Intuitionstic (Q, L)-Fuzzy Subhemirings of a Hemiring

M. LATHA and N. ANITHA

ABSTRACT

In this paper, we made an attempt to study the algebraic nature of an Intuitionstic (Q, L)-fuzzy subhemirings of a hemi ring.

Key words: Intuitionstic fuzzy set , Intuitionstic **(Q, L)-**fuzzy sub hemi ring, Intuitionstic pseudo **(Q, L)-**fuzzy coset.

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INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R; + ;.). Some of them in particular, nearrings and several kinds of semi rings have been proven very useful. Semirings (called also halfrings) are algebras (R; + ;.) share the same properties as a ring except that (R; +) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R; + ;.) is said to be a semiring (R; +) and (R ;.) are semigroups satisfying a. (b+c) =a.b+a.c and (b+c).a=b.a+c.a for all a,b and c in R. A semiring R is said to be additively commutative if a+b=b+a for all a,b an c in R. A semiring R may have an identity 1, defined by 1.a=a=a.1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh[15], several researchers explores on the generalization of the concept of fuzzy sets. The notion of anti left h-ideals in hemiring was introduced by Akram.M and K.H.Dar [1].The notion of homomorphism and anti-

Department of Mathematics, Karpagam University, Coimbatore - 641021
Department of Mathematics, Periyar University, PG Extension Centre, Dharmapuri - 05

homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[7].Osman Kanzanci ,Sultan yamark ans serife yilmaz in [6] have introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear-rings) in a near –ring and investigated some related properties. Solairaju.A and R.Nagarajan, have given a new structure in construction of Q-fuzzy groups and subgroups [11] and [12]. In this paper, we introduce the properties and some theorems intuitionistic (Q,L)-fuzzy subhemirings of a hemiring.

1.1 Definition: An intuitionistic (Q,L)fuzzy subset (IQLFS) A in X is defined as an object of the form $A = \{< (x, q), \mu_A(x,q), \nu_A(x,q) > / x \in X \text{ and } q \text{ in } Q\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.2 Definition: Let R be a hemiring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic (Q,L) fuzzy subhemiring (IQLFSHR) of R if it satisfies the following conditions:

(i) $\mu_A(x+y,q) \geq \left((\mu_A(x,q) \wedge \mu_A(y,q)\right)$

(ii) $\mu_A(xy,q) \geq \left((\mu_A(x,q) \wedge \mu_A(y,q)\right)$

(iii) $\nu_A(x+y,q) \leq \left((\nu_A(x,q) \vee \nu_A(y,q)\right)$

(iv) $\nu_A(xy,q) \leq \left((\nu_A(x,q) \vee \nu_A(y,q)\right)$, for all x and y in R, and q in Q.

1.3 Definition: Let (R, +, .) and $(R', +, .)$ be any two hemirings. Let $f: R \to R'$ be any function and A be an intuitionistic (Q,L) fuzzy subhemiring in R, V be an intuitionistic (Q,L)-fuzzy subhemiring in $f(R) = R'$, defined by $\mu_V(y,q) = \mathbf{sup}_{x \in f^{-1}(y)} \mu_A(x,q)$ and $\nu_V(y,q) = \inf_{x \in f^{-1}(y)} \mu_A(x,q)$ for all x in R and y in R' and q in Q. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.4 Definition: Let A be an intuitionistic (Q,L) fuzzy subhemiring of a hemiring (R, +, .) and a in R, Then the pseudo intuitionistic (Q,L) fuzzy coset $(aA)^p$ is defined by $\left((a\mu_A)^P\right)(x,q) = p^{(a)}\mu_A(x,q)$, $((a\nu_A)^P)(x,q) = p^{(a)}\nu_A(x,q)$, for every x in R, q in Q and for some p in P.

