COMPOSITION GRAPHS OF INTUITIONISTIC FUZZY IDEAL OF M*Γ* GROUP IN NEAR RINGS

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Abstract: From the graphs of Intuitionistic fuzzy ideal (GIFI) of M*Γ*group in near rings (NR), its composition graphs are found. Also, some examples, important theorems with their proofs and properties are explained in detail.

Keywords: Intuitionistic fuzzy (IF) ideal of M*Γ*group in near rings, Graph of IF ideal of M*Γ*group in near rings, Composition graph of IF ideal.

I. INTRODUCTION

The fuzzy subset in non-empty set was first presented by Zadeh [11] in 1965. After that, many generalisations Intuitionistic fuzzy set was developed by Atanassov [1, 2] and he initiated IF set as one of its extensions.

Subsequently, Jun and Lee [4] studied fuzzy *Γ* rings. Zhan and Davvaz[12] then explained fuzzy ideals of near rings with many theorems. . Bhavanari and Kuncham [3] explained fuzzy cosets of *Γ* near rings as an extension. Parvathi *et al.* [9] defined the various graph operations in IF graphs for the first time. Mala and Shanmugapriya [5, 6] defined IF ideal of M*Γ*group in near rings and extended it as GIFI of M*Γ* groups in NR explaining their properties.

Now in this paper we study composition of IF ideal graphs of M*Γ* group in near rings.

II. PRELIMINARIES

Definition 2.1

An additive group Gin *M*, is a ***Γ*-near ring module**, if there is a mapping between
$M \times \Gamma \times G \to G$ given by

(i) $(u_1 + u_2)\alpha_1 g = u_1\alpha_1 g + u_2\alpha_1 g$ and

(ii) $(u_1\alpha_1 u_2)\alpha_2 g = u_1\alpha_1 (u_2\alpha_2 g)$

for all u_1, u_2 ∈M, α_1, α_2∈*Γ*, and g∈G an M*Γ*-module.

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Definition 2.2

A membership function ʮ$_F$: M → [0, 1] of non-empty fuzzy set F in M, is defined by ʮ$_F$(x).

Definition 2.3

The level set of ʮ is given as set U(ʮ, ƫ) = {ϰ∈ M | ʮ(ϰ) ≥ƫ} for a fuzzy set ʮ in a ring M and ƫ∈ [0, 1].

Definition 2.4

A fuzzy set ʮ in a G - ring M is called a fuzzy left ideal of M, if it conforms to:

(i) ʮ(u - v) ≥ ʮ(u) ∧ʮ(v),

(ii) ʮ(u α v) ≥ ʮ(v)

for all u, v∈ M and α ∈ Γ

Definition 2.5

Let an IF set A be in nonempty fixed set Ꭓ with A = {[u, ʮ$_A$(u),Һ$_A$(u)] /u ∈Ꭓ}, with ʮ$_A$: Ꭓ → [0, 1] and Һ$_A$: Ꭓ → [0, 1] give the degree of membership and non-membership of u∈Ꭓ and 0 = ʮ (u) = 1 for every u∈Ꭓ. Then, A = {[u, ʮ$_A$(u),Һ$_A$(u)] /u∈Ꭓ}.

Definition 2.6

I is an IFI of G in NR and ʮ$_I$: G→ [0,1] and Һ$_I$: G→ [0,1] defined on I which satisfies the following conditions.

i. ʮ$_I$ (u + v) ≥ min {ʮ$_I$ (u), ʮ$_I$ (v)}
ii. ʮ$_I$ (u + v – u) ≥ ʮ$_I$ (v)
iii. ʮ$_I$ (u) = ʮ$_I$ (-u)
iv. ʮ$_I$ (nα (a + u) – n α a) ≥ ʮ$_I$ (u)
v. Һ$_I$ (u + v) ≤ max {Һ$_I$ (u), Һ$_I$ (v)}
vi. Һ$_I$ (u + v – u) ≤ Һ$_I$ (v)
vii. Һ$_I$ (u) = Һ$_I$ (- u)
viii. Һ$_I$ (n α (a + u) – n α a) ≤ Һ$_I$ (u) every n ϵ N, α ϵ Γ, a, u, v ϵ I.

