TRIANGULAR SWITCH VERTEX GRAPH ON GRAPH LABELLING

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Abstract

A node labelling of a graph G_1 is a mapping g : $V_1(G_1) \rightarrow$ {-1,1} with persuaded edge labelling g* : $E_1(G_1) \rightarrow$ {-1,1} delineated by g*(ab) = g(a)g(b) is named a SPCL if $|v_g(-1) - v_g(1)| \leq 1$ and $|e_{g*}(-1) - e_{g*}(1)| \leq 1$, $v_g(-1)$ and $v_g(1)$ are the count of nodes labelled with -1 and 1 and $e_{g*}(-1)$ and $e_{g*}(1)$ are the count of edges labelled with -1 and 1. Here we concentrate on the signed product cordial labelling for the TSV graph and discuss Eulerian and Hamiltonian paths in TSV. Also, we introduced the concept of prime composite cordial labelling. The researcher concludes TSV graph is Prime Composite cordial labelling.

Keywords :Triangular switch vertex graph, Signed product cordial labelling and Prime Composite cordial labelling.

I. INTRODUCTION

A node labelling of a graph G_1 is a mapping g from the set of nodes of G_1 to a set of elements, often integers. Each edge ab has a label that depends on the nodes a and b and their labels g(a) and g(b). A.Rosa has proposed the postulation of graph labelling in (1967). I. Cahit (1987)[1] established the perception of cordial labelling. M.Sundaram, R.Ponraj [2]and S. Somasundaram have established the perception of PCL and TPCL in (2004)[3] and (2006)[4]. J.Baskar Babujee and L.Shobana [5] launched the perception of SPCL in (2011). The concept of TSPCL was proposed by M.Santhi[6] and A.James Albert (2015). discussed the Eulerian graph, Hamiltonian graph and proper colouring.

Here, the researcher defined and introduced the perception of Prime Composite cordial labeling (PCCL) [7]and also, we proved the Triangular Switch Vertex (TSV) graph is a Prime composite cordial graph.

II. PRELIMINARIES

2.1. Graph Labelling

A graph labelling is an assignment of integers to the nodes or edges or both (edge and node) subject to specific conditions. If the mapping domain is the set of nodes (or edges), then the labelling is called node(or edge) labelling.

2.2. Cordial Labelling

Cordial labelling of a graph G_1 with node-set V_1 is a mapping g from the nodes of G_1 to {0,1} in order that if each edge ab is assigned the label $|g(a) - g(b)|$, the count of nodes labelled '0' and the count of nodes labelled '1' vary almost by one and also the count of edges labelled '0' and the count of edges labelled '1' vary at most by one.

2.3. Product Cordial Labelling

A PCL of a graph G_1 with node-set V_1 is a mapping g from $V_1(G_1)$ to {0,1} in order that if each edge ab is assigned the label g(a)g(b), the count of nodes labelled '0' and the count of nodes labelled '1' vary at most by one, and the count of edges labelled '0' and the count of edges labelled '1' vary at most by one. A graph that admits PCL is named cordial product graph.

2.4. Node Product Cordial Labelling

A binary node labelling of a graph G_1 with persuaded edge labelling g*: $E_1(G_1) \rightarrow$ {0,1}

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delineated by g*(e=ab) = g(a)g(b) is called a NPCL if

$|v_g(1) - v_g(0)| \leq 1$ and $|e_{g*}(1) - e_{g*}(0)| \leq 1$, $v_g(1)$ and $v_g(0)$ are the count of nodes labelled with 1 and 0 and $e_{g*}(1)$ and $e_{g*}(0)$ are the count of edges labelled with 1 and 0.