1.5 Definition: Let (R, +, .) be a hemiring. An intuitionistic (Q,L) fuzzy subhemiring A of R is said to be an intuitionistic (Q,L) fuzzy normal subhemiring (IQLFNSHR) of R if it satisfies the following conditions: (i) $\mu_A(xy) = \mu_A(yx)$, (ii) $\nu_A(xy) = \nu_A(yx)$, for all x and y in R.

2. Properties of Intuitionstic (Q, L)-Fuzzy Subhemirings of a Hemiring

2.1 Theorem: If A and B are two intuitionistic (Q,L))-fuzzy subhemirings of the hemirings R_1 and R_2 respectively,then anti product AXB is an intuitionistic (Q,L))-fuzzy subhemiring of $R_1 X R_2$.

Proof: Let A and B be two (Q,L)- fuzzy subhemirings of the hemirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then $((x_1, y_1), q)$ and $((x_2, y_2),q)$ are in $R_1 X R_2$.Now, $\mu_{AXB}[((x_1, y_1) + (x_2, y_2)), q)] \geq \{(\mu_A(x_1, q) \wedge \mu_A(x_2, q)) \wedge ((\mu_B(y_1, q) \wedge \mu_B(y_2, q))\} \geq (\mu_{AXB}(x_1, y_1), q) \wedge \mu_{AXB}(x_2, y_2), q))$ .Therefore,$\mu_{AXB}[((x_1, y_1) + (x_2, y_2)), q)] \geq (\mu_{AXB}(x_1, y_1), q) \wedge \mu_{AXB}(x_2, y_2), q))$. Also, $\mu_{AXB}[((x_1, y_1)(x_2, y_2)), q)] \geq \{(\mu_A(x_1, q) \wedge \mu_A(x_2, q)) \wedge ((\mu_B(y_1, q) \wedge \mu_B(y_2, q))\} \geq (\mu_{AXB}(x_1, y_1), q) \wedge \mu_{AXB}(x_2, y_2), q))$. Therefore, $\mu_{AXB}[((x_1, y_1)(x_2, y_2)), q)] \geq (\mu_{AXB}(x_1, y_1), q) \wedge \mu_{AXB}(x_2, y_2), q))$ .Let x_1 and x_2 be in R_1 , y_1 and y_2 be in R_2. Then $((x_1, y_1), q)$ and $((x_2, y_2),q)$ are in $R_1 X R_2$.Now, $\nu_{AXB}[((x_1, y_1) + (x_2, y_2)), q)] \leq \{(\nu_A(x_1, q) \vee \nu_A(x_2, q)) \vee ((\nu_B(y_1, q) \vee \nu_B(y_2, q))\} \leq (\nu_{AXB}(x_1, y_1), q) \vee \nu_{AXB}(x_2, y_2), q))$.Therefore,$\nu_{AXB}[((x_1, y_1) + (x_2, y_2)), q)] \leq (\nu_{AXB}(x_1, y_1), q) \vee \nu_{AXB}(x_2, y_2), q))$. Also, $\nu_{AXB}[((x_1, y_1)(x_2, y_2)), q)] \leq \{(\nu_A(x_1, q) \vee \nu_A(x_2, q)) \vee ((\nu_B(y_1, q) \vee \nu_B(y_2, q))\} \leq (\nu_{AXB}(x_1, y_1), q) \vee \nu_{AXB}(x_2, y_2), q))$.Therefore,$\nu_{AXB}[((x_1, y_1)(x_2, y_2)), q)] \leq (\nu_{AXB}(x_1, y_1), q) \vee \nu_{AXB}(x_2, y_2), q))$.Hence AXB is an intuitionistic (Q,L))-fuzzy subhemiring of a hemiring $R_1 X R_2$.

2.2 Theorem: Let A is an intuitionistic (Q,L))-fuzzy subhemiring of a hemiring (R, +, .),then the pseudo intuitionistic (Q,L))-fuzzy coset $(aA)^p$ is intuitionistic (Q,L))-fuzzy subhemiring of a hemiring R, for every a in R.