Definition 2.7

Let $G = G_1 \circ G_2 = (V_1 \times V_2, E)$ be the composition of two graphs G_1 and G_2, where $E = \{(l,l_2)(l,k_2): l \in V_1, l_2k_2 \in E_2\} \cup \{(l_1,w)(k_1,w): w \in V_2, l_1k_1 \in E_1\} \cup \{(l_1,l_2)(k_1,k_2): l_1k_1 \in E_1, l_2 \neq k_2\}$. Then, the composition of IFGs G_1 and G_2, denoted by $G = G_1 \circ G_2$ is an IFG defined by

(i) $(\mu_1 \circ \mu_1')(l_1,l_2) = \min(\mu_1(l_1), \mu_1'(l_2))$ for every $(l_1, l_2) \in V_1 \times V_2$ and

$(\nu_1 \circ \nu_1')(l_1,l_2) = \max(\nu_1(l_1), \nu_1'(l_2))$ for every $(l_1, l_2) \in V_1 \times V_2$.

(ii) $(\mu_2 \circ \mu_2')(l,l_2)(l,k_2) = \min(\mu_1(l), \mu_2(l_2k_2))$ for every $l \in V_1$, and $l_2k_2 \in E_2$

$(\nu_2 \circ \nu_2')(l,l_2)(l,k_2) = \max(\nu_1(l), \nu_2(l_2k_2))$ for every $l \in V_1$, and $l_2k_2 \in E_2$

$(\mu_2 \circ \mu_2')(l_1,w)(k_1,w) = \min(\mu_1(w), \mu_2(l_1k_1))$ for every $w \in V_2$, and $l_1,k_1 \in E_1$

$(\nu_2 \circ \nu_2')(l_1,w)(k_1,w) = \max(\nu_1(w), \nu_2(l_1k_1))$ for every $w \in V_2$, and $l_1k_1 \in E_1$

$(\mu_2 \circ \mu_2')(l_1, l_2)(k_1,k_2) = \min(\mu_1'(l_2), \mu_1'(k_2), \mu_2(l_1,k_1))$ for every $(l_1,l_2)(k_1,k_2) \in E - E''$

$(\nu_2 \circ \nu_2')(l_1,l_2)(k_1, k2) = \max(\nu_1'(l_2), \nu_1'(k_2), \nu_2(l_1,k_1))$ for every $(l_1,l_2)(k_1,k_2) \in E - E''$

where $E'' = \{(l, l_2)(l,k_2) : l \in V_1$, for every $l_2k_2 \in E_2\} \cup \{(l_1, w)(k_1, w) : w \in V_2$, for every $l_1k_1 \in E_1\}$.

III. THEROEMS ON COMPOSITION GIFI OFMΓ IN NR

Definition 3.1

Let $G_1(VI_1, EI_1, \mu I_1, \gamma I_1)$ and $G_2(VI_2, EI_2, \mu I_2, \gamma I_2)$ be two graphs of IF ideal I_1 and I_2 of MΓ group in near ring then, $G_1 \circ G_2 = (VI, EI, \mu I, \gamma I)$ is called composition graph of IF ideal structure.

Here, $VI = VI_1 \times VI_2$ with $\mu I(\eta,\beta) = \mu I_1(\eta) \vee \mu I_2(\beta)$ for all $(\eta,\beta) \in VI = VI_1 \times VI_2$ and

$\gamma I(\eta,\beta) = \gamma I_1(\eta) \wedge \gamma I_2(\beta)$ for all $(\eta,\beta) \in VI$.

$EI = \{((\eta,\beta)(\eta',\beta')) / \eta=\eta'$ and $\beta,\beta' \in EI_2$ (or) $\beta = \beta'$ and $\eta, \eta' \in EI_1\}$

Also, $\mu I((\eta,\beta)(\eta',\beta')) = \{ \mu I_1(\eta) \vee \mu I_2(\beta\beta')$ where $\eta = \eta'$ & $\beta\beta' \in EI_2$

$\mu I_2(\beta') \vee \mu I_1(\eta\eta')$ where $\beta=\beta'$ & $\eta\eta' \in EI_1$ and

$\mu I_1(\eta) \vee \mu I_2(\beta') \vee \mu I_1(\eta\eta')$ where $\beta=\beta'$ & $\eta\eta' \in E - EI_1$

$\gamma I((\eta,\beta)(\eta',\beta')) = \{ \gamma I_1(\eta) \wedge \gamma I_2(\beta\beta')$ where $\eta = \eta'$ & $\beta\beta' \in EI_2$

