A Novel Approach For Reduction Of Linear Discrete Systems Using An Expansion Method

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Abstract

In this paper, certain novel algebraic schemes are proposed for obtaining reduced order models of a given stable higher order linear time invariant discrete systems (LTIDS), represented in the form of transfer functions. In this scheme, the given discrete system in the z domain is transferred to p domain by applying linear transformation, and then the p domain higher order model is equated with that of the reduced order model transfer function considered. To minimize the computations in model reduction, an expansion scheme is proposed. The equations formulated are solved to obtain the unknown parameters of the reduced order model. Then by applying inverse transformation the p domain reduced order system is transferred to z domain for the chosen performance index. The unit step response of the proposed model is compared with other techniques. To show the merits of the proposed analytical schemes, an example is illustrated and the necessary algorithm is also provided.

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Key words: Model reduction, expansion method, Liner Transformation and LTIDS.

1. INTRODUCTION

Today the importance of reduced order model has vastly increased, since there is a great development in design, implementation and analysis of complex control systems. Several authors have proposed variety of model reduction methods in the time domain, frequency domain or combination of both. Based on the modified Routh stability criterion, Shamash and Feinmesser [1] have introduced two methods for reduction of linear discrete time system. Shamash model are found to be applicable to discrete system with bilinear transformation, Prasad [2] has suggested a mixed method for model reduction for discrete time system using stability equation method and weighted time moments which involves the application of bilinear transformation.

This model reduction technique involves in the transformation of the given stable higher order LTIDS into an equivalent model in a different plane as suggested by Shamash [3]. This transformed system is suitably used to extract reduced order models using the proposed scheme. By applying the inverse transformation the reduced order model is transformed back to discrete model. Simplification of Z-transfer functions by continued fraction method was proposed by Shih [4]. Pade approximation method by Chung [5] and Lucas [6-8]. Moment matching by Shih et. al. [9] and Parthasarathy et.al. [10-11] improved the work of Shamash [3] using modified Cauer form. Method of Optimally approximating the given system in respect of the squared output errors of impulse response was given by Aplevich [12]. Chung et. al. [13] reported a method where the concept of Edgar [14] used for order reduction of continues system was improved and extended to discrete system. Prasad, Fernandez [15] developed a method of model reduction using optimization and Routh approximation method. Mukerjee [16] has suggested order reduction of linear discrete system using Eigen Spectrum Analysis and Farsi [17] reported a method to obtain reduced model using the stability type algorithm. C. Hwang [18] has given model reduction via bilinear Routh approximation technique.

In this proposed scheme, the method of model reduction of linear system using an expansion method is presented. The paper is organized as follows. The statement of the problem is presented in section 2. Section 3 deals with the method of model reduction. Section 4 and 5 give the model selection criterion and algorithm. The results are shown in section 6 and concluding remarks are presented in section 7.

2. STATEMENT OF PROBLEM

Basic Concepts:

Consider an n^{th} order linear time invariant system described by

$$G(z)=\frac{A_{n-1}z^{n-1}+\dots\dots\dots+A_2z^2+A_1z+A_0}{B_nz^n+\dots\dots\dots+B_2z^2+B_1z+B_0} \quad \text{----------------(1)}$$

The discrete transfer function G(z) given by equation (1) is transferred to G(p) by applying Shamash Linear transformation, z=p+1. The transformation of the transferred discrete system in p plane is represented by the equation (2).

$$G(p)=\frac{A_{n-1}p^{n-1}+\dots\dots\dots+A_2p^2+A_1p+A_0}{B_np^n+\dots\dots\dots+B_2p^2+B_1p+B_0} \quad \text{----------------(2)}$$

The corresponding k^{th} order reduced model is of the form

$$R(p)=\frac{a_{k-1}p^{k-1}+\dots\dots\dots+a_2p^2+a_1p+a_0}{b_kp^k+\dots\dots\dots+b_2p^2+b_1p+b_0} \quad \text{----------------(3)}$$

In the system described by equation (2), the problem is to find a reduced order model of the system described in equation (2) to the form of equation (3) such that the reduced order model retains the important characteristics of the original system and approximates its response as closely as possible for the same type of inputs.

3. METHOD OF MODEL REDUCTION

a. The procedure for determining the reduced order model:

The n^{th} order original system equation (2) is approximated to the k^{th} order-reduced model considered in equation (3). That is $G(p)\cong R(p)$

$$\frac{A_{n-1}p^{n-1}+\dots\dots\dots+A_2p^2+A_1p+A_0}{B_np^n+\dots\dots\dots+B_2p^2+B_1p+B_0}\cong \frac{a_{k-1}p^{k-1}+\dots\dots\dots+a_2p^2+a_1p+a_0}{b_kp^k+\dots\dots\dots+b_2p^2+b_1p+b_0}$$