Proof: Let A is an intuitionistic (Q,L))-fuzzy subhemiring of a hemiring R. For every x and y in R, we have $((a\,\mu_A)^p)(x + y, q) \geq p(a)\left(\mu_A(x, q) \wedge \mu_A(y, q)\right) \in p(a)\mu_A(x, q) \wedge p(a)\mu_A(y, q) = ((a\,\mu_A)^p(x, q) \wedge (a\,\mu_A)^p(y, q)$, Therefore,$((a\,\mu_A)^p)(x + y, q) \geq ((a\,\mu_A)^p(x, q) \wedge (a\,\mu_A)^p(y, q))$.
Now , $((a\,\mu_A)^p)(xy, q) \geq p(a)\left(\mu_A(x, q) \wedge \mu_A(y, q)\right) = p(a)\mu_A(x, q) \wedge p(a)\mu_A(y, q) = ((a\,\mu_A)^p(x, q) \wedge (a\,\mu_A)^p(y, q)$. Therefore,$((a\,\mu_A)^p)(xy, q) \geq ((a\,\mu_A)^p(x, q) \wedge (a\,\mu_A)^p(y, q))$. For every x and y in R, we have$(a\,\nu_A)^p(x + y, q) \leq p(a)(\nu_A(x, q) \vee \nu_A(y, q)) \in p(a)\nu_A(x, q) \vee p(a)\nu_A(y, q) = ((a\,\nu_A)^p(x, q) \vee (a\,\nu_A)^p(y, q)$. Therefore,$((a\,\nu_A)^p)(x + y, q) \leq ((a\,\nu_A)^p(x, q) \vee (a\,\nu_A)^p(y, q))$.
Now$((a\,\nu_A)^p)(xy, q) \leq p(a)(\nu_A(x, q) \vee \nu_A(y, q)) = (p(a)\nu_A(x, q) \vee p(a)\nu_A(y, q) = ((a\,\nu_A)^p(x, q) \vee (a\,\nu_A)^p(y, q))$. Therefore,$((a\,\nu_A)^p)(xy, q) \leq ((a\,\nu_A)^p(x, q) \vee (a\,\nu_A)^p(y, q))$. Hence $(aA)^p$ is an intuitionistic (Q,L))-fuzzy subhemiring of a hemiring R .

2.3 Theorem: Let (R, +,.) and (R', +, .) be any two hemirings. The homomorphic image of an intuitionistic (Q,L))-fuzzy subhemiring of R is an intuitionistic (Q,L))-fuzzy subhemiring of R'.
Proof: Let $f: R \to R'$ be homomorphism.Then,$f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let V= f(A), where A is an intuitionistic (Q,L)) fuzzy subhemiring of R. Now, for $f(x)$, $f(y)$ in R', $\mu_V((f(x) + (f(y)), q)) \geq \mu_A(x + y, q) \geq (\mu_A(x, q) \wedge \mu_A(y, q)$,which implies that $\mu_V((f(x) + (f(y)), q)) \geq (\mu_V(f(x), q) \wedge (\mu_V(f(y), q)$.Again, $\mu_V((f(x)\,f(y), q)) \geq \mu_A(xy, q) \geq (\mu_A(x, q) \wedge \mu_A(y, q)$,which implies that $\mu_V((f(x)(f(y)), q)) \geq (\mu_V((f(x), q) \wedge \mu_V(f(y), q))$. Now, for $f(x)$, $f(y)$ in R', $\nu_V((f(x), q) + (f(y), q)) \leq \nu_A(x + y, q) \leq (\nu_A(x, q) \vee \nu_A(y, q)$,which implies that$\nu_V(f(x) + f(y)), q) \leq (\nu_V((f(x), q) \vee \nu_V(f(y), q))$.Again,

$\nu_V((f(x)\,f(y),q)) \le \nu_A(xy,q) \le (\mu_A(x,q) \vee \mu_A(y,q)$,which implies that $\nu_V((f(x)f(y),q)) \le (\nu_V((f(x),q) \vee \nu_V(f(y),q))$. Hence V is an intuitionistic (Q,L))-fuzzy subhemiring of hemiring R'.