2.5. Edge Product Cordial Labelling

For a graph G_1, the edge labelling function is delineated as g : $E_1(G_1) \rightarrow \{0,1\}$ and persuaded node labelling function g* : $V_1(G_1) \rightarrow \{0,1\}$ is given as if $e_1, e_2, \ldots, e_k$ are all the edges incident to the node V_1 then g*(v) = g(e_1)g(e_2)g(e_3)…g(e_k). Let $v_g(i)$ be the count of the nodes of G_1 having label i under g* and $e_g(i)$ be the count of edges of G_1 having label i under g for i = 0,1. g is called an EPCL of graph G_1 if

$|v_g(0) - v_g(1)| \leq 1$ and $|e_g(0) - e_g(1)| \leq 1$. A graph is named an edge product cordial if it admits an edge product cordial labelling.

2.6. Total Product Cordial Labelling

A TPCL of a graph G_1 with node set V_1 is a mapping g from V_1 to {0,1} in order that if each edge ab is assigned the label g(a)g(b), the count of nodes and edges labelled with ‘0’ and the count of nodes and edges labelled with ‘1’ vary atmost by one.

2.7. Signed Product Cordial Labelling

A node labelling of a graph G_1 is a mapping g : $V_1(G_1) \rightarrow \{-1,1\}$ with persuaded edge labelling g* : $E_1(G_1) \rightarrow \{-1,1\}$ delineated by g*(ab)=g(a)g(b) is named a SPCL if $|v_g(-1) - v_g(1)| \leq 1$ and $|e_{g*}(-1) - e_{g*}(1)| \leq 1$, $v_g(-1)$ and $v_g(1)$ are the count of nodes labelled with ‘-1’ and ‘1’ and $e_{g*}(-1)$ and $e_{g*}(1)$ are the count of edges labelled with ‘-1’ and ‘1’.

2.8. Total Signed Product Cordial Labelling

Let g : $V_1(G_1) \rightarrow \{-1,1\}$ with persuaded edge labelling g* : $E_1(G_1) \rightarrow \{-1,1\}$ delineated by g*(ab) = g(a)g(b). Then g is named a TSPCL if

$|(v_g(-1) + e_{g*}(-1)) - (v_g(1) + e_{g*}(1))| \leq 1$. A graph with a TSPCL is named a total signed product cordial graph.

2.9. Graph Coloring

Graph colouring is an assignment of colours to the vertices of a graph G such that no two adjacent vertices have the same colour., no two vertices of an edge should be the same colour.

2.10. Chromatic Number

The minimum number of colours needed to colour a graph G_1 is named its chromatic number. It is denoted by X(G_1).

2.11. Prime-Composite Cordial Labelling

A node labelling of a graph G_1 is a mapping g : $V_1(G_1) \rightarrow \mathbb{N}$ with a persuaded edge labelling g* : E_1 (G_1) → {0,1} delineated by g*(e=a+b) = {1 $if\ e\ is\ prime$ 0 $if\ e\ is\ composite$ is named Prime-Composite Cordial Labelling (PCCL) if $|eg*(0) - eg*(1)| \leq 1$, $eg*(0)$ and $eg*(1)$ are the count of edges labelled with 0 and 1. A graph with prime-composite cordial labelling is called a prime-composite cordial graph.

Triangular Switch Vertex Graph

Consider a cycle graph $\mathcal{G}$ with 3 nodes. Let $(1 \leq i \leq n)$ be the same graph of $\mathcal{G}$. Connect all the apex nodes by the edges as follows $e1 = uu1$, $e2 = u1u2$, $e3 = u2u3, \ldots, ei = ui{-}1ui (1 \leq i \leq n)$, where u and ui are the apex nodes of $\mathcal{G}$ and $\mathcal{G}i$ respectively. The resulting graph is called the Triangular Switch Vertex (TSV) Graph and G. indicates it The node set of G is $V_T = \{u, u_1, u_2, \ldots, u_i\} \cup V'$ (V′ is the set of all remaining nodes in G other than the apex nodes.) The edge set of G is

$E_T = \{e_1, e_2, e_3, \ldots, e_i\} \cup E'$ (E′ is the set of all remaining edges other than $e_i's$.).

Properties of TSV Graph

- A Triangular Switch Vertex Graph is a connected graph.

