expands the curve towards the boundaries. In order to stop the curve propagating on the edges a function of the image features of interest is multiplied to the velocity function

$$\frac{\partial C(p,t)}{\partial t} = g(I)(V+K)\overrightarrow{N} \qquad (2)$$

where C(p,t) denotes the closed evolving contour, V e" 0, k is the curvature, N is the unit normal vector(inward or outward) along the evolving contour. Important issue of these models is the selection of stopping function g(I). Several kinds of stopping forces [10] are suggested to meet accurate segmentation.

2. Stopping Force

Efficiency of segmentation process using level set is depending on the stopping force for the propagating surface. Earlier research called this as " leakage prevention" techniques because they tried to prevent any bleeding or boundaries during propagation. Level sets equations are classified depending upon the kind of stopping force designed. Some of the stopping force and their drawback are discussed below. The proposed stopping force technique is discussed in the next section.

2.1 Stopping force due to Gradient

This model was proposed by Caselles et al.[1] and Malladi [7]. If ϕ (x, t) was a 2D scalar function that embedded the zero level curve (ZLC), then the geometric active contour was given by

$$\frac{\partial \phi}{\partial t} = c(x)(k + V_0) \mid \nabla \phi \mid \qquad (3)$$

Where k is the level set curvature, V_0 is a constant and c(x) is the stopping term (type-1) based on image gradient and is given as

$$c(x) = \frac{1}{1 + \mid \nabla [G_\sigma x) * I(x)] \mid} \qquad (4)$$

The limitation of this model is that the stopping term was not robust and hence could not stop the bleeding or leaking of the boundaries. The pulling back feature is not strong.

2.2 Stopping Force due to Edge Strength:

Kichenassamy *et. al.*[6] tried to solve the above problem by introducing the extra stopping term so-called "Pull Back Term". This can be expressed as:

$$\frac{\partial \phi}{\partial t} = c(x)(k + V_0)\,|\nabla \phi| + \nabla_c \bullet \nabla_\phi \qquad (5)$$

where $(\nabla c \bullet \nabla \phi)$ is stopping term (type-2) denotes the projection of an attractive force vector on the normal to the surface. The weakness of this model is that, it still suffers from boundary leaking for medical images and natural images.

2.3 Stopping Force due to area Minimization

Siddiqui et.al.·[9] *has* suggested a model by adding an extra term to the previous model i.e.

$$\frac{\partial \phi}{\partial t} = c(x)(k + V_0)\,|\nabla_\phi| + (\nabla_c, \nabla_\phi) + \frac{v_0}{2} x(\nabla_c \,|\, \nabla_\phi) \qquad (6)$$

where $\frac{V_0}{2} x(\nabla_c)\,|\nabla_\phi|$ was the area minimizing term and is mathematically equal to the product of the divergence of the stopping term (type-3) times the gradient flow. The weakness of this model is that the system does not take the advantage of the regional neighborhood of the propagation or evolution of level sets.

3. Fuzzy Rule Based Stopping Force

In section 2, different stopping force models and their limitations were discussed. To overcome some of the weaknesses of the previous models we are proposing a Fuzzy stopping force using the membership functions by considering region pixel distribution, closeness to a region's center and pixel spatial relations. The above three-membership function helps in defining a stopping force. Each membership function has a corresponding membership value for every region, which indicates the degree of belongingness to that region.

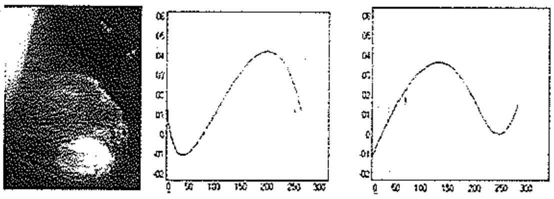
3.1 Membership function for region pixel distribution

In gray level image every region has distinctive pixel distribution, which characterizes to some extent the region's properties. The approach adopted here is to automatically define the membership function including its structure from the pixel distribution of that particular region. This can be achieved in three steps:

1. Segment the original image into a desired number of regions by applying a clustering algorithm such as FCM [3].
2. Generate the gray level pixel intensity histogram [4] for every region and normalize the frequency of each gray level into the range [0, 1]
3. Use a polynomial representation to approximate each region. The polynomial value of a region for every gray level pixel corresponds to the membership value of that pixel in that region.