By cross multiplying and simplifying

$$\begin{aligned}
&A_{n-1}p^{n-1}+\dots+A_2p^2+A_1p+A_0)*\\
&(b_kp^k+\dots+b_2p^2+b_1p+b_0)=\\
&(B_np^n+\dots+B_2p^2+B_1p+B_0)*\\
&(a_{k-1}p^{k-1}+\dots+a_2p^2+a_1p+a_0)\\
&A_{n-1}b_kp^{n+k-1}+\dots+A_{n-1}b_1p^n+\\
&A_{n-1}b_0p^{n-1}+\dots+A_1b_kp^{k+1}+\dots+A_1b_1p^2\\
&+A_1b_0p+A_0b_kp^k+..+A_0b_1p+\\
&A_0b_0=B_na_{k-1}p^{n+k-1}+..+B_na_1p^{n+1}\\
&+..+B_na_0p^n+..+B_1a_{k-1}p^k+..+B_1a_0p\\
&+B_0a_{k-1}p^{k-1}+..+B_0a_1p+B_0a_0
\end{aligned}$$

Equating the co-efficient of corresponding terms in the last equation, the following relations are obtained.

$$*A_0b_0 = B_0a_0$$
$$p(A_0b_1 + A_1b_0) = p(B_0a_1 + B_1a_0)$$
$$*A_0b_1 + A_1b_0 = B_0a_1 + B_1a_0$$
$$p^2(A_0b_2 + A_1b_1 + A_2b_0) =$$
$$p^2(B_0a_2 + B_1a_1 + B_2a_0)$$
$$*A_0b_2 + A_1b_1 + A_2b_0 =$$
$$B_0a_2 + B_1a_1 + B_2a_0$$

etc....up to,

$$p^{n+k-1}(A_{n-1}b_k) = p^{n+k-1}(B_na_{k-1})$$
$$*A_{n-1}b_k = B_na_{k-1}$$

-----------------(4)

b. Proposed guidelines for the selection of general expansion point $'a_0'$:

Choose a general expansion point $'a_0'$ as,

$$'a_0' = \frac{\text{Sum of poles} \pm \text{sum of zeros}}{\text{Number of poles} \pm \text{number of zeros}}$$

-----------------(5)

$'a_0'$ can be evaluated for the system represented by equation (2). Four possible values of($'a_0'$($'a_{01}', 'a_{02}', 'a_{03}'$ and $'a_{04}'$) can be obtained for expansion. The unknown co-efficients (a_1, b_0, b_1 and b_2) are determined by taking any positive value of $'a_0'$ using relations (4). Further, if required the expansion point

$$'a_0' = \frac{a_{01} + a_{02} + a_{03} + a_{04}}{4}$$

can be used to determine unknown co-efficients. Using the proposed scheme, the reduced order model is obtained.

4. CRITERION FOR MODEL SELECTION

For the purpose of comparison of model responses, an error index ' J ' is chosen and is defined as

$$J = \sum_{i=0}^{N} (Y_0 - Y_r)^2$$

-----------------(6)

where

Y_0 = Output of the system at the i[th] instant of time,

Y_r = Output of the reduced order system at the i[th] instant of time, and

J = Cumulative error index for 'N' Seconds.

5. GENERAL ALGORITHM FOR THE SECOND MODEL ORDER REDUCTION FROM GIVEN HIGHER ORDER SYSTEM.

Steps:

Step 1: Transfer function of the given higher order system is obtained first.

Step 2: Using Linear Transformation, z domain transfer function is converted to p domain transfer function.

Step 3: The p domain transfer function is reduced to the reduced order transfer function of the form,

$$\frac{a_1p + a_0}{b_2p^2 + b_1p + b_0}$$

Step 4: Simplify and find the relation between the constants.

Step 5: The general expansion point $'a_0'$ is evaluated.

Step 6: The value of

$$'a_0' = \frac{a_{01} + a_{02} + a_{03} + a_{04}}{4}$$

is used if it is required.

Step 7: The other unknown coefficients (a_1, b_0, b_1 and b_2) are determined by using the relations obtained from step 4.

Step 8: By applying inverse transformation, the p domain system is transferred to z domain system.

Step 9: Obtain required reduced order model.

Step 10: Using the above procedure obtain the unit step responses.

6. ILLUSTRATIONS

Example 1:

As an illustration to the above method, the eighth order system described by the transfer function has been considered [3]:

$$G(z)=\frac{N(z)}{D(z)} \qquad \text{-----------------(7)}$$

$$N(z)=1.682z^7+1.116z^6-0.21z^5+0.152z^4-0.516z^3-0.262z^2+0.044z-0.018$$

$$D(z)=8z^8-5.046z^7-3.348z^6+0.63z^5-0.456z^4+1.548z^3+0.786z^2-0.132z+0.018$$

The given original system in z domain is transferred to the p domain by applying linear transformation, $z = p+1$ to G(z). G(p) is obtained as

$$G(p) = \frac{N(p)}{D(p)} \qquad \text{-----------------(8)}$$

$$N(p)=1.682p^7+12.89p^6+47.808p^5+74.712p^4+79.182p^3+49.064p^2+16p+1.988$$

$$D(p)=8p^8+58.354p^7+185.33p^6+322.576p^5+335.864p^4+21045p^3+76.808p^2+16p+2$$

The equation (8) is equated to the 2nd order reduced model considered:

$$R(p)=\frac{a_1p+a_0}{b_2p^2+b_1p+b_0}$$

That is $\quad G(p) \cong R(p) \qquad$ -----------------(9)

where a_0, a_1, b_0, b_1 and b_2 are unknown co-efficients. By using the equation (5), the four possible values of 'a_0' are obtained and the values are 1.0022, 0.2942, 15.0327and 0.0196. The other unknown coefficients (a_1, b_0, b_1 and b_2) are determined by cross-multiplying both sides and equating the coefficients of corresponding terms in equation (9). Thus the second order reduced models for all possible values are obtained and shown in Table 1.1.