- A Triangular Switch Vertex Graph is a simple graph and acyclic graph.
- TSV graph is a Semi-Eulerian graph.
- TSV graph is not a Hamiltonian graph.
- TSV graph is a finite graph and not a null graph.
- TSV graph is not a regular graph.

  TSV graph is not a complete graph.
- TSV graph is 3-chromatic.
- A Triangular Switch Vertex Graph admits signed cordial product labelling.
- A Triangular Switch Vertex Graph admits total signed product cordial labelling.
- A Triangular Switch Vertex Graph admits prime-composite cordial labelling

III. MAIN RESULTS

Theorem 3.1

Triangular Switch Vertex Graph (TSV Graph) admits SPCL.

Proof

Allow G be a Triangular Switch vertex Graph with 3n+3 count of nodes and 4n+3 count of edges.

The node set of G is V_T= $\{u, u_1, u_2, \dots, u_i\} \cup V'$ and the edge set of G is E_T=$\{e_1, e_2, e_3, \dots, e_i\} \cup E'$, $1 \le i \le n$.

Define the node labelling g : V_T(G) → {-1,1} with persuaded edge labelling

g* : E_T(G) → {-1,1} delineated by g*(ab)=g(a)g(b) then the ensuing two different occurrences are to be observed.

Case-(i)

while n is odd

$v_g(1) = n + \mathbb{N},\ \mathbb{N} \ge 2$

$v_g(-1) = n + \mathbb{N},\ \mathbb{N} \ge 2$

and $e_{g^*}(1) = 2n + 1$

$e_{g^*}(-1) = 2n + 2$

Hence, we have

$|v_g(-1) - v_g(1)| \le 1$ *and* $|e_{g^*}(-1) - e_{g^*}(1)| \le 1$.

Case-(ii)

while n is even

$v_g(1) = n + \mathbb{N},\ \mathbb{N} \ge 3$

$v_g(-1) = n + \mathbb{N},\ \mathbb{N} \ge 2$

and $e_{g^*}(1) = 2n + 1$, $e_{g^*}(-1) = 2n + 2$

Hence, we have

$|v_g(-1) - v_g(1)| \le 1$ *and* $|e_{g^*}(-1) - e_{g^*}(1)| \le 1$,

where $v_g(-1)$ is the count of nodes labelled

with '-1', $v_g(1)$ is the count of nodes labelled with '1',

$e_g(-1)$ is the count of edges labelled with '-1', and $e_g(1)$ is the count of edges labelled with '1'.

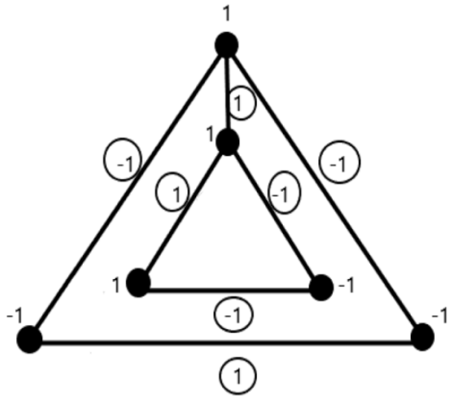
By the above cases, we have

$|v_g(-1) - v_g(1)| \le 1$ *and* $|e_{g^*}(-1) - e_{g^*}(1)| \le 1$.

Therefore,

Triangular Switch Vertex Graph admits SPCL.

Example 3.2 Triangular Switch Vertex with one iteration



Theorem – 3.3

TSV graph admits TSPCL.

Proof

Allow G be a TSV graph with a 3n+3 count of nodes and a 4n+3 count of edges.

The node-set of G is V_T

$= \{u, u_1, u_2, \dots, u_i\} \cup V'$

and the edge set of G is

$E_T = \{e_1, e_2, e_3, \dots, e_i\} \cup$

E', $1 \leq i \leq n$.

Define the node labelling g :

V_T(G) → {1,-1} with

persuaded edge labelling g* :

E_T(G) → {1,-1} delineated

by g*(ab)=g(a)g(b)

then the ensuing two different occurrences are to be observed.