The pixel distribution of each region is used to produce the corresponding membership function and the gray level intensity histogram is generated for both the regions, with the frequencies of occurrences normalized. A polynomial is then approximates histogram of each region. For example $F(x)=a_0+a_1x+a_2x^2+a_3x^3$ where x is an independent variable (8-bit gray level pixel intensity). The coefficients a_0, a_1, a_2, a_3 are computed by applying a least square fit to histogram of each region. Values of F(x) are constrained between 0 and 1, and it represent the membership values of each gray level pixel. The degree of belonging to a region of a candidate pixel, i.e. the pixel to be classified, is determined from the respective membership function. In a general case, (refer Figure 2) if $P_{s,t}$ is the pixel with gray level at location (s,t) then the two membership functions $\mu_{DR_1}(P_{s,t})$ and $\mu_{DR_2}(P_{s,t})$ for the pixel distributions of region R_1 and R_2

are expressed as $\mu_{DR_1}(P_{s,t}) = f_{R_1}(P_{s,t})$ and $\mu_{DR_2}(P_{s,t}) = f_{R_2}(P_{s,t})$ where $f_{R_1}(P_{s,t})$ and $f_{R_2}(P_{s,t})$ are the respective polynomials of regions R_1 and R_2. In figure 1(a) having a gray value of 145, the membership values for regions R_1 and R_2 can be determined from the respective polynomials as 0.40 and 0.14 respectively and it is as shown in the figure 2(b) and 2(c) respectively.



(a) (b) (c)

Figure 2. (a) Mammography image (b) Membership function for affected cancer area (region R_1) (c) Membership function for cancer unaffected area (region R_2)

3.2 Membership function to measure the closeness of a region

This membership function represents similarity between a candidate pixel and the center of the region based on gray level pixel intensity and is measured using *City Block Distance*. The pixel must always be closer to the belonging region than any other region and the degree of belongingness of candidate pixel to a region is determined from the k-means clustering algorithm. When a candidate pixel joins its nearest region, the center of that particular region is recomputed. The centroid of a region R_j is defined as $c(R_i) = \frac{1}{N_j}\sum_{i=1}^{N_i} P_j(i)$ where N_j is the number of pixels and $P_j(i)$ represents the i^{th} pixel gray level intensity in the j^{th} region. A membership function should reflect the axiom that " the closer a pixel is to a region, the larger the membership value that pixel should have". Hence the membership function μ_{CR_j} $\mu_{CR_j}(P_{s,t})$ which determines the degree of belongingness of candidate pixel $P_{s,t}$ at location (s,t) to a region R_j is defined as

$$\mu_{CR}P_{s,t} = 1 - \frac{C(R_j) - P_{s,t}}{D} \qquad (7)$$

where D is a constant equal to the difference between the maximum and minimum gray level intensity values in an image[D=255 for 8 bit gray scale].

3.3 Membership function for spatial relation

The principles of proximity and good continuation are used to define this particular membership function. Where the pixels are close together exhibit relatively smooth variations, there is an obvious expectation that strong spatial relationships will exist between neighboring pixels within that region. Considering the neighborhood relationship between the candidate pixel and its surrounding pixels reduces the number of overlapped pixels. By using fixed size neighborhood around a candidate pixel their distance from a candidate pixel has to be calculated. The neighborhood pixel configuration for r=1,r=2 and r=4 are as shown below

$$\begin{array}{ccc} & 1 & \\ 1 & 0 & 1 \\ & 1 & \end{array} \qquad \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array} \qquad \begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{array}$$

(a) (b) (c)

where 0 and 1 represent candidate and neighborhood pixels respectively. The number of neighbors will be $(r+1)^2$ for r=1 and $(r+1)^2 - 1$ otherwise. It is assumed that the variation of neighboring pixels in a region is limited to some threshold (T). To construct a membership function, the number of neighborhood pixels and their distances from the candidate pixel must be considered. The membership function μ should posses the property such as, $\mu \alpha N$ and $\mu \alpha 1/d(P_{x,y}, P_{s,t})$ where, N represents the number of neighbors and $d(P_{x,y}, P_{s,t})$ is the distance between pixels $P_{x,y}$ & $P_{s,t}$. The summation of the inverse distances of a region R is