$\gamma I_2(\beta') \wedge \gamma I_1(\eta\eta')$ where $\beta=\beta'$ & $\eta\eta' \in EI_1$ and

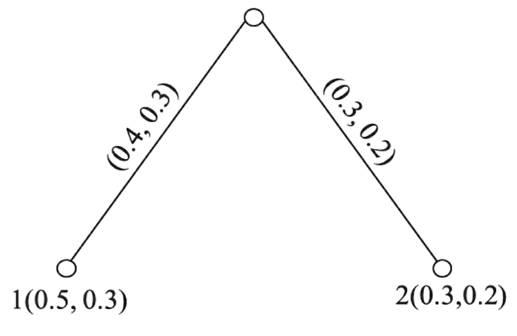
$\gamma I_1(\eta) \wedge \gamma I_2(\beta') \wedge \gamma I_1(\eta\eta')$ where $\beta=\beta'$ & $\eta\eta' \in E - EI_1$

Here EI (edges set) has edges either if the first coordinate or the second coordinate is same with an already existing edge in G1 or G2 or there is no edge in G_1 or G_2.

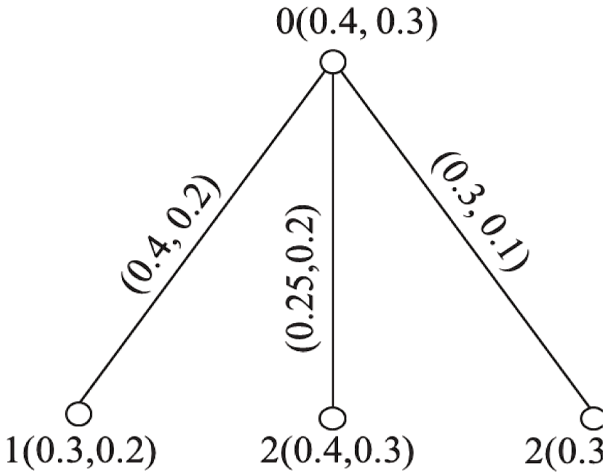
Example 3.2

Consider graph of IF ideal$I_1 = \{0\}$ and $I_2 = \{0,1,2\}$ of MΓgroup in near rings belongs to Z_3 as G_1 and G_2 then, we get $G_1 \circ G_2$ as a maximal intuitionistic graph as to follows: $G_1 \circ G_2$ has vertex set $V_I = V_{I_1} \times V_{I_2} = \{0,1,2\} \times \{0,1,2\} = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$ and Edges set E_I has edges with either first co-ordinate same or second co-ordinate same in V_I along with the remaining coordinate either exists or not as an edge in their corresponding graphs. Thus, $G_1 \circ G_2$ is represented as follows:

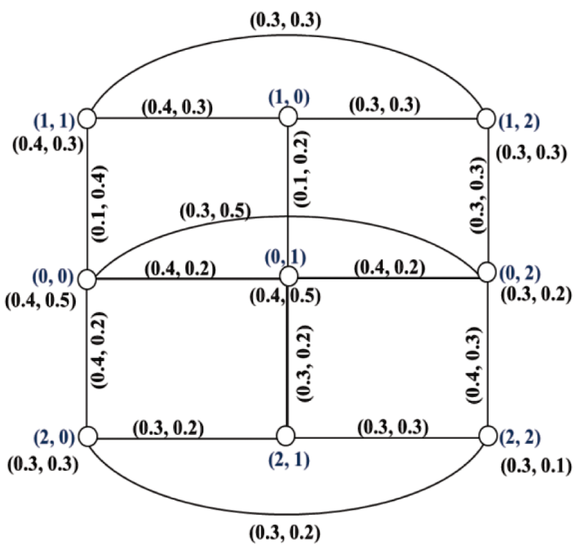
Graph G₁:



Graph G_2 :



Graph $G_1 o\, G_2$:



Definition 3.3

A GIFI of MΓ group in NR G_I (V_I, E_I, μ_I, γ_I) is μ_I – γ_I strong if μ_I ((η,β) (η',β')) = min {μ_I(η,β) , μ_I (η'β')},γ_I ((η,β) (η',β')) = max {γ_I (η,β) , γ_I (η'β')} where, (η,β), (η',β') ϵ V_I

and it is same for all edges in G_I then, G_I is called μ_I – γ_I strong graph of IF ideal.

Theorem 3.4

Composition of two strong GIFI is also a strong graph of IF ideal of MΓ group in NR.