Case-(i)

While n is odd

$v_g(-1) = n + \mathbb{N},\ \mathbb{N} \geq 2$

$v_g(1) = n + \mathbb{N},\ \mathbb{N} \geq 2$ and

$e_{g^*}(-1) = 2n + 2$

$e_{g^*}(1) = 2n + 1$

Hence, we have

$|(v_g(-1) + e_{g^*}(-1)) - (v_g(1) + e_{g^*}(1))| \leq 1.$

Case-(ii)

While n is even

$v_g(-1) = n + \mathbb{N},\ \mathbb{N} \geq 2$

$v_g(1) = n + \mathbb{N},\ \mathbb{N} \geq 3$ and

$e_{g^*}(-1) = 2n + 2$

$e_{g^*}(1) = 2n + 1.$

Hence, we have

$|(v_g(-1) + e_{g^*}(-1)) - (v_g(1) + e_{g^*}(1))| \leq 1,$

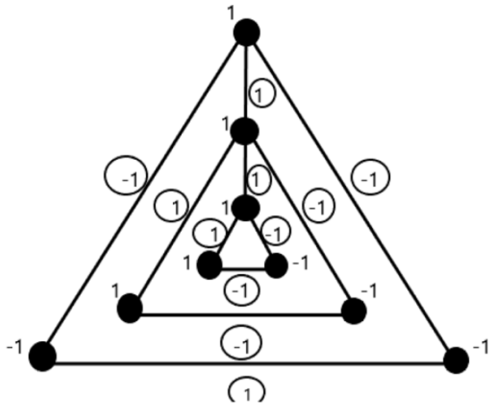
where $v_g(-1)$ is the count of nodes labelled with '-1', $v_g(1)$ is the count of nodes labelled with '1', $e_g(-1)$ is the count of edges labelled with '-1' and $e_g(1)$ is the count of edges labelled with '1' respectively.

By the above cases, we have

$|(v_g(-1) + e_{g^*}(-1)) - (v_g(1) + e_{g^*}(1))| \leq 1.$

Therefore, the TSV graph admits TSPCL. Hence TSV graph is a total signed cordial product graph.

Example 3.4 Triangular Switch Vertex with two iteration



Theorem – 3.5

TSV graph admits prime-composite cordial labelling.

Proof

Allow G be a TSV graph with 3n+3 count of nodes and 4n+3 count of edges.

The node-set of G is

V_T_T= $\{u, u1, u2, \dots, ui\} \cup V_T'$ and the edge set of G is E_T= $\{e1, e2, \dots, ei\}$ ∪ E′, $1 \le i \le n$

Define the node labelling g : V_T(G) → ℕ with persuaded edge labelling g* : E_T(G) → {0,1} defined by

$$g^*(e=a+b) = \begin{cases} 1 & if\ e\ is\ prime \\ 0 & if\ e\ is\ composite \end{cases}$$

The ensuing two occurrences are to be observed.

Case-(i)

While n is odd

$e_{g*}(1) = 2n + 2$ *and*
$e_{g*}(0) = 2n + 1.$

Hence, we have

$|e_{g*}(1) - e_{g*}(0)| \le 1.$

Case-(ii)

While n is even

$e_{g*}(1) = 2n + 2$ *and*
$e_{g*}(0) = 2n + 1.$

Hence, we have

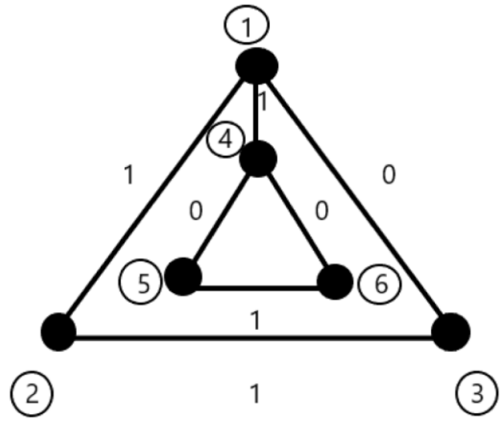
$|e_{g*}(1) - e_{g*}(0)| \le 1,$

where $e_{g*}(1)$ and $e_{g*}(0)$ are the counts of edges labelled with 1 and 0.