$$G_{R_j} = \sum_{i=1}^{N_j} \frac{1}{d_i(P_{x,y}, P_{s,t})} \quad (8)$$

Where $N_j = |\zeta(P_{st}, r)|$ is the number of neighborhood pixels of the candidate pixel $P_{s,t}$ in the region R_j and $d_i(P_{x,y}, P_{st})$ is the distance between the i^{th} pixel $P_{x,y}$ of region R_j and the candidate pixel $P_{s,t}$. Considering the number neighbors N_j and the sum of their inverse distances G_{R_j} from the candidate pixel $P_{s,t}$, the membership function $\mu_{NR_j}(P_{s,t}, r)$ of the region R_j becomes

$$\mu_{NR_j}(P_{s,t,r}) = \frac{N_j X G_{R_j}}{\sum_{j=1}^{R} N_j X G_{R_j}}$$

where R is the number of segmented image regions.

3.4 Fuzzy rule definition

The definition of fuzzy rule is the most important and challenging aspect of Fuzzy Rule based Stopping Force, as its effectiveness is vital to the overall performance. In this paper, the fuzzy rule is heuristically defined using the three-membership function. The overall membership value $\mu_{AR_j}(P_{s,t})$ of a pixel $P_{s,t}$ for region R_j represents the overall degree of belonging to that region and defined by the weighted average of the above three individual membership function values

$$\mu_{DR_j}(P_{s,t}), \mu_{CR_j}(P_{s,t}), \mu_{NR_j}(P_{s,t}) \text{ i.e.,}$$

$$\mu_{AR_j}(P_{s,t}) = \frac{W_D \mu_{DR_j}(P_{s,t}) + W_C \mu_{CR_j}(P_{s,t}) + W_N \mu_{NR_j}(P_{s,t})}{W_D + W_C + W_N} \quad (10)$$

W_D, W_C and W_N are the weights of the membership values for pixel distribution, closeness to the cluster centers and neighborhood relations respectively. The overall membership value $\mu_{AR_j}(P_{s,t})$ is used in the antecedent condition of the fuzzy **IF-THEN rule as:**

IF $(P_{s,t})$ supports region R_j THEN pixel $P_{s,t}$ belongs to region R_j

The important weighting factors are W_D, W_C and W_N as their values represent a trade-off between gray level pixel intensity and spatial relationship. Prominence was initially given to W_C because, it contributes more to the human visual perception and for this reason W_C was set to greater than 1 and others are set to 1.

3.5 Image Segmentation Procedure using Fuzzy stopping force for the level sets equation

The steps used in the experimentation of fuzzy rule stopping force for level sets to segment the Region of Interest (ROI) in a given image is as shown below

1. Classify the pixels of an image into a desired number of regions using any appropriate clustering algorithm.
2. Derive the key weight, threshold and membership function for each pixel distribution
3. Initialize the center of all regions required to define the membership function and put initial contour around the center.
4. Select an unclassified pixel from the image and calculate each membership function value in each region for that pixel.
5. Classify the unclassified pixel into a desired region by applying level set equation with fuzzy stopping force.
6. Continue until no more pixels join the region of interest.

4. Conclusions And Results

The new fuzzy stopping force for the segmentation of images using level sets is experimented with different variety of images namely natural, synthetic, mammography and CT images. In order to measure the performance quantitative evaluation is carried out using confusion matrix M that contains the results of the classification experiments [3]. It is an R X R square matrix, where M_{ij} denotes the number of j^{th} region pixels wrongly classified in the i^{th} region by the segmentation algorithm. A frictional confusion matrix is written as a matrix C in which the entry C_{ji} shows the probability of assigning a sample to class C_j given that the true class was C_i.

A model based error estimation procedure is used to evaluate the performance of the algorithm. The probability of error, which is also error rate of a classifier, can be obtained if the density function for each class is known. When the true class of a sample is C_i the probability that it will correctly classified is equal to the fraction of the samples belonging to class C_i that fall inside the decision region R_i for C_i.