2.4 Theorem: Let (R, +,.) and (R', +,.) be any two hemirings. The homomorphic preimage of an intuitionistic (Q,L))-fuzzy subhemiring of R' is an intuitionistic (Q,L))-fuzzy subhemiring of R.

Proof: Let $f: R \to R'$ be a homomorphism.Then,$f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let V= f(A) where V is an (Q,L) fuzzy subhemiring of R'. Now, for x, y in R, $\mu_A((x+y,q)) = \mu_V((f(x)+f(y)),q) \ge (\mu_V(f(x),q) \wedge \mu_V(f(y),q)$ $= (\mu_A(x,q) \wedge \mu_A(y,q)$, which implies that $\mu_A((x+y,q)) \ge (\mu_A(x,q) \wedge \mu_A(y,q)$. Again, $\mu_A((xy,q)) = \mu_V((f(x)f(y),q) \ge (\mu_V(f(x),q) \wedge \mu_V(f(y),q) = (\mu_A(x,q) \wedge \mu_A(y,q)$, which implies that $\mu_A((xy,q)) \ge (\mu_A(x,q) \wedge \mu_A(y,q)$.Let V= f(A) where V is an anti (Q,L) fuzzy subhemiring of R'. Now, for x, y in R, $\nu_A((x+y),q)) = \nu_V((f(x)+f(y)),q) \le (\nu_V(f(x),q) \vee \nu_V(f(y),q) = (\nu_A(x,q) \vee \nu_A(y,q)$, which implies that $\nu_A((x+y),q)) \le (\nu_A(x,q) \vee \nu_A(y,q)$. Again, $\nu_A((xy,q)) = \nu_V((f(x)f(y)),q) \le (\nu_V(f(x),q) \vee \nu_V(f(y),q) = (\nu_A(x,q) \vee \nu_A(y,q)$, which implies that $\nu_A((xy,q)) \le (\nu_A(x,q) \vee \nu_A(y,q)$. Hence A is an intuitionistic (Q,L))-fuzzy subhemiring of hemiring R.

2.5 Theorem: Let (R, +, .) and (R', +, .) be any two hemirings. The anti-homomorphic image of an intuitionistic (Q,L))-fuzzy subhemiring of R is an intuitionistic (Q,L))-fuzzy subhemiring of $\mathbf{R'}$.

Proof: Let $f: R \to R'$ be a anti-homomorphism.Then,$f(x+y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R. Let V= f(A) where A is an (Q,L)- fuzzy subhemiring of R. Now, for f(x) , f(y) in R', $\mu_V((f(x)+f(y),q)) \ge \mu_A(y+x,q) \ge (\mu_A(y,q) \wedge \mu_A(x,q) = (\mu_A(x,q) \wedge \mu_A(y,q)$ which implies that $\mu_V((f(x)+f(y),q)) \ge (\mu_V((f(x),q) \wedge \mu_V(f(y),q))$. Again,$\mu_V((f(x)f(y),q)) \ge \mu_A(yx,q) \ge (\mu_A(y,q) \wedge \mu_A(x,q) = (\mu_A(x,q) \wedge \mu_A(y,q)$ which implies that $\mu_V((f(x)f(y),q)) \ge (\mu_V((f(x),q) \wedge \mu_V(f(y),q))$.Let V=f(A) where A is an intuitionistic (Q,L))- fuzzy subhemiring of R.

Now, for f(x) , f(y) in R', $\nu_V((f(x)+f(y),q)) \le \nu_A(y+x,q) \le (\nu_A(y,q) \vee \nu_A(x,q) = (\nu_A(x,q) \vee \nu_A(y,q)$,which implies that $\nu_V((f(x)+f(y),q)) \le (\nu_V((f(x),q) \vee \nu_V(f(y),q))$. Again,$\nu_V((f(x)f(y),q)) \le \nu_A(yx,q) \le (\nu_A(y,q) \vee \nu_A(x,q) = (\nu_A(x,q) \vee \nu_A(y,q)$ which implies that $\nu_V((f(x)f(y),q)) \le (\nu_V((f(x),q) \vee \nu_V(f(y),q))$. Hence V is an intuitionistic (Q,L))-fuzzy subhemiring of hemiring R'.

2.6 Theorem: Let (R, +,.) and (R′, +, .) be any two hemirings. The anti-homomorphic preimage of an intuitionistic (Q,L))-fuzzy subhemiring of R′is an intuitionistic (Q,L))-fuzzy subhemiring of R.

Proof: Let V = f(A)where V is an (Q, L) − fuzzysubhemiring of R′. Let x and y in R.Then $\mu_A((x+y,q)) = \mu_V((f(x)+f(y)),q) \ge (\mu_V(f(y),q) \wedge \mu_V(f(x),q) = (\mu_A(x,q) \wedge \mu_A(y,q)$, which implies that $\mu_A((x+y,q)) \ge (\mu_A(x,q) \wedge \mu_A(y,q)$. Again, $\mu_A((xy,q)) = \mu_V(f(x)\,f(y),q)$ $\ge (\mu_V(f(y),q) \wedge \mu_V(f(x),q) = (\mu_A(x,q) \wedge \mu_A(y,q)$,Let V = f(A)where V is an anti (Q, L) fuzzy subhemiring of R′. Let x and y in R.Then $\nu_A((x+y,q)) = \nu_V(f(x)+f(y)),q) \le (\nu_V(f(y),q) \vee \nu_V(f(x),q) = (\nu_A(x,q) \vee \nu_A(y,q)$, which implies that $\nu_A((x+y,q)) \le (\nu_A(x,q) \vee \nu_A(y,q)$. Again,$\nu_A((xy,q)) = \nu_V((f(x)\,f(y),q) \le (\nu_V(f(y),q) \vee \nu_V(f(x),q) = (\nu_A(x,q) \vee \nu_A(y,q)$, which implies that $\nu_A((xy,q)) \le (\nu_A(x,q) \vee \nu_A(y,q)$. Hence A is an intuitionistic (Q,L))-fuzzy subhemiring of hemiring R.

In the following Theorem is the composition operation of functions :

2.7 Theorem: Let A be an intuitionistic (Q,L))-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H. Then A∘ f is an intuitionistic (Q,L))-fuzzy subhemiring of R.
Proof: Let x and y in R . Then we have, $(\mu_A \circ f)((x+y, q)) = \mu_A((f(x) + (f(y)), q) \geq (\mu_A(f(x), q) \wedge \mu_A(f(y), q) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$.which implies that $(\mu_A \circ f)((x+y, q)) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$. And, $(\mu_A \circ f)((xy, q)) = \mu_A((f(x)f(y)), q) \geq (\mu_A(f(x), q) \wedge \mu_A(f(y), q) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$, which implies that $(\mu_A \circ f)((xy, q)) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$. Let x and y in R . Then we have, $(\nu_A \circ f)((x+y, q)) = \nu_A((f(x) + f(y)), q) \leq (\nu_A(f(x), q) \vee \nu_A(f(y), q) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$.which implies that $(\nu_A \circ f)((x+y, q)) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$. And, $(\nu_A \circ f)((xy, q)) = \nu_A((f(x) f(y), q) \leq (\nu_A(f(x), q) \vee \nu_A(f(y), q) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$, which implies that $(\nu_A \circ f)((xy, q)) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$. Therefore A∘f is an intuitionistic (Q,L))-fuzzy subhemiring of hemiring R.