Therefore, the TSV graph admits prime-composite cordial labelling.

Hence TSV graph is a prime-composite cordial graph.

Example 3.6



Theorem – 3.7

Triangular Switch Vertex Graph (TSV Graph) is 3-chromatic.

Proof

Allow G to be a triangular switch vertex graph with 3n+3 nodes and 4n+3 edges.

Every node in G is coloured correctly with 3 different colours, say c_1, c_2 *and* c_3, respectively.

By the definition of the Triangular Switch Vertex graph, We know that TVS graph start with cycle graph with 3 nodes.

uppose that the apex node u of $\mathcal{G}$ coloured with c_1 then the other two nodes are colored with c_2 *and* c_3 (*or*) c_3 *and* c_2 respectively.

Since each node in $\mathcal{G}$ is adjacent to every other node.

The apex node u_1 *of* $\mathcal{G}_1$ is adjacent to u. Therefore u_1 is coloured with c_2 then the other two nodes in $\mathcal{G}_1$ are coloured with c_1 *and* c_3 (*or*) c_3 *and* c_1.

Now the apex node u_2 *of* $\mathcal{G}_2$ is adjacent to u_1. Therefore u_2 is colored with c_3. The other two nodes are coloured with c_1 *and* c_2 (*or*) c_2 *and* c_1.

By continuing this process,

Every node in G

can be coloured

appropriately

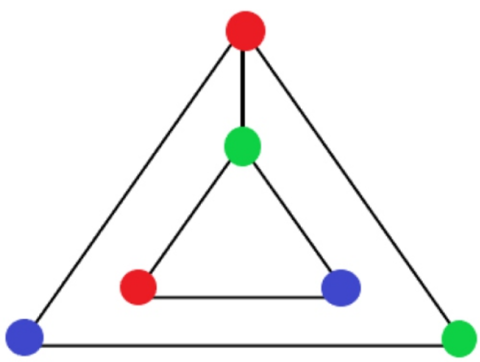
with 3 different

colours.

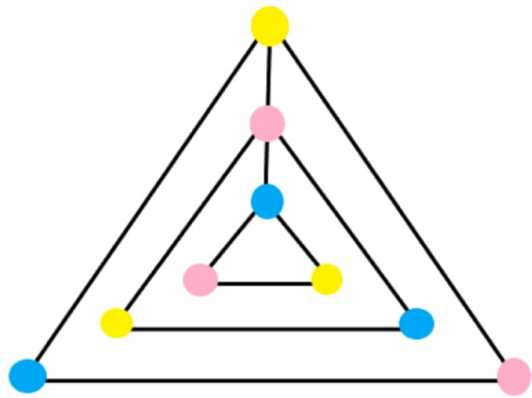
Therefore, G is

3-chromatic.

Example



Graph -1



Graph - 2

Theorem – 3.8

A Triangular Switch Vertex Graph (TVS Graph) is a Semi-Eulerian graph.

Proof

Allow G to be a triangular switch vertex graph with 3n+3 nodes and 4n+3 edges.

We have to prove that G is a Semi-Eulerian graph.

It is enough to show that G contains exactly 2 nodes of degree odd.

We know that, The degree of the apex node u of $\mathcal{G}$ is odd. Also, the degree of apex node

u_i *of* $\mathcal{G}_i$ $(1 \leq i \leq n)$ is 3.

Hence G has precisely two nodes of degree 3. Hence G contains exactly 2 nodes of degree odd.

Therefore, G is a Semi-Eulerian graph.

Remark

- TSV graph is not an Eulerian graph since it contains only Eulerian path, not Eulerian circuit.

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