Let C be the event that a given sample is classified correctly and let ε be the event that the sample is classified erroneously and it is given by $P(C \mid Ci = \sum_{X \in Ri} P(X \mid Ci)$ where $X \in Ri$ means the summation is performed over all values of X in the decision region R_i. Similarly the probability of error for members of class C_i is equal to the probability of X falling out side the R_i decision region $P(\varepsilon \mid Ci) = \sum_{X \notin Ri} P(X \mid Ci)$ where $X \notin Ri$ means the summation or integral is performed over all values of X that are not in the decision region R_i. The overall probability of a correct decision for a random sample from the population, averaged over all the classes is given by $P(C) = \sum_{i=1}^{k} P(Ci)P(C \mid Ci) = \sum_{i=1}^{k} P(Ci) \sum_{x \notin Ri} P(X \mid Ci)$ The overall probability of error P $(\in)$ is given by

$$P(\in) = 1 - P(C) = 1 - \sum_{i=1}^{k} P(Ci) \sum_{x \in Ri} P(X) \mid Ci$$

$= \sum_{i=1}^{k} P(Ci) \sum_{x \notin Ri} P(X) \mid Ci$ An advantage of the model based method is that the error rate estimates are exact if the densities are known; but the disadvantage is that if the assumed density functions do not fit the data well, the reliability of the error estimate will decrease.

Percentage of error of all i^{th} region pixels that are not classified in the i^{th} region is given by

$$ErrorI = \frac{\left(\sum_{j=1}^{R} C_{ji} - C_{ii} \right)}{\sum_{j=1}^{R} C_{ji}} X \ 100$$

$$ErrorII = \frac{\sum_{j=1}^{R} (C_{ij} - C_{ii})}{\sum_{j=1}^{R} \sum_{j=1}^{R} C_{ij} - \sum_{j=1}^{R} C_{ji}} X \ 100$$

Initialization of the center of the region of interest is to be performed randomly. The maximum number of iterations, minimum level of improvement and the value of fuzzifier gives the performance evaluation of the different stopping force function for the segmentation of medical images.

4.1 Segmentation of CT/Mammography Images

The experiment is carried out for the segmentation of masses from the CT/Mammography images. We have used a Fuzzy rule based stopping force for the level sets equation for segmentation of mammography images. This algorithm has been experimentally tested upon various different CT/Mammography images using K-means clustering algorithm. The results will prove the effective segmentation helps in extracting the parameters such as Density, Size, Shape and Margins needed for cancer detection in mammography images. The results obtained using the above algorithm for r= 2 and r=4 are shown in figure 3(e). Dense breast containing a malignant mass and initial contour placed inside the mass is shown in the figure 3(a) and the segmented output for type1, type2 and type3 stopping forces are shown in figure 3(b),3(c) and 3(d) respectively.

Table 1

Error percentage in segmentation of ROI in Mammography, CT & Natural images

Shopping Force Type	Region of Interest in different images	%of Error	
		Type I	Type II
Type-1	1. Mammographic 2. CT Image	26.00 25.00	15.00 16.00
Type-2	1. Mammographic 2. CT Image	24.00 22.00	17.00 18.00
Type-3	1. Mammographic 2. CT Image	15.00 16.00	21.00 22.00
Proposed Type i. (Fuzzy) for r=2	1. Mammographic 2. CT Image 3. Natural image	10.00 09.00 02.00	22.00 23.00 01.00
ii. (Fuzzy) for r=4	1. Mammographic 2. CT Image 3. Natural image	08.00 07.00 01.00	24.00 25.00 01.00

The results exhibits better segmentation for larger values of neighborhood radius and it is as shown in the figure 3(e). The error rates are compared in the *TABLE 1* with respect to the manually segmented reference regions of the image. The results are more promising for fuzzy stopping force used for the level sets compared to other type of stopping forces as discussed in the section 2. The fuzzy value of the three membership functions are calculated in parallel since they are independent from one another to address the time complexity issues.

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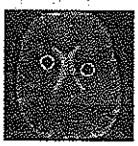
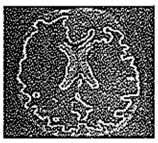
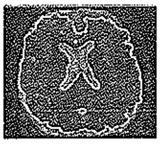
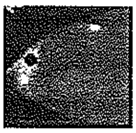
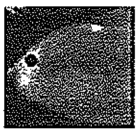
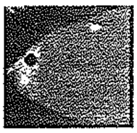
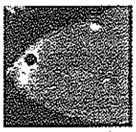
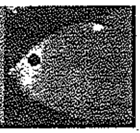
Type of image	Input image with initial contour	Segmented output of type1 Stopping force	Segmented output of type2 Stopping force	Segmented output of type3 Stopping force	Segmented output of new **Fuzzy shopping force** r=2	r=4
CT image						
Mammograms						
Natural Image