Proof:

Let $G_1(V_{I_1}, E_{I_1}, \mu_{I_1}, \gamma_{I_1})$ and $G_2(V_{I_2}, E_{I_2}, \mu_{I_2}, \gamma_{I_2})$ be two strong graphs of IF ideal of MΓ group in near rings. Then their edges exist by the definition of composition graphs and have its membership and non-membership values as follows:

μ_I((η,β) (η',β')) = { μ_{I_1} (η) ∨ μ_{I_2} (ββ') where η = η'&ββ' ϵ E_{I_2}

μ_{I_2} (β') ∨ μ_{I_1} (ηη') where β=β'&ηη' ϵ E_{I_1}

μ_{I_1}(η, η')∨μ_{I_2}(β, β') : ηβ∈E_I, η'≠β'}

and γ_I ((η,β) (η',β')) = { γ_{I_1} (η) ∧ γ_{I_2} (ββ') where η = η'&ββ' ϵ E_{I_2}

γ_{I_2} (β') ∧ γ_{I_1} (ηη') where β=β'&ηη' ϵ E_{I_1}

γ_{I_1}(η, η')∨γ_{I_2}(β, β') : ηβ∈E_I, η'≠β}

Let $G_1(V_{I_1}, E_{I_1}, \mu_{I_1}, \gamma_{I_1})$ and $G_2(V_{I_2}, E_{I_2}, \mu_{I_2}, \gamma_{I_2})$ be two graphs of the ideals I_1 and I_2 and are strong graphs of IF ideal then,

$G_1 o\, G_2$ = (V_I, E_I, μ_I, γ_I) is the composition of two strong IF ideal graphs with the structure $V_I = V_{I_1}$ X V_{I_2} and

E_I = {(η, β) (η',β') / η = η', ββ' ϵ E_{I_2} or β = β', ηη' ϵ E_{I_1}}

with μ_I (η,β) = μ_{I_1} (η) ∨ μ_{I_2} (β) for all (η, β) ϵ $V_I = V_{I_1}$ X V_{I_2}

γ_I (η, β) = γ_{I_1} (η) ∧ γ_{I_2} (β) for all (η, β) ϵ $V_I = V_{I_1}$ X V_{I_2}

Then, by the definition of composition graph we have,

$\mu_I((\eta,\beta)(\eta'\beta')) = \{\mu_{I_1}(\eta) \vee \mu_{I_2}(\beta\beta')$

$\mu_{I_1}(\eta) \vee \mu_{I_2}(\beta) \vee \mu_{I_2}(\beta')$ where $\eta = \eta'$ & $\beta\beta' \epsilon E_{I_2}$

$= \mu_{I_2}(\beta') \vee \mu_{I_1}(\eta\eta')$

$\mu_{I_2}(\beta') \vee \mu_{I_1}(\eta) \vee \mu_{I_1}(\eta)$ where $\beta=\beta'$ & $\eta\eta' \epsilon E_{I_1}$

$= \mu_{I_1}(\eta, \eta') \vee \mu_{I_2}(\beta, \beta')$

$\mu_{I_1}(\eta) \vee \mu_{I_1}(\eta') \vee \mu_{I_2}(\beta) \vee \mu_{I_2}(\beta')$: $\eta\beta \in E_I$, $\eta' \neq \beta'$

and $\gamma_I((\eta,\beta)(\eta',\beta')) = \{\gamma_{I_1}(\eta) \wedge \gamma_{I_2}(\beta\beta')$

$\gamma_{I_1}(\eta) \wedge \gamma_{I_2}(\beta) \wedge \gamma_{I_2}(\beta')$ where $\eta = \eta'$ & $\beta\beta' \epsilon E_{I_2}$

$= \gamma_{I_2}(\beta') \wedge \gamma_{I_1}(\eta\eta')$

$\gamma_{I_2}(\beta') \wedge \gamma_{I_1}(\eta) \wedge \gamma_{I_1}(\eta')$ where $\beta=\beta'$ & $\eta\eta' \epsilon E_{I_1}$

$= \gamma_{I_1}(\eta, \eta') \vee \gamma_{I_2}(\beta, \beta')$

$\gamma_{I_1}(\eta) \vee \gamma_{I_1}(\eta') \vee \gamma_{I_2}(\beta) \vee \gamma_{I_2}(\beta')$: $\eta\beta \in E_I$, $\eta' \neq \beta'$

Hence, it satisfies the definition of strong IF ideal graphs as G_1 and G_2 are strong IF ideal graphs.