2.8 Theorem: Let A be an intuitionistic (Q,L))-fuzzy subhemiring of hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then A∘ f is an intuitionistic (Q,L))-fuzzy subhemiring of R.
Proof:Let x and y in R . Then we have, $(\mu_A \circ f)((x+y, q)) = \mu_A((f(y) + f(x)), q) \geq (\mu_A(f(x), q) \wedge \mu_A(f(y), q) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$. which implies that $(\mu_A \circ f)((x+y, q)) \geq ((\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q))$. And, $(\mu_A \circ f)((xy, q)) = \mu_A((f(y) f(x)), q) \geq (\mu_A(f(x), q) \wedge \mu_A(f(y), q) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$,which implies that $(\mu_A \circ f)((xy, q)) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$.Let x and y in R . Then we have, $(\nu_A \circ f)((x+y, q)) = \nu_A((f(y) + f(x)), q) \leq (\nu_A(f(x), q) \vee \nu_A(f(y), q) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$. which implies that $(\nu_A \circ f)((x+y, q)) \leq ((\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q))$. And, $(\nu_A \circ f)((xy, q)) = \nu_A((f(y) f(x), q) \leq (\nu_A(f(x), q) \vee \nu_A(f(y), q) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$,which implies that $(\nu_A \circ f)((xy, q)) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$. Therefore A∘f is an intuitionistic (Q,L))-fuzzy subhemiring of hemiring R.

REFERENCE

1. Akram.M and K.H.Dar On anti fuzzy left h-ideals in hemi rings, International Mathematical Forum, 2(46); 2295-2304, 2007.

2. Anitha.N and Arjunan.K, Homomorphism in Intuitionistic fuzzy subhemirings of a hemi ring, International Journal of Mathematical Science and Applications (IJMSEA),'Vol.4 (V); 165-172, 2010.

3. Anthony.J.M and H.Sherwood, fuzzy groups Redefined, Journal of mathematical analysis and applications, 69; 124-130, 1979.

4. Asok kumer Ray, on product of fuzzy subgroups, fuzzy sets and systems, 105; 181-183, 1999.

5. Biswas.R, Fuzzy subgroups and anti-fuzzy subgroups, fuzzy sets and systems, 35; 121-124, 1990.

6. Osman kazanci, sultan yamark and serife yilmaz, 2007.On intuitionistic Q-fuzzy R-subgroups of near rings, International mathematical forum, 2(59):2899-2910.

7. Palaniappan.N&K.Arjunan, The homomorphism, anti homomorphism of a fuzzy and an anti-fuzzy ideals of a ring, Varahmihir Journal of Mathematical Sciences,6(1);181-006,2008.

8. Palaniappan.N &K.Arjunan Operation on fuzzy and anti fuzzy ideals, Antartical J.Math, 4(1); 59-64, 2007.

9. Palaniappan.N &K.Arjunan, Some properties of intuitionistic fuzzy subgroups, Acta Ciencia Indica, VolXXXIIII (2); 321-328, 2007.

10. Rajesh Kumar, fuzzy Algebra, University of Delhi Publication Division,Volume 1, 1993. 11.Solairaju .A and R.Nagarajan, 2008.Q-fuzzy left R-subgroups of near rings w.r.t T-norms, Antarctica journal ofmathematics, 5:1-2.

12. Solairaju .A and R.Nagarajan, 2009.A new structure and construction of Q-fuzzy groups, Advances in fuzzy mathematics, Volume4 (1):23-29.

13. Vasantha Kandasamy.W.B., Smarandache fuzzy algebra, American research press, Rehoboth, 2003.

14. Xueling MA.Jianming ZHAN, On fuzzy h-ideals of hemi rings, journal of Systems science & Complexity, 20; 470-478, 2007.

15. Zadeh.L.A, Fuzzy sets Information and control, 8; 338-353, 1965.