Table –1.1 Different values of 'a_0' and reduced model in 'p domain.

Values of'a_0'	Reduced model in 'p' domain
1.0022	$\frac{2.3245p+1.0022}{10.7870p^2+2.7930p+1.0022}$
0.2942	$\frac{0.6824p+0.2942}{3.1666p^2+0.8199p+0.2942}$
15.0327	$\frac{34.8672p+15.0327}{161.8019p^2+41.8939p+15.0327}$
0.0196	$\frac{0.0455p+0.0196}{0.2110p^2+0.0546p+0.0196}$

Note: The other possible value of 'a_0' is,

$$a_0 = \frac{a_{01}+a_{02}+a_{03}+a_{04}}{4}$$

and the result is presented in Table - 1.2.

Table -1.2 Alternative value of 'a_0' and reduced model in 'p domain.

Valueof 'a_0'	Reduced model in 'p' domain
4.087	$\frac{9.4800p+4.0870}{43.9919p^2+11.3904p+4.0870}$

By applying inverse linear transformation, $p = z - 1$, the reduced model in p domain are transferred back to z domain. The resulting reduced order models in z domain and corresponding error Index ' J ' are calculated and shown in Table 1.3.

Table –1.3 Different values of 'a_0' and error index 'J' in 'z'domain

Values of 'a_0'	Reduced model in 'z'domain	err Index 'J'
1.0022	$\frac{0.21549z - 0.12258}{z^2 - 1.74109z + 0.83398}$	37e-2
0.2942	$\frac{0.21549z - 0.12259}{z^2 - 1.74107z + 0.83398}$	38e-2
15.0327	$\frac{0.21549z - 0.12258}{z^2 - 1.74167z + 0.83390}$	24e-2
0.0196	$\frac{0.21563z - 0.12275}{z^2 - 1.74123z + 0.83410}$	37e-2
4.087	$\frac{0.21549z - 0.12259}{z^2 - 1.741079z + 0.83398}$	38e-2

The cumulative error index is calculated for 10 seconds. From the Table -1.3, it is observed that the reduced order model obtained using the proposed scheme gives minimum error index ' J ' for the value of 'a_0' (=15.0327) is 0.0024. The Table–1.4 gives the comparison of different reduction methods for example 1. The minimum error index '' of proposed method is compared with that of other techniques and shown in Table -1.4. The unit step response is plotted and compared with other techniques, and the responses are shown in Figure 1.

Table –1.4: Comparison of different reduction methods with error index

Method	Reduced order model in 'z' domain	c.error Index 'J'
Prasad Model	$\frac{0.085160z - 0.031231}{z^2 - 1.730351z + 0.784280}$	2.0597
Sastry et.al. Model	$\frac{0.316666z - 0.277340}{z^2 - 1.683176z + 0.722779}$	0.3476
Proposed Method	$\frac{0.21549z - 0.12258}{z^2 - 1.74167z + 0.83390}$	0.0024

Since the proposed method yields the minimum error index (0.0024) compared to the other models, for the value of 'a_0' = 15.0327, this value may be chosen for further analysis and design.

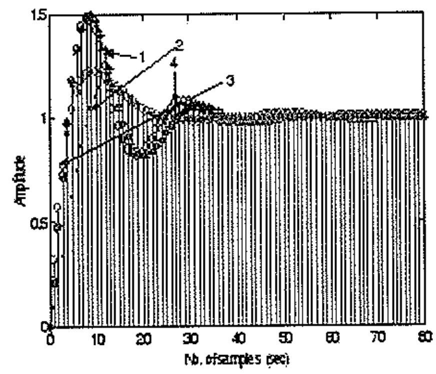

Figure 1. Unit step responses of original and reduced order systems for example 1.

Figure details:

1. Original eighth order system
2. Prasad Model
3. Sastry et.al. Model
4. Proposed Method

Inferences

The step response of original system and reduced models shown in figure 1 depicts the following features.

1) The Proposed method is far better than that of Prasad model and Sastry et.al. model

2) The Proposed reduced model response is almost matching the original system response.

7. CONCLUSION

In this paper, the proposed scheme is employed through Shamash linear transformation for obtaining the reduced order model for a stable LTIDS. The unit step response of reduced order model is chosen for comparison with the existing results. It is observed that the reduced order model obtained by the proposed method is better compared to other techniques. The cumulative error index obtained is minimum. Further, the model reduction of Multi-Input Multi-Output system is under processing using the proposed scheme.

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Appendix:

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