Result 3.5

The composition graph is a strong graph of an IF ideal and it is independent of their products.

The composition graph $G = G_1 \circ G_2$ may be a strong IF ideal graph when G_1 and G_2 are not strong IF ideal graphs.

Case (i):

$\eta = \eta'$ and $\beta\beta' \epsilon E_{I2}$. Then

$\mu_I((\eta,\beta)(\eta',\beta')) = \mu_{I_1}(\eta) \vee \mu_{I_2}(\beta\beta')$

$= \mu_{I_1}(\eta) \vee [\mu_{I_2}(\beta) \wedge \mu_{I_2}(\beta')]$

$= [\mu_{I_1}(\eta) \vee \mu_{I_2}(\beta)] \wedge [\mu_{I_1}(\eta') \vee \mu_{I_2}(\beta')]$

$= \mu_I(\eta\beta) \wedge \mu_I(\eta'\beta')$

Case (ii):

$\beta = \beta'$ and $\eta\eta' \epsilon E_{I_1}$. Then,

$\mu_I((\eta,\beta)(\eta',\beta')) = \mu_{I_2}(\beta') \vee \mu_{I_1}(\eta\eta')$

$= \mu_{I_2}(\beta') \vee [\mu_{I_1}(\eta) \wedge \mu_{I_1}(\eta')]$

$= [\mu_{I_2}(\beta') \vee \mu_{I_1}(\eta)] \wedge [\mu_{I_2}(\beta') \vee \mu_{I_1}(\eta')]$

$= \mu_I(\eta\beta) \wedge \mu_I(\eta'\beta')$

Case (iii):

$\mu_I((\eta,\beta)(\eta',\beta')) = \mu_{I_1}(\eta, \eta') \vee \mu_{I_2}(\beta, \beta')$: $\eta\beta \in E_I$, $\eta' \neq \beta'$

$= \mu_{I_1}(\eta) \vee \mu_{I_1}(\eta') \vee \mu_{I_2}(\beta) \vee \mu_{I_2}(\beta')$: $\eta\beta \in E_I$, $\eta' \neq \beta'$

for all the edges of the composition graph.

Similarly, we find non-membership values for the edges as follows:

Case (i):

$\eta = \eta'$ and $\beta\beta' \epsilon E_{I_2}$. Then,

$\gamma_I((\eta,\beta)(\eta',\beta')) = \gamma_{I_1}(\eta) \wedge \gamma_{I_2}(\beta\beta')$

$= \gamma_{I_1}(\eta) \wedge [\gamma_{I_2}(\beta) \vee \gamma_{I_2}(\beta')]$

$= [\gamma_{I_1}(\eta) \wedge \gamma_{I_2}(\beta)] \vee [\gamma_{I_1}(\eta') \wedge \gamma_{I_2}(\beta')]$

$= \gamma_I(\eta\beta) \vee \gamma_I(\eta'\beta')$

Case (ii):

$\beta = \beta'$ and $\eta\eta' \epsilon E_{I_1}$. Then

$\gamma_I((\eta,\beta)(\eta',\beta')) = \gamma_{I_2}(\beta') \wedge \gamma_{I_1}(\eta\eta')$

$= \gamma_{I_2}(\beta') \wedge [\gamma_{I_1}(\eta) \vee \gamma_{I_1}(\eta')]$

$= [\gamma_{I_2}(\beta') \wedge \gamma_{I_1}(\eta)] \vee [\gamma_{I_2}(\beta') \wedge \gamma_{I_1}(\eta')]$

$= \gamma_I(\eta\beta) \vee \gamma_I(\eta'\beta')$

Case (iii):

γ_I ((η,β) (η',β')) = γ_{I_1}(η, η')∨γ_{I_2}(β, β') : ηβ∈E_1, η'≠β'

=γ_{I_1}(η)∨γ_{I_1}(η')∨γ_{I_2}(β)∨γ_{I_2}(β') : ηβ∈E_1, η'≠β'

for all the edges of the composition graph.

Hence G = $G_1 \circ G_2$ is a strong IF ideal graph when G_1 and G_2 are not strong IF ideal graphs.

IV. CONCLUSION

We have considered GIFI of MΓ groups in NR and defined the composition of these graphs and proved few important theorems on composition in it with an